APPH 4200 Physics of Plasmas: In-Class Worksheet

Answer the following without looking at your notes or textbooks.

Question

Find an expression for fluid flow around a cylinder. Take the fluid to be inviscid, incompressible, and two-dimensional. Then, with $\mathbf{U} = \{U_r, U_\theta, U_z\} = \{0, U_\phi(r), 0\}$, derive an expression for $U_\theta(r)$ having constant circulation.

You may use the velocity potential, $\mathbf{U} = \nabla \phi_v$, and the equations

$$\begin{aligned} \nabla \cdot \mathbf{U} &= 0 \\ \nabla \times \mathbf{U} &= 0 \end{aligned}$$

Define the fluid circulation as

$$\Gamma \equiv \oint \mathbf{U} \cdot d\mathbf{s}$$

wher the line integral surrounds the cylinder. What boundary condition did you assume as $r \to \infty$?

Recall the gradient and Laplacian of a scalar in cylindrical coordinates are

$$\begin{aligned} \nabla^2 \phi &= \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} \\ \nabla \phi &= \hat{\mathbf{r}}\frac{\partial \phi}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial \phi}{\partial \theta} + \hat{\mathbf{z}}\frac{\partial \phi}{\partial z} \end{aligned}$$

Answer

The steady invisid fluid flow is . . .