

APPH 4200 Physics of Plasmas: In-Class Worksheet

Answer the following without looking at your notes or textbooks. (This is one of the questions at the end of Chapter 3.)

Question

Consider steady axisymmetric flow of a *compressible* fluid. The equation of mass continuity is given in cylindrical coordinates (r, θ, z) as

$$\frac{\partial}{\partial r}(\rho r U_r) + \frac{\partial}{\partial z}(\rho r U_z) = 0$$

Question: define a streamfunction for the compressible fluid so that the equation of continuity is satisfied automatically.

You may use the definitions:

$$\begin{aligned}\nabla f &= \hat{r} \frac{\partial f}{\partial r} + \frac{\hat{\theta}}{r} \frac{\partial f}{\partial \theta} + \hat{z} \frac{\partial f}{\partial z} \\ \nabla \cdot (\mathbf{A} \times \mathbf{B}) &= \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}\end{aligned}$$

Answer

The proof of the identities is ...