Reflex-Klystron Oscillators*

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Summary—**A comprehensive analysis of reflex klystrons is developed by considering the electrons as particles acted upon by forces which modify their motion. The analysis is similar to earlier explanations of electron bunching in a field-free drift space and predicts a similar current distribution when bunching takes place in a reflecting field. The effect of the bunched electron beam is treated qualitatively by considering the effect of the beam admittance upon a simple equivalent circuit. A quantitative mathematical analysis based upon oscillator theory is also derived and the results are presented in a series of universal curves which are used to explain the operating characteristics of these tubes. Power output, efficiency, starting current, electronic tuning, and modulation properties are discussed. Some general remarks on reflex-oscillator design considerations are also included.**

Introduction

REFLEX-klystron oscillators are an important member of an extensive family of velocity-modulation tubes invented independently by R. H. Varian and W. W. Hansen at Stanford University, W. C. Hahn and G. F. Metcalf at Schenectady, and O. Heil in Germany. Velocity-modulation tubes are now quite generally known as klystrons, and perform the same functions at frequencies in the microwave region that triodes and pentodes do at lower frequencies. The mechanism of energy conversion is different, but analogies between klystrons and the electrical circuits used with conventional tubes are often useful. Klystrons with one or more resonators are used as oscillators, and multiresonator klystrons often replace conventional vacuum tubes for other applications.

A reflex klystron utilizes a single resonator, and obtains feedback by reflecting the electron beam so that it passes through the resonator a second time. This type of oscillator was described briefly by Hahn and Metcalf,[1] and has been discussed in greater detail in other papers.[2–4] The operation of these tubes can be explained by a ballistic or kinematic analysis; i.e., the electrons may be considered as particles which follow Newton's laws of motion. An understanding of the application of such a kinematic analysis to the principles of operation of the ordinary two-resonator klystron will be assumed. These principles have been presented in papers by Varian[5] and Webster,[6,7] and a similar analysis will be developed for the reflex-klystron oscillator.

The analysis has been subdivided into two parts. The first section derives the transit-time relationships for the reflex type of klystron from the laws of motion. Then these relations are expanded to explain electron bunching, and the similarity between reflection-field bunching and bunching in a field-free drift space is shown. A second section applies these relationships to a derivation of the efficiency, power output, and electronic tuning of a reflex-klystron oscillator. The dependence of these characteristics on the beam current, beam voltage, reflector voltage, load, and other klystron design factors will be shown.

Operating Principles of a Reflex Klystron

A simplified drawing of a reflex klystron is shown in Fig. 1. The tube is a figure of revolution about the axis AA. The cathode surface K provides a source of electrons when it is indirectly heated by F. The electrons are accelerated by the voltage E_0, which is known as the beam voltage, or as the acceleration voltage because it determines the velocity which the electrons have acquired when they reach the anode plane. The emission current is controlled by the voltage E_g which is applied to the grid G. The cylindrical portion of the control-grid structure acts as a focusing element and gives a collimated beam which continues along the axis of the tube past the anode plane. In many klystron designs, the grid is not used and this electrode is only a focusing

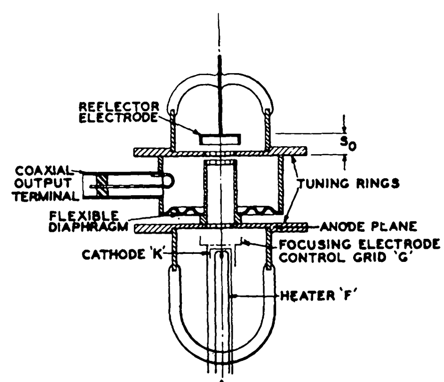

Fig. 1—Cross-section view of a reflex klystron.

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† Sperry Gyroscope Company, Inc., Garden City, Long Island, N. Y.

[1] W. C. Hahn and G. F. Metcalf, "Velocity-modulated tubes," Proc. I.R.E., vol. 27, pp. 106–117; February, 1939.

[2] A. E. Harrison, "Klystron Technical Manual," Sperry Gyroscope Company, Inc., Great Neck, Long Island, New York, 1944.

[3] A. E. Harrison, "Kinematics of reflection oscillators," *Jour. Appl. Phys.*, vol. 15, pp. 709–711; October, 1944.

[4] J. R. Pierce, "Reflex oscillators," Proc. I.R.E., vol. 33, pp. 112–118; February, 1945.

[5] R. H. Varian and S. F. Varian, "A high-frequency oscillator and amplifier," *Jour. Appl. Phys.*, vol. 10, pp. 321–327; May, 1939.

[6] D. L. Webster, "Cathode-ray bunching," *Jour. Appl. Phys.*, vol. 10, pp. 501–508; July, 1939.

[7] D. L. Webster, "The theory of klystron oscillations," *Jour. Appl. Phys.*, vol. 10, pp. 864–872; December, 1939.

element. Fig. 2 shows the reflex klystron connected to the proper power supplies. The standard diagram for velocity-modulation tubes has been used, and the operating voltages have been labeled with the designations which will be used throughout this discussion.

This electron-gun structure is quite similar to a triode tube; it has a cathode, a control grid, and an anode. In a klystron, however, the electron gun is merely the source of an electron beam and the radio-frequency portion of the klystron is independent of the electron source.

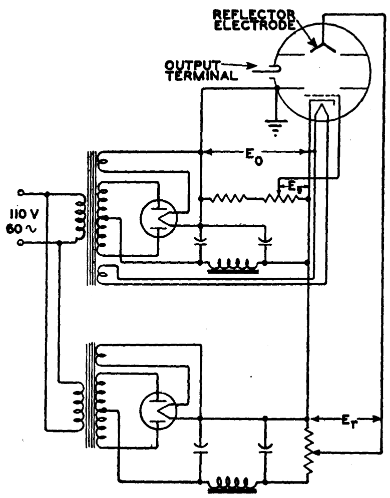

Fig. 2—Circuit diagram for a reflex oscillator and power supply.

The beam travels along the axis of the tube beyond the anode plane with a uniform velocity corresponding to E_0, the acceleration voltage, until it reaches the resonator gap. A radio-frequency voltage across the resonator gap will modify the velocity of the electrons in the beam. Some electrons will be speeded up when the field has a direction which will accelerate the beam. Other electrons will be slowed down during another part of the radio-frequency cycle, and the velocity of some electrons will not be changed because they pass the gap when the resonator voltage is zero. The velocity variation will be assumed to be small, and the average velocity of the electrons in the beam will be identical to the velocity corresponding to the acceleration voltage, since an equal number of electrons will be slowed down and speeded up during one radio-frequency cycle.

Beyond the resonator gap, the electrons encounter a retarding electric field produced by the potential between the reflector and the anode (E_0+E_r). This reflecting field brings the electrons to rest and returns them to the cavity resonator. The shape of the reflector electrode is designed to preserve the focus of the beam. The beam current is constant when the beam leaves the resonator gap, but electron bunching takes place while the electrons are in the reflection space, and the beam is density modulated when it returns to the cavity resonator.

If space-charge effects and the focusing action of the reflector shape are neglected, the bunching action is analogous to the motion of objects in a gravitational field.[3] An Applegate diagram, in Fig. 3, is a convenient method of illustrating the bunching action. This diagram represents the resonator-gap voltage as a function of time, and plots the position in the reflection space of a number of electrons which pass the resonator gap at selected intervals during a complete cycle. The opposite action of the radio-frequency field on the electrons leaving the resonator and those returning to the resonator after bunching has been shown on the diagram.

An electron which has been speeded up by the action of the radio-frequency field will travel farther into the reflecting field and will take longer than the average

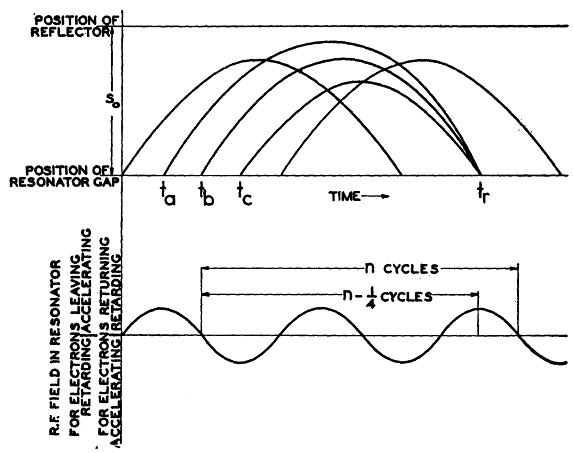

Fig. 3—Applegate diagram for a reflex-klystron oscillator.

time to return to the resonator. This behavior is similar to throwing a ball into the air; the harder the ball is thrown, the longer it takes to return to the ground. Reference to Fig. 3 will show that an electron which passes the resonator gap early in the cycle at time t_a is accelerated and requires a longer time to return than an electron leaving at time t_b when the radio-frequency field is zero. The electrons which leave at time t_c later in the cycle require less than the average transit time and all of these electrons return to the resonator in a bunch at time t_r. Bunching of the electron beam is the result, and the uniform flow of beam current is converted into an equivalent direct current with a superimposed alternating component.

The arrival time t_r of a group of electrons returning to the resonator depends upon the physical dimensions of the klystron, and also depends upon the acceleration voltage and the reflector voltage. In general, the transit time for the electron with average velocity, leaving at time t_b, may correspond to any number of cycles of the radio-frequency field, and this number need not be an integer. But in order to sustain oscillations, the electron bunch must arrive during the time when the radio frequency is retarding the returning electrons, so that the electron velocity is reduced and some of the kinetic energy of the electrons is transferred into electromagnetic energy in the cavity-resonator field.

The electron which is to become the center of the bunch leaves at the time t_b when the radio-frequency voltage is zero and changing from acceleration to deceleration. At an integral number of cycles later, the radio-frequency voltage will again be zero, but for the returning electron, the field will be changing from deceleration to acceleration. This time is indicated by n cycles in Fig. 3. Since a maximum retarding field is required for maximum energy transfer from the bunched electron beam, the transit time for an electron which enters the reflecting field with average velocity must correspond to one-quarter cycle less than an integral number of cycles. This transit-time requirement may be verified by inspection of Fig. 3.

Most of the electrons are collected by the metal walls of the tube after they have given up energy to the resonator field. Other electrons may have been lost by interception by the grid structures. A few electrons may survive these chances of getting collected and will be decelerated near the cathode surface, then reaccelerated with the newly emitted electrons. Upon re-entering the reflection space, these electrons will behave differently from the electrons which are going through the round-trip cycle for the first time. These electrons which make multiple transits may produce undesirable effects, but in most cases the effect of these electrons may be neglected. More important factors, such as space-charge debunching forces, will be neglected in order to simplify the analysis. *This theory is not intended for designing klystrons, but to help in understanding many of the phenomena which occur.*

Transit-Time Relationships in the Reflection Space

It was mentioned previously that the electrons which pass the resonator gap when the radio-frequency voltage is zero enter the reflecting field without any change in velocity, and are defined as electrons with average velocity. Electrons which pass the resonator gap at a time t_b (see Fig. 3) when the radio-frequency field is changing from accleration to deceleration, become the center of the bunch. The electrons in the bunch have different velocities, and these velocities are continually changing during the time the electrons are in the reflection space; however, it is convenient to consider that the bunch moves as a unit along a path determined by the electron which is to become the center of the bunch. Note that the lines in Fig. 3 representing electrons leaving at times t_a, t_b, and t_c appear to converge about the center of the bunch.

A brief review of electron ballistics will derive the equations which are useful in determining the relationships between the transit time and the tube-design parameters. The calculation of the transit time from the tube voltages and the reflector-electrode spacing will not be accurate because the effect of the nonuniform field and the effect of space charge have been neglected. Although the effects of space charge are quite important, the assumption simplifies the analysis considerably, and the result is quite useful.

In the derivations which follow, the terminology will be defined as it is introduced. In addition, a glossary of symbols is included in an appendix. The average electron velocity v_0 is determined by the acceleration voltage E_0, and the relation may be obtained from the fact that the kinetic energy gained by an electron of mass m and charge e is equal to the potential energy which accelerates the electron. This relation may be stated

$$1/2(mv_0^2) = E_0 e. \qquad (1)$$

Equation (1) is then rewritten in the form

$$v_0 = \sqrt{\frac{2e}{m} E_0}. \qquad (2)$$

Other laws of motion of particles may be used to determine the transit time. If the deceleration is denoted by a, then the position of a particle as a function of time is given by

$$s = v_0 t - 1/2(at^2). \qquad (3)$$

When t is equal to the average transit time T_0 the electron has returned to the resonator, the electron velocity is again v_0, but in the opposite direction, and s is equal to zero; i.e.,

$$0 = v_0 T_0 - 1/2(aT_0^2). \qquad (4)$$

There are two solutions to (4). T_0 equal to zero corresponds to an electron which has not traversed the reflection space, and is disregarded. The other solution is

$$T_0 = \frac{2v_0}{a}\cdot \qquad (5)$$

The deceleration a may be evaluated from the familiar equation for the force acting on a particle. This force is given by the product of the charge on the electron and the gradient of the potential between the anode and the reflector electrode. If the reflector field is assumed to be uniform, the gradient is simply the sum of the voltages on the reflector electrode divided by s_0, the reflector spacing. Therefore,

$$F = ma - e\,\frac{E_0 + E_r}{s_0}\cdot \qquad (6)$$

Substitution of (6) and (2) in (5) gives

$$T_0 = \frac{2v_0}{\dfrac{e}{m}\,\dfrac{E_0 + E_r}{s_0}} = 4s_0\,\frac{\sqrt{\dfrac{m}{2e} E_0}}{E_0 + E_r} \qquad (7)$$

for the average transit time.

It is usually more convenient to express the transit time in terms of a number of oscillation cycles rather than as a time interval. This equivalent number of cycles will be designated N, and is defined by

$$N = fT_0 \tag{8}$$

where f is the frequency of oscillation. Equation (7) may therefore be written

$$N = 4fs_0 \frac{\sqrt{\frac{m}{2e}E_0}}{E_0 + E_r}. \tag{9}$$

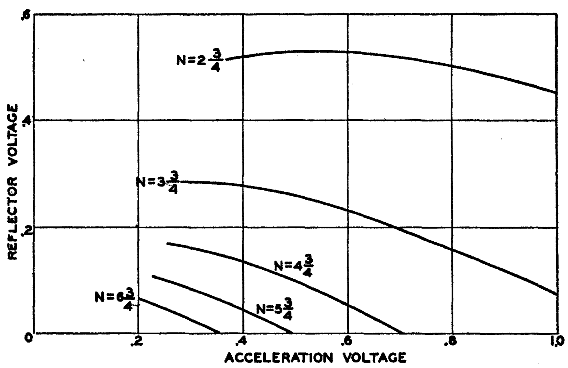

Fig. 4—Family of curves showing voltage modes in a reflex oscillator.

If oscillation is to be at maximum strength, the number of cycles during the transit time in the reflection space must satisfy the relation mentioned in the discussion of Fig. 3; i.e.,

$$N = n - 1/4 \tag{10}$$

where n is any integer greater than zero. Oscillation at the same frequency will occur for a number of values of N, and each value of N may be provided by the proper choice of the acceleration voltage and the reflector voltage. A series of curves showing the reflector voltage required to give constant frequency for any value of acceleration voltage is shown in Fig. 4. Each curve represents a different value of N. The value of N may be estimated from the frequency, reflector spacing, and voltages involved. These transit times are an important factor in the behavior of reflex klystrons, and the importance of transit time will be discussed in greater detail in the sections which follow. In practice, transit time corresponding to values of N between $1\frac{3}{4}$ and $10\frac{3}{4}$ cycles are typical.

Electron-Bunching Relationships

It is obvious that electron bunching must occur in a reflex klystron because the velocity variation introduced by the resonator voltage produces a variation of the transit times of electrons which pass the resonator gap at different times during a cycle. This variation of transit time may be expected from (5), which may be rewritten in terms of a varying velocity instead of the average velocity, and becomes

$$T = \frac{2v}{a} \tag{11}$$

when T and v are varying quantities. The current distribution in the bunched beam is similar to the bunching in a two-resonator klystron, but the manner in which the electrons become grouped is different and there is a phase difference of 180 degrees between the two types of bunching.

These differences between reflection-field bunching and field-free bunching are introduced because the transit time is proportional to the electron velocity in a reflex klystron; while the transit time in the field-free drift space between the resonators in a two-resonator klystron is inversely proportional to the velocity. As a result, the electron bunch in a reflex klystron is formed around the electron which passed the resonator gap when the radio-frequency voltage was changing from acceleration to decleration. In contrast, the bunch in a two-resonator klystron forms around the electron which passed the input resonator gap when the radio-frequency field was changing from deceleration to acceleration.[3]

The existence of a field-free bunching space in addition to the reflection space requires a modification of this analysis. A discussion of this effect is given in a number of references[1–3] and will not be repeated here.

An analysis of the bunching process in a reflex klystron may be made, following the method used by Webster[6] for the two-resonator type of klystron. Negligible transit time across the resonator gap will be assumed in the preliminary analysis, and the factors which must be modified when this assumption is invalid will be discussed in a later section.

The electrons approach the resonator gap with average velocity v_0, which is determined by the acceleration voltage E_0 as shown in (2). The velocity of the electrons will be modified by the radio-frequency voltage at the resonator gap, and after passing the gap the velocity will be

$$v = \sqrt{\frac{2e}{m}}\sqrt{E_0 + E_1 \sin \omega t_1} \tag{12}$$

where E_1 is the peak value of the radio-frequency voltage at the resonator gap, ω is the angular frequency and equal to $2\pi f$, and t_1 is the time required for an electron to pass the resonator gap. The transit time of an electron will be given by (11), and may be rewritten in a form similar to (7).

$$T = 4s_0 \frac{\sqrt{\frac{m}{2e}E_0}}{E_0 + E_r}\sqrt{1 + \frac{E_1}{E_0}\sin \omega t_1}. \tag{13}$$

Equation (7) may be substituted in (13) to give

$$T = T_0 \sqrt{1 + \frac{E_1}{E_0} \sin \omega t_1}. \qquad (13a)$$

When the ratio of E_1/E_0 is small, an approximate form of (13a) may be used

$$T = T_0 \left(1 + \frac{E_1}{2E_0} \sin \omega t_1\right). \qquad (14)$$

Returning electrons will arrive at the resonator gap at a time t_2, which will be the sum of the transit time (T) and the departure time (t_1).

$$t_2 = t_1 + T_0 \left(1 + \frac{E_1}{2E_0} \sin \omega t_1\right). \qquad (15)$$

The number of electrons which return to the resonator during a time interval dt_2 will be equal to the product of the instantaneous beam current in the reverse direction

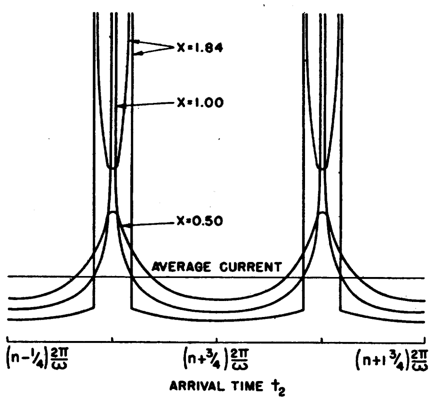

Fig. 5—Instantaneous beam current. Two complete cycles are shown and three values of the bunching parameter are represented.

I_2 and the time interval dt_2. This same number of electrons originally passed the resonator gap during an interval dt_1, when the beam current in the forward direction was equal to I_0, the direct beam current. If these expressions for the number of electrons are equated,

$$I_2 dt_2 = I_0 dt_1 \qquad (16)$$

and the instantaneous bunched current is given by

$$I_2 = I_0 dt_1/dt_2. \qquad (17)$$

Differentiating both sides of (15) gives

$$dt_2 = dt_1 \left(1 + \omega T_0 \frac{E_1}{2E_0} \cos \omega t_1\right) \qquad (18)$$

or

$$dt_2 = dt_1 \left(1 + \pi f T_0 \frac{E_1}{E_0} \cos \omega t_1\right). \qquad (18a)$$

Substituting (8) in (18a) gives

$$dt_2 = dt_1 \left(1 + \pi N \frac{E_1}{E_0} \cos \omega t_1\right) \qquad (18b)$$

which may be rewritten

$$dt_2 = dt_1(1 + x \cos \omega t_1). \qquad (18c)$$

The quantity x is known as the bunching parameter, and is defined by

$$x = \pi N \frac{E_1}{E_0}. \qquad (19)$$

Other expressions for the bunching parameter may be obtained by substitution in (19), but these expressions will not be similar to the other equations for the bunching parameter when bunching occurs in a field-free drift space.

Substituting (18c) in (17) gives

$$I_2 = \frac{I_0}{1 + x \cos \omega t_1}. \qquad (20)$$

Equation (20) is identical in form to the expression for the bunched current in a double-resonator klystron.[6]

The equations for the instantaneous current express this current as a function of t_1, the departure time of the electrons when they enter the reflecting field. It is more desirable to know the relation between the instantaneous current and t_2, the arrival time of the returning electrons. This relationship is easily obtained if a curve of t_1 versus t_2 is available, and a family of such curves is illustrated in Fig. 5 for several different values of the bunching parameter x. This graphical representation of the relationship is necessary because (15) cannot be solved explicitly for t_1. Rewriting (15) in terms of the bunching parameter x gives a form which is convenient for computation of the curves in Fig. 5. Equation (15) then becomes

$$t_2 = t_1 + T_0 + \frac{x}{\omega} \sin \omega t_1. \qquad (21)$$

Note that the slope of the curves in Fig. 5 may become negative when the bunching parameter is greater than unity. This negative slope corresponds to a negative value of I_2 indicated by (20) when x is greater than unity. The beam current never becomes negative; this sign merely means that electrons departing at a later time return before electrons which left earlier but traveled farther into the reflecting field. Since electrons leaving at three different times may arrive simultaneously, the beam current is the sum of the absolute magnitudes of the values obtained from (20) for the three values of t_1. Additional discussion of this point, based on an analysis of bunching in a field-free drift space, has been published.[8]

Curves of instantaneous current, corresponding to the

[8] D. L. Webster, "Velocity modulation currents," *Jour. Appl. Phys.*, vol. 13, pp. 786–787; December, 1942.

t_1 versus t_2 curves in Fig. 5, are shown in Fig. 6. The current peaks when the bunching parameter is unity, or greater, are quite large, but are not infinite if the transit time in the resonator gap is finite.[9,10] However, it is

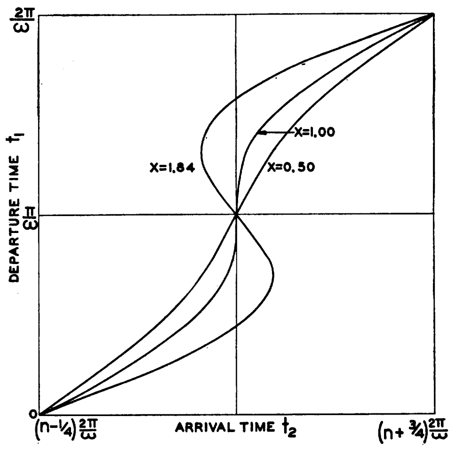

Fig. 6—Electron-arrival-time curves for three values of the bunching parameter.

convenient to treat the gap length as infinitesimal, and correction factors which can be applied when the transit time through the gap is appreciable will be given in the next section.

Since the instantaneous beam current is identical to that given by Webster[6] for the field-free case, the current may be expressed by a Fourier series with coefficients which are Bessel functions of the first kind.

$$\begin{aligned} I_2 = I_0[1 &+ 2J_1(x) \sin(\omega t_2 - 2\pi N) \\ &+ 2J_2(2x) \sin 2(\omega t_2 - 2\pi N) + \cdots \\ &+ 2J_n(nx) \sin n(\omega t_2 - 2\pi N)]. \end{aligned} \tag{22}$$

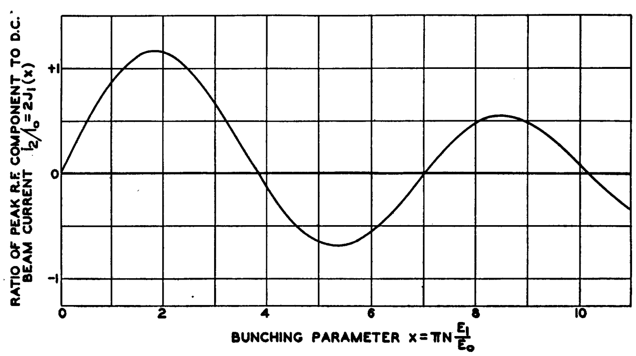

Fig. 7—Radio-frequency component of the bunched beam current.

Only the second term is of particular interest in an oscillator, and the fundamental component of the radio-frequency current in the beam, which will be designated i_2, is given by

$$i_2 = 2I_0 J_1(x) \sin(\omega t_2 - 2\pi N). \tag{23}$$

[9] L. J. Black and P. L. Morton, "Current and power in velocity-modulation tubes," Proc. I.R.E., vol. 32, pp. 477–482; August, 1944.
[10] A. E. Harrison, "Graphical methods for analysis of velocity-modulation bunching," Proc. I.R.E., vol. 33, pp. 20–33; January, 1945.

The higher harmonics are unimportant because reflex klystrons are designed to operate with a high effective Q.

Fig. 7 shows the peak value of the radio-frequency component of the bunched beam current as a function of the bunching parameter. The peak value has been divided by I_0 so that the ordinates of the curve are equal to $2J_1(x)$. This Bessel function output curve is characteristic of klystron tubes, and may be considered analogous to the plate-current versus grid-voltage characteristic of conventional tubes.

Transit-Time Effects in the Resonant Cavity

The previous discussion has ignored the effect of the transit time of the electrons in the resonator gap. If the electron crosses the gap in a small fraction of an oscillation cycle, then the change in kinetic energy will be determined by the potential difference across the gap at that instant. However, if an electron requires a full cycle to traverse the resonator gap, the electron will be accelerated during half of the cycle and decelerated during the remainder of the cycle. As a result, the net change in kinetic energy will be zero if the gap voltage is very small compared to the beam voltage.

This effect may be expressed in terms of a "beam coupling coefficient" of the gap. The expression for the bunching parameter in (19) must be modified by this beam coupling coefficient β when the transit time across the resonator gap is an appreciable fraction of a cycle, and

$$x = \beta\pi N \frac{E_1}{E_0} \tag{24}$$

gives the correct value for the bunching parameter. Equations (24) and (19) become identical when β has a value of unity.

It is necessary to know the transit time across the gap in order to evaluate β. If the distance is d, and the electron velocity has the average value v_0, then the transit time is d/v_0. The transit angle δ is given by

$$\delta = 2\pi f d/v_0. \tag{25}$$

If the averaging process mentioned in the previous paragraph is performed, the value β may be shown to be

$$\beta = \frac{\sin \delta/2}{\delta/2}. \tag{26}$$

In practice, β is always less than unity, but in many cases it is convenient to assume it is equal to unity. Since this coefficient appears in most of the equations which describe the behavior of reflex-klystron oscillators, it will be referred to frequently in the next section on oscillator theory, which will utilize the fundamental principles derived here to explain the electrical characteristics of these tubes.

General Theory of Oscillators

The basic principles of electron bunching discussed in the preceding sections can be used to derive the typical electrical characteristics of reflex-klystron oscillators. The analysis is quite similar to the analysis of oscillators in the more familiar radio-frequency region. Certain outstanding differences will be apparent; the most important is the dependence of the frequency of oscillation on the input voltages. These differences are the result of the dependence of the bunching action on transit time, and emphasize the fact that analogies to conventional vacuum tubes cannot always be used to describe the behavior of klystrons, although some of the concepts and terminology are equally useful in discussion of velocity-modulation tubes.

There are several methods which might be used to analyze the operation of a reflex-klystron oscillator. All of these methods are essentially the same and merely represent differing viewpoints in approaching the problem. Pierce[4] has described a method which equates the admittance of the resonator of a reflex oscillator and the transadmittance of the bunched electron beam. A variation of this method, using impedances instead of admittances, was used in an analysis of double-resonator klystron oscillators.[11] This variation of the analysis is desirable for a double-resonator klystron oscillator because the relation between the output current and input voltage in tightly coupled tuned circuits is usually given in the form of a transfer impedance. A reflex-klystron oscillator is much simpler to analyze because a single resonator is used.

The effect of the reflected beam in a reflex-klystron oscillator can be explained quite easily by assuming that the radio-frequency component of the bunched beam introduces an admittance Y_2 in parallel with the resonant circuit. This method reduces the analysis to a simple circuit problem in which a change in the value of Y_2 may change the resonant frequency or losses in the circuit. The results are correct; in fact, it can be shown that the various methods of analysis are mathematically identical. The advantage of the method to be used here is primarily convenient in visualizing the problem, since the effect of varying components in a circuit is often more easily understood than the effect of varying parameters in an equation.

The Equivalent Circuit of a Reflex-Klystron Oscillator

An equivalent circuit for a reflex-klystron oscillator based on the method outlined above, is shown in Fig. 8. The cavity resonator and its coupled load are represented by the parallel resistance-inductance-capacitance circuit. The copper losses and other resonator losses such as loading caused by the beam itself or secondary electrons, are represented by an equivalent shunt resistance R_S, and the coupled load or output circuit considered as another parallel resistance R_L. Then the effective resistance R_{SL} would be given by the expression for two resistances in parallel

$$R_{SL} = \frac{R_S R_L}{R_S + R_L}. \tag{27}$$

The equivalent capacitance C represents the capacitance of the resonator gap. The value of this capacitance can be estimated to a satisfactory approximation from the formula for a parallel-plate capacitor, using the area and spacing of the resonator grids forming the gap. The value of the equivalent inductance L is chosen to make the resonant frequency of the equivalent circuit equal to the resonant frequency of the cavity.

If the reflex klystron is oscillating, or if energy is coupled into the cavity resonator from an external source, then a voltage will exist across the resonator gap. This voltage is represented by the voltage E across the capacitance C in the equivalent diagram in Fig. 8, and the value of E is given by

$$E = E_1 \sin \omega t \tag{28}$$

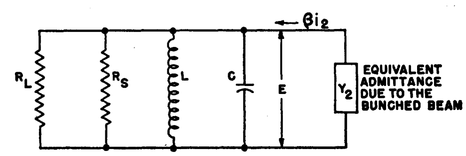

Fig. 8—Equivalent circuit for a reflex-klystron oscillator.

where E_1 is the peak value of the voltage across the resonator gap, and ω and t represent the angular frequency of oscillation and time.

The bunching action produces a radio-frequency current i_2 which depends upon the beam current I_0 and the bunching parameter x, as shown in (23).

$$i_2 = 2I_0 J_1(x) \sin (\omega t - 2\pi N). \tag{23}$$

N represents the number of oscillation cycles during the time an electron is in the reflection space. A current βi_2 is shown flowing out of the "fictitious" admittance Y_2, which represents the effect of the bunched beam current in the equivalent diagram. This direction for the current is chosen because Y_2 represents the source of power. The beam coupling coefficient β is introduced in order to include the effect of the decreased energy transfer from the beam to the resonator when the gap transit time is large. This factor must be included in each step of the derivation in which it should appear; but a value of unity, corresponding to negligible-gap transit time, will be assumed in most cases in order to simplify the discussion of this analysis.

[11] A. E. Harrison, "Klystron oscillators," *Electronics*, vol. 17, pp. 100–107; November, 1944.

Admittances are used in this discussion because admittances can be added when considering parallel circuits. Equation (27), expressed in the form of the sum of two conductances (the real part of an admittance), would be written

$$\frac{1}{R_{SL}} = \frac{1}{R_S} + \frac{1}{R_L} \cdot \tag{29}$$

The total admittance of the resonator Y_S would include the susceptance terms for the inductance and capacitance as well as the conductance terms in (29).

$$Y_S = \frac{1}{R_S} + \frac{1}{R_L} - \frac{j}{\omega L} + j\omega C. \tag{30}$$

This form will be quite convenient in the analysis of a reflex oscillator because real and imaginary terms may be considered separately.

An evaluation of the admittance Y_2 which is added to the resonator admittance may be obtained from the fact that a voltage E must cause a current βi_2 to flow. The magnitude of Y_2 will be determined by the ratio of the peak value of βi_2 and the peak voltage E_1.

$$Y_2 = \frac{2\beta I_0 J_1(x)}{E_1} \cdot \tag{31}$$

The phase of Y_2 is determined by the transit time in the reflection field. If the transit time corresponds to $(n-\frac{1}{4})$ cycles, where n is an integer, then the electrons in the bunch will be retarded, and the beam will transfer energy to the radio-frequency field in the resonator. This relation was explained in the discussion of Fig. 3. Under these conditions Y_2 will be a pure negative conductance. A transit time of $(n+\frac{1}{4})$ cycles corresponds to a transfer of energy from the radio-frequency field to the electron beam, and in this case Y_2 is a positive conductance; i.e., the beam represents an additional loss in the circuit.

Other values of transit time cause Y_2 to be complex since the radio-frequency component of the bunched beam current will not be in phase with the resonator voltage. The phase angle of i_2 will be represented by ϕ, and ϕ will be considered zero when the transit time in the reflection field corresponds to $(n-\frac{1}{4})$ cycles. The expression for i_2 in (23) may be rewritten

$$i_2 = 2I_0J_1(x) \sin [\omega t - 2\pi(n - 1/4) - \phi]. \tag{32}$$

Comparison of (23) and (32) shows that the phase angle ϕ is defined by

$$\phi = 2\pi N - 2\pi(n - 1/4). \tag{33}$$

N may have any value and is determined by the transit time in the reflection space, but n is always an integer. If the transit time is correct for maximum output, then the phase angle ϕ is zero, and N is given by (10).

$$N = n - 1/4. \tag{10}$$

Decreasing either the acceleration voltage or the reflector voltage increases the transit time in the reflection space and increases the angle ϕ.

It will be convenient to express i_2 in the vector form instead of the sinusoidal form in (32).

$$i_2 = 2I_0J_1(x)[\cos \phi - j \sin \phi]. \tag{34}$$

Since Y_2 is a negative admittance when ϕ is equal to zero, as defined in the discussion following (31), the complex admittance is

$$Y_2 = \frac{-\beta i_2}{E_1} = \frac{2\beta I_0 J_1(x)}{E_1} [-\cos \phi + j \sin \phi]. \tag{35}$$

Both components of the admittance are plotted in Fig. 9. The conductance, which is the real term in (35), is shown as a solid line, and the susceptance is a dash line. The vertical scale in Fig. 9 is purely arbitrary, since I_0, $J_1(x)$ and E_1 are unspecified.

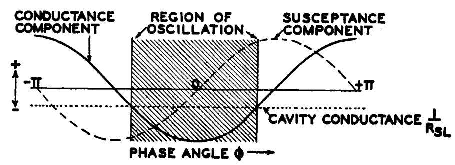

Fig. 9—Conductance and susceptance components of the beam admittance.

A qualitative analysis of a reflex oscillator may be obtained from inspection of Fig. 9. As the phase angle is increased from a negative value toward zero, the conductance changes from a positive value, indicating a loss, to a negative value representing a source of power. Oscillation will occur when the negative conductance is equal in magnitude to the conductance of the cavity; i.e., when the source of power is just sufficient to supply the losses in the resonator and the load. The magnitude of the circuit conductance is indicated by the horizontal dotted line in Fig. 9. The shaded portion shows the region in which oscillation will occur.

When ϕ is equal to zero, corresponding to the transit time for maximum output, the beam susceptance is zero and the tube will oscillate at the natural frequency of the cavity resonator. Note that the equivalent capacitance of the resonator corresponds to a positive susceptance in (30). Increasing the phase angle until ϕ is positive introduces an additional positive susceptance in parallel with C, and the frequency of oscillation becomes less than the natural frequency of the resonator. A negative susceptance might be considered a negative capacitance which decreases the effect of C, or it might be viewed as an inductance in parallel with L. Either viewpoint indicates that the resonant frequency of the system will be increased when ϕ is negative.

The value of this analysis can be demonstrated by experimental verification of the theory. If the beam current is kept quite small so that oscillation does not occur, the magnitude of the beam conductance and susceptance components will be sinusoidal, as shown by Fig. 9, and the effective Q and resonant frequency of the cavity will vary as the phase of the feedback is changed by varying the reflector voltage. These changes were measured; the results of the experiment are shown in Fig. 10 and agree quite closely with the theoretical prediction.

A casual inspection of Figs. 9 and 10 might suggest that the tuning effect becomes small for large values of the phase angle ϕ near the points where oscillation fails to occur, because the sine function is not changing

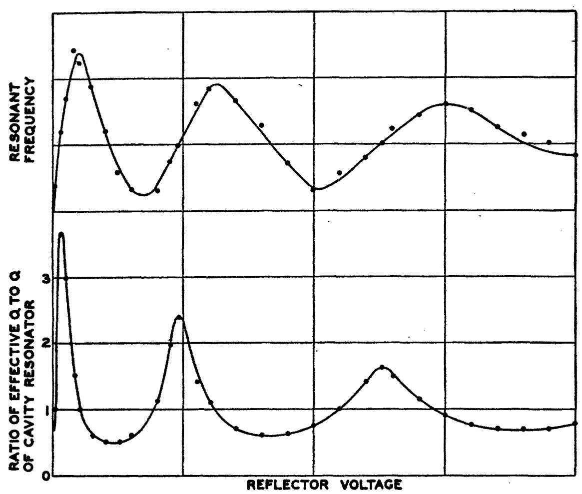

Fig. 10—Experimental curves showing the effect of the beam admittance.

rapidly. This behavior is correct for the conditions represented by Fig. 10, but when the beam current is large enough to maintain oscillation; i.e., when the beam current is much greater than the starting current, the sinusoidal variation of frequency does not occur. Actually, the scale in Fig. 9 depends upon the ratio $J_1(x)/E_1$, and this ratio decreases as the strength of oscillation increases. As a result, the tuning effect decreases rapidly as the transit time in the reflection space approaches the value required to make the phase angle ϕ equal to zero, and the frequency deviation is actually proportional to the tangent of the phase angle rather than the sine. This effect will be apparent from the quantitative analysis which follows.

Analysis of Reflex-Oscillator Characteristics

If the shunt resistance of a resonator is independent of frequency, the analysis is simplified because the power output and efficiency relations are obtained by considering only the conductance component of the beam admittance. After the strength of oscillation has been determined, the frequency of oscillation can be obtained from the magnitude of the beam susceptance. If the beam conductance is greater than the value required to supply the losses in the resonator and its load, the strength of oscillation will increase until the value of the negative beam conductance is reduced to the conductance of the resonator and its load. This means that the conductance of the system is zero when the klystron is oscillating. The sum of the susceptances must also be zero, and this relation determines the frequency of oscillation.

The starting current is one of the important characteristics of an oscillator, and will be used to illustrate this method of analysis. The starting current is the lowest value of beam current I_0 which will allow oscillation to exist. The sum of the cavity conductance and the beam conductance from (35) must be zero for oscillation to occur.

$$\frac{1}{R_{SL}} - \frac{2\beta I_0 J_1(x)}{E_1}\cos\phi = 0. \tag{36}$$

The peak resonator voltage E_1 and x are related, and the analysis is simplified if x is used as the variable. E_1 may be expressed in terms of x by rewriting (24).

$$x = \beta\pi N\frac{E_1}{E_0} \tag{24}$$

$$E_1 = \frac{E_0 x}{\beta\pi N}\cdot \tag{37}$$

Substituting (37) in (36) and rearranging terms gives

$$\frac{x}{2J_1(x)} = \frac{\beta^2\pi N I_0 R_{SL}}{E_0}\cos\phi \tag{38}$$

$$\frac{x}{2J_1(x)} = \frac{\beta^2\pi N I_0 R_S R_L}{E_0(R_S+R_L)}\cos\phi. \tag{38a}$$

Weak oscillation corresponds to extremely small resonator voltage, and the bunching parameter x is almost zero under these conditions. The $J_1(x)$ Bessel function is equal to $x/2$ for small values of x, therefore the left side of the equation will be unity when I_0 is equal to the starting current. The current will be a minimum for the starting conditions only if the phase is correct; i.e., $\cos\phi$ must be a maximum and ϕ is equal to zero, the phase for maximum output. When these conditions are imposed on (38), we obtain an expression for the starting current.

$$I_{\text{Start}} = \frac{E_0}{\beta^2\pi N R_{SL}}\cdot \tag{39}$$

Reasonable values which might be substituted into (39) in order to give some idea of the current required for oscillation follow:

$$\beta^2 = 1.0$$
$$E_0 = 300 \text{ volts}$$
$$N = 4\tfrac{3}{4} \text{ cycles}$$
$$R_{SL} = 20{,}000 \text{ ohms}.$$

Representative values have been chosen, and indicate that a beam current of one milliampere will maintain oscillation.

The term $x/2J_1(x)$ in (38) and (38a) is one form of a very important parameter in the analysis of any oscillator. It was used and explained in an article[11] on double-resonator klystron oscillators and will appear as a co-ordinate in many of the illustrations which follow. The basic parameter, which applies to conventional vacuum tubes as well as velocity-modulation types, may be defined as the magnitude of the ratio of the small-signal transadmittance of a tube to the large-signal transadmittance. This ratio is a measure of the saturation effect at high input levels and the term "transreduction factor" has been proposed for this ratio. The term is not limited to analysis of oscillators but is equally useful in amplifiers and other vacuum-tube circuits. When used in an analysis of klystron operation based on small variations of velocity, the value of the parameter has the convenient mathematical equivalent $x/2J_1(x)$, which has been mentioned.

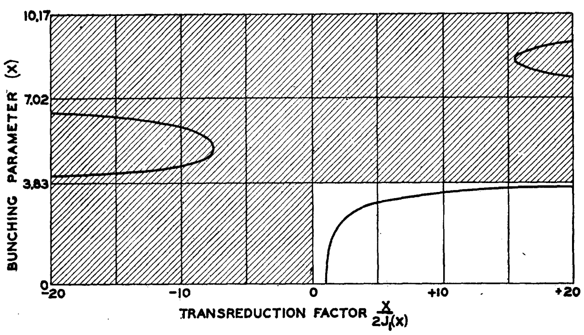

Fig. 11—Bunching parameter x as a function of beam current and other variables. The unshaded portion is the normal operating region for a reflex oscillator.

Increasing the beam current above the starting current value will greatly increase the output. This can be shown by deriving the expression for the power delivered to the resonator and load. This power will be designated P_2, and is the power delivered by the bunched beam to the shunt resistance R_{SL}. The value of P_2 is given by one half of the product of E_1, the peak resonator voltage, and the peak value of the in-phase component of i_2. This product must be reduced by the beam coupling coefficient β, in order to include the effect of finite transit time across the resonator gap.

$$P_2 = 1/2(E_1\beta i_2 \cos\phi) = \beta E_1 I_0 J_1(x) \cos\phi. \quad (40)$$

Substituting the expression for E_1 in (37) into (40)

$$P_2 = \frac{E_0 I_0 \cos\phi}{\pi N} x J_1(x). \quad (41)$$

In order to compute P_2, it is necessary to know the dependence of E_1 or x upon the beam current, I_0. Equations (24) or (37) do not furnish this information, but the relation can be obtained indirectly from (38). Values may be substituted in (38) or (38a) to obtain the value of the transreduction factor $x/2J_1(x)$ corresponding to the assumed value of the beam current I_0. The relation between the bunching parameter x and $x/2J_1(x)$ can be obtained from a table of Bessel functions, or from Fig. 7, which is a curve of $2J_1(x)$ as a function of x. This relation between x and $x/2J_1(x)$ is

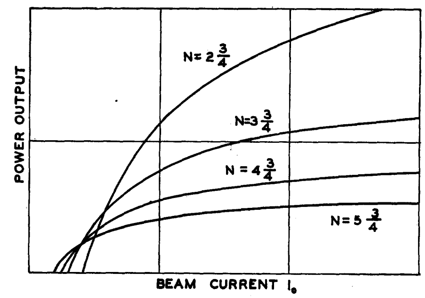

Fig. 12—Power output as a function of beam current. Several modes corresponding to different transit times are shown.

given in Fig. 11 for all values of x between zero and 10.17, corresponding to the third zero of the Bessel function, but only the unshaded region is of importance in the normal operation of a reflex-klystron oscillator. The value of $x/2J_1(x)$ computed from (38) or (38a) is used with Fig. 11 to obtain values for x and $J_1(x)$ corresponding to the assumed value of the beam current I_0, and the power can then be computed from (41).

Curves of power delivered by the bunched beam as a function of beam current I_0, computed in the manner described above, are shown in Fig. 12 for various values of N. Those curves not only show the increase of power as the current is increased above the starting value, but also indicate that the maximum power from a reflex oscillator and the starting current are inversely proportional to N, the number of cycles during transit in the reflection field. In other words, increasing the number of cycles required for bunching, either by reducing the reflector voltage or actually changing the tube design by increasing the reflector spacing, will decrease the output which can be obtained but will permit the tube to be operated with a smaller beam current.

It would be interesting to investigate the region in Fig. 11 where $x/2J_1(x)$ has a negative value. The negative sign has the same significance as the negative portion of the Bessel-function curve; i.e., when the bunching parameter x is greater than 3.83, the Bessel function becomes negative and the phase of the bunched beam is shifted 180 degrees.[10] Reference to (35) will illustrate the effect of this phase shift. Oscillation can occur only when the equivalent beam conductance is negative. Normally, this condition is met when the phase angle ϕ is zero and the Bessel function has a positive value. However, if the phase angle is 180 degrees, corresponding to the usual region of nonoscillation, but the resonator voltage is made large enough to give a negative value of $J_1(x)$, then the beam conductance defined by (35) is also negative. Oscillation would not be self-starting, but *might* be maintained if the beam current was sufficiently high and the correct value of resonator voltage was obtained by overdriving the resonator.

If any of the variables other than current are changed,

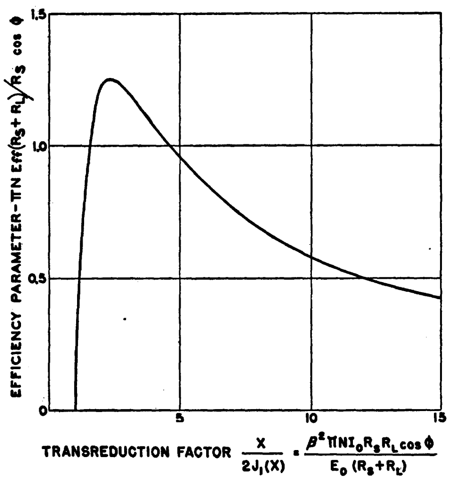

Fig. 13—Universal curve for the efficiency of a reflex-klystron oscillator.

such as the load resistance or the phase angle ϕ, the use of curves to show the effect of each variable becomes quite complicated. Fortunately, all of the variables can be combined into dimensionless parameters and the characteristics can be presented in a universal curve as illustrated by Fig. 13. The transreduction factor $x/2J_1(x)$ in (38) is one example of a useful dimensionless parameter and the efficiency parameter to be derived below is another example.

The power delivered by the bunched beam, defined by (41), is not all useful power since some is absorbed by the resonator losses. We are more interested in the power delivered to the load, which will be designated P_L. Then

$$P_L = \frac{R_S}{R_L + R_S} P_2 = \frac{R_S E_0 I_0 \cos\phi}{\pi N (R_L + R_S)} x J_1(x). \qquad (42)$$

If we divide the power output by the beam power input we obtain the efficiency of the klystron oscillator. Equation (42) can be rearranged so that the efficiency (abbreviated "Eff.") and the other factors involved are related to a dimensionless efficiency parameter $xJ_1(x)$.

$$xJ_1(x) = \frac{\pi N}{\cos\phi} \frac{R_L + R_S}{R_S} \text{Eff.} \qquad (43)$$

Fig. 13 combines these two dimensionless parameters in a single curve which relates the output characteristics of a reflex-klystron oscillator to the design factors which may be varied. The vertical co-ordinate is $xJ_1(x)$ and $x/2J_1(x)$ is the horizontal co-ordinate.

Effect of Voltage, Current, and Load on Klystron Output

Most of the output characteristics which are typical of reflex-klystron oscillators can be predicted by inspection of Fig. 13. Consider the case when the load, beam current, and acceleration voltage remain fixed, but the reflector voltage is varied. Assume that the phase angle ϕ is $\pi/2$ for zero reflector voltage; i.e., when the reflector electrode is at cathode potential. Cos ϕ will be zero, corresponding to an operating point at the origin in Fig. 13. Increasing the negative reflector voltage will decrease ϕ and cos ϕ will vary from zero to a maximum of unity and then decrease again. The value of N will also vary, but if N is large this variation is not important in a qualitative analysis, and N will be assumed a constant for the range of each voltage mode.

When cos ϕ is zero, the transreduction factor $x/2J_1(x)$ is also zero, since the value of $x/2J_1(x)$ is determined by (38) or (38a).

$$\frac{x}{2J_1(x)} = \frac{\beta^2 \pi N I_0 R_{SL}}{E_0} \cos\phi. \qquad (38)$$

Oscillation will not occur until cos ϕ has increased until the value of $x/2J_1(x)$ is unity. As cos ϕ increases beyond this point, the output will increase as shown by Fig. 13. When cos ϕ is unity, $x/2J_1(x)$ will have its maximum value and the output will also be maximum. This is true for the region where the efficiency curve is decreasing because the cos ϕ term increases faster than the efficiency parameter in Fig. 13 decreases. As the reflector voltage is increased beyond the value giving maximum output, the phase angle becomes negative, and cos ϕ decreases until the output is again zero.

As the reflector voltage is increased further, the sign of cos ϕ will become negative and the beam-conductance term in (35) has a positive value. This positive beam conductance represents an additional loss, therefore

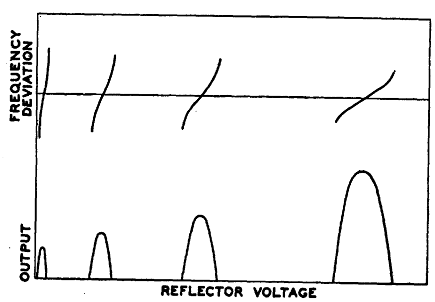

Fig. 14—Power-output and frequency characteristics when the reflector voltage of a reflex klystron is varied.

oscillation does not occur. When the transit time has changed by an amount equivalent to one complete cycle, the phase is again correct for oscillation and another output mode will occur. Normally, there are several of these voltage modes, and oscillation does not occur in the region between modes where the phase angle is incorrect. This behavior is illustrated by Fig. 14.

The higher reflector-voltage modes correspond to smaller values of N and the output is greater for two reasons: first, the ordinate $xJ_1(x)$ in Fig. 13 becomes greater as N is decreased, since decreasing N corresponds to moving from right to left on the curve in Fig. 13; second, the efficiency for a particular value on the curve is inversely proportional to N. Eventually it is no longer possible to observe modes with higher reflector voltage because N has become so low that the starting current is greater than the beam current. The last mode observed may have the highest output of the series, or it may have less output than the previous mode. The latter case corresponds to a point in Fig. 13 to the left of the maximum of the curve.

The maximum theoretical efficiency of a reflex-klystron oscillator is less than the value for a double-resonator oscillator, and is inversely proportional to N. The efficiency for any value of N can be calculated from Fig. 13. If most of the power is transferred to the load and the phase angle is adjusted for maximum output, then (43) may be rewritten

$$\text{maximum efficiency} = \frac{xJ_1(x)}{\pi N} = \frac{1.25}{\pi N} \cdot \tag{44}$$

The assumptions used in this derivation are not valid for small values of N, and theoretical efficiencies between 20 and 30 per cent are indicated when better approximations are made in the computation of efficiency for values of N less than two.

It is interesting to note that the efficiency obtainable for any mode is independent of the beam coupling coefficient. If the transit time across the resonator gap is large, making the value of β less than unity, then it is theoretically possible to overcome this disadvantage by increasing the beam current. The power output will be greater because the same maximum efficiency requires more power input. If sufficient beam current is available so that the load resistance R_L is small in comparison with the shunt resistance of the resonator R_S, the effect of a small value of β may be counteracted by decreasing the load; i.e., increasing the value of R_L.

If the output load impedance is varied (by varying the length of the output line or some other method of impedance transformation), the output will increase to a maximum, then decrease suddenly and the klystron may refuse to oscillate for certain load impedances. This effect occurs first for the higher reflector-voltage modes because the starting current is higher for these modes. When the beam current is constant the load required for maximum output is different for each mode. Heavier loading is required for maximum output from the modes corresponding to the larger values of N.

This effect can be demonstrated conveniently with a dynamic method of observing the output. An alternating voltage can be superimposed upon the reflector voltage, causing the output to be swept through several modes periodically. The output voltage is applied to a cathode-ray oscilloscope with the sweep synchronized with the reflector-voltage modulation. A pattern similar to Fig. 14 will be observed. If the klystron is lightly loaded, all of the modes will be small, but the higher reflector-voltage modes will increase until the mode with the smallest value of N corresponds to the point of maximum efficiency on Fig. 13. Increasing the load further will decrease the output from the highest voltage mode until it disappears when the transreduction factor becomes less than unity. The other modes with larger values of N will continue to increase in output, with the modes disappearing successively until the load is so great that the klystron cannot oscillate at any reflector voltage.

Analysis of Electronic Tuning

The qualitative analysis based on Fig. 9 predicted that the frequency of oscillation would change as the phase of the bunched beam was varied by changing the acceleration voltage or the reflector voltage. This effect is known as electronic tuning. The power output and efficiency relationships were obtained by considering only the conductance components of the beam and cavity admittances. Similarly, the electronic-tuning analysis requires the sum of the susceptances to be zero. The magnitude of the beam susceptance depends upon the strength of oscillation, however. As a result, the imaginary component of the beam admittance depends upon the magnitude of the real component.

Equation (36) may be rewritten

$$\frac{2\beta I_0 J_1(x)}{E_1} = \frac{1}{R_{SL}\cos\phi} \cdot \tag{45}$$

Then (45) may be substituted in the imaginary term of (35) to obtain the value of the beam susceptance in terms of the phase angle ϕ.

$$\frac{2\beta I_0 J_1(x)}{E_1}\sin\phi = \frac{\sin\phi}{R_{SL}\cos\phi} = \frac{\tan\phi}{R_{SL}} \cdot \tag{46}$$

Equating all susceptance terms in the resonator and beam admittances to zero gives an expression which may be used to determine the frequency of oscillation. If ω is the angular frequency of oscillation for any phase angle, and ω_0 is the angular frequency corresponding to zero phase; i.e., the resonant frequency of the cavity, then

$$-\frac{1}{\omega L} + \omega C + \frac{\tan\phi}{R_{SL}} = 0. \tag{47}$$

Rearranging terms gives

$$\frac{R_{SL}}{\omega_0 L}\left(\frac{\omega_0}{\omega} - \omega\omega_0 LC\right) = \tan\phi. \tag{47a}$$

But $R_{SL}/\omega_0 L$ is equal to the loaded Q of the resonator, Q_L and LC is equal to $1/\omega_0^2$, therefore

$$Q_L\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)=\tan\phi. \tag{47b}$$

When ω and ω_0 do not differ by more than a few per cent $((\omega_0/\omega)-(\omega/\omega_0))$, may be rewritten

$$\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}=2\,\frac{\omega_0-\omega}{\omega}=2\,\frac{\Delta f}{f} \tag{48}$$

and (47b) becomes the familiar expression for the phase of a parallel-resonant circuit.

$$2Q_L\frac{\Delta f}{f}=\tan\phi. \tag{49}$$

The term 2 $Q_L\Delta f/f$ is a convenient frequency-deviation parameter which is often used in universal curves for resonant circuits. It relates the actual frequency deviation to the loaded Q of the circuit.

Equation (42) and Fig. 13 allow the power output to be calculated as a function of the phase angle ϕ, and the frequency deviation from the resonant frequency of the cavity can be obtained from (49). However, it is more useful to know these characteristics as a function of voltage instead of phase. Equation (9), repeated below,

$$N=4fs_0\frac{\sqrt{\dfrac{m}{2e}E_0}}{E_0+E_r} \tag{9}$$

may be substituted into (33) to obtain a value of ϕ, and this value of ϕ may then be substituted into (42) and (49), giving the output power and frequency characteristics as a function of reflector voltage. Fig. 14 was obtained in this manner.

Fig. 15 repeats the characteristics shown in Fig. 14 for a single mode and a number of different values of loaded Q. The curves for heavy loading correspond to a load which is almost great enough to prevent oscillation. Curves are also shown for the loading which gives maximum output, and very light loading when most of the power is absorbed by the resonator losses.

A number of interesting conclusions are illustrated by Fig. 15. The slope of the linear portion of the frequency characteristic is inversely proportional to the loaded Q of the resonator. This fact is apparent from (49), but only the trend is indicated by Fig. 15, since actual values of Q_L are not given. Increasing the Q by decreasing the load does not decrease the electronic-tuning bandwidth as might be expected, since this change will increase the bunching and the phase angle may be varied over a larger range before the output decreases appreciably. The bandwidth between zero-output points actually increases as the loading is decreased, and the bandwidth between half-power points is decreased only slightly. Decreased loading causes the amplitude characteristic to become more uniform over a large range of voltage, but the frequency-deviation curve becomes quite nonlinear.

These qualitative conclusions are interesting, but a method of calculating the bandwidths is more valuable. The desired equations may be obtained by evaluating

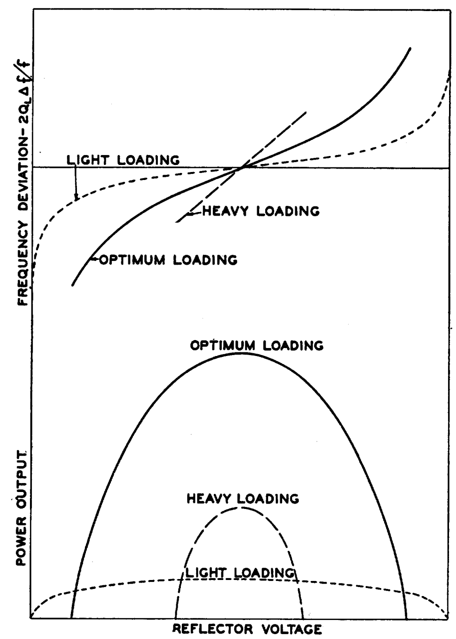

Fig. 15—Power-output and frequency characteristics for different loads.

the phase angle ϕ for the output being considered, and substituting this value of ϕ in (49). This process will be carried out for the zero-power point and also the half-power point. Equation (38a) may be rewritten

$$\cos\phi=\frac{E_0(R_S+R_L)}{\beta^2\pi NI_0R_SR_L}\,\frac{x}{2J_1(x)}\,. \tag{50}$$

For zero output, the value of $x/2J_1(x)$ is unity; therefore

$$\cos\phi_0=\frac{E_0(R_S+R_L)}{\beta^2\pi NI_0R_SR_L}, \tag{51}$$

and

$$\tan\phi_0=\sqrt{\frac{1}{\cos^2\phi_0}-1}. \tag{52}$$

Note that $\cos\phi_0$; i.e., the cosine of the phase angle when the output is zero, has a value equal to the reciprocal of the transreduction factor $x/2J_1(x)$ for the operating conditions when the phase angle is zero, corresponding to maximum output. Therefore, (52) may be rewritten

$$\tan \phi_0 = \sqrt{\left(\frac{x}{2J_1(x)}\right)^2 - 1}. \qquad (52a)$$

The bandwidth between zero-output points is obtained by substituting (52) in (49). However, the frequency deviation $\Delta f/f$ is measured from the point of maximum output; therefore, the bandwidth between zero-output points will be twice the value indicated by (49). The term $(2\Delta f/f)_0$ will be introduced to avoid confusion between the bandwidth between the two zero-output points and the frequency deviation from the frequency corresponding to maximum output. Then

$$2Q_L\left(\frac{2\Delta f}{f_0}\right) = 2 \tan \phi_0 = 2\sqrt{\frac{1}{\cos^2 \phi_0} - 1}. \qquad (53)$$

Evaluation of the bandwidth between half-power points is somewhat more complicated, and requires the determination of the bunching-parameter value which corresponds to one half of the maximum output. The power output for any operating condition is the square of the peak voltage E_1 divided by twice the load resistance R_L.

$$P_L = \frac{E_1^2}{2R_L} = \frac{E_0^2 x^2}{2\beta^2\pi^2 N^2 R_L}. \qquad (54)$$

Equation (37) has been substituted for E_1 in (54). The value of the bunching parameter x for maximum output can be obtained from Fig. 11 with $\cos \phi$ equal to unity. This maximum output does not necessarily correspond to the point of optimum efficiency in Fig. 13, but is the maximum output for the given conditions of load and input when the phase angle is zero. These conditions determine the value of $x/2J_1(x)$ and x is then determined from Fig. 11. This value of x divided by $\sqrt{2}$ is the value of the bunching parameter which corresponds to the half-power points. Substituting this value of the bunching parameter in (50) gives

$$\cos \phi_{1/2} = \frac{E_0(R_S + R_L)}{\beta^2\pi N I_0 R_S R_L} \frac{x/\sqrt{2}}{2J_1(x/\sqrt{2})}. \qquad (55)$$

Equation (55) may also be written

$$\cos \phi_{1/2} = \frac{2J_1(x)}{x} \frac{x/\sqrt{2}}{2J_1(x/\sqrt{2})}. \qquad (55a)$$

A definition for the bandwidth between half-power points, similar to the definition for zero-output conditions, gives

$$2Q_L\left(\frac{2\Delta f}{f}\right)_{1/2} = 2 \tan \phi_{1/2} = 2\sqrt{\frac{1}{\cos^2 \phi_{1/2}} - 1}. \qquad (56)$$

These expressions may appear complicated, but the evaluation of $\cos \phi_{1/2}$ from Fig. 11 is quite simple. The method can be illustrated by a sample calculation. Assume that $x/2J_1(x)$ equal to 2.30 corresponds to the operating conditions when the phase angle is zero. This corresponds to maximum output from the tube. The bunching parameter x for this value of $x/2J_1(x)$ is 2.40, as indicated by the curve in Fig. 11. The value of x for the half-power point would be $2.40/\sqrt{2}$ or 1.70, and corresponds to $x/2J_1(x)$ equal to 1.47. Cos $\phi_{1/2}$ is then 1.47/2.30, or 0.64. Substitution of this value of $\cos \phi_{1/2}$ in (56) gives a value of 2.40 for $2Q_L(2\Delta f/f)_{1/2}$.

The calculations for bandwidths between zero-output and half-power points have been made and the results are plotted in Fig. 16 as a function of $x/2J_1(x)$, the transreduction factor. A dotted line has been drawn

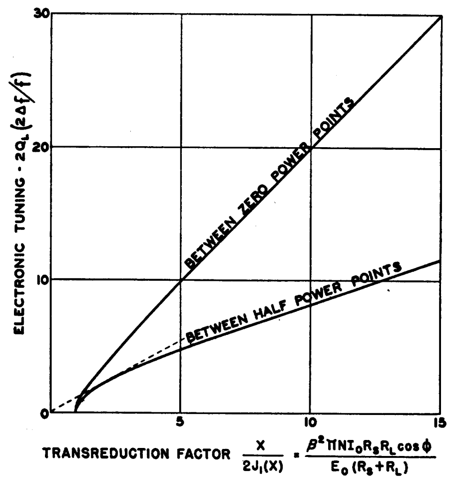

Fig. 16—Universal curves for the electronic tuning of a reflex oscillator.

through the origin and tangent to the curve for the bandwidth between half-power points. Since Q_L is proportional to $R_S R_L/(R_S+R_L)$, this dotted line is proportional to Q_L and $(2\Delta f/f)_{1/2}$ will be a maximum at the point of tangency. In other words, the maximum bandwidth between half-power points occurs when the conductance parameter has a value of approximately 2.30, the same as the value required for optimum output from the tube.

It is interesting to note that the bandwith between half-power points for a single resonant circuit is 2.00 when using these co-ordinates for the frequency deviation. The value for a reflex-klystron oscillator with the load adjusted for maximum bandwidth is 2.40, or 20 per cent greater than the bandwidth associated with the loaded Q of the resonator. Increasing the bunching by increasing the beam current, decreasing the loading, or in any other manner which increases the value of the transreduction factor, will increase the value of $2Q_L(2\Delta f/f)_{1/2}$. However, it is not correct to state that the electronic tuning of a reflex klystron is independent of the loaded Q of the resonator. The frequency deviation in the linear region is inversely proportional to Q_L, but increasing Q_L by reducing the load causes overbunching and the tube can oscillate over a wider range of voltage variation. As a result, the half-power point is extended into the nonlinear region of the frequency-deviation characteristic and the actual frequency bandwidth $(2\Delta f/f)_{1/2}$ decreases only slightly from the maximum bandwidth when the oscillator is loaded to give maximum output.

Effect of Load Variations

Some of the effects of varying the load have been mentioned in the discussion of Figs. 13 and 15. The previous discussion assumed that the load can be represented by an equivalent shunt resistance R_L. This equivalent resistance has a magnitude similar to the shunt resistance of the cavity resonator; i.e., R_L is usually several thousand ohms. The characteristic impedance of the coaxial output line is very much smaller, usually in the order of magnitude of 100 ohms for convenient physical dimensions, and the coupling loop must be designed to transform an impedance of perhaps 100 ohms to the required value of several thousand ohms. Some tubes are manufactured with coupling loops which are fixed in size and position; in this case, the equivalent load resistance can be changed only by changing the load itself or by using some type of impedance transformer between the load and the coaxial output terminal. Other tube types may also permit variation of the size or position of the coupling loop as a means of adjusting the load.

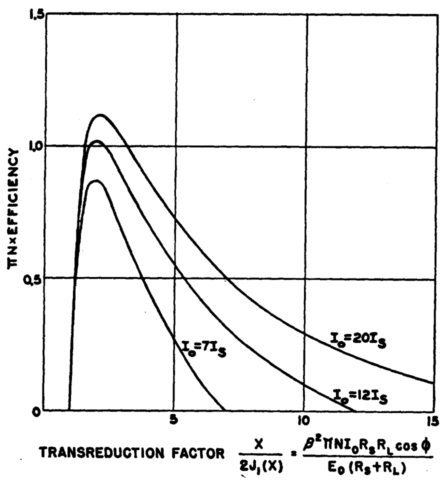

Fig. 17—Efficiency of a reflex oscillator as a function of load. Curves for three values of beam current are shown.

If a variable length of line is used as an impedance transformer, the resistive component of the load can be varied, if there are standing waves in the line, but a reactive component may also be introduced. This reactive component will affect the frequency of oscillation. The analysis of this effect will not be considered in detail in this paper. However, the effect is quite important and should not be overlooked when using these tubes.

Frequency changes may also be caused by changing only the resistive component of the load if the phase angle ϕ is not zero. Consider a case illustrated by Fig. 15 when the reflector voltage does not correspond to the adjustment for maximum output and the frequency deviation is not zero. Decreasing the load will decrease the frequency deviation. This effect is also indicated by the magnitude of the beam susceptance in (47). Decreasing the load corresponds to increasing the load resistance R_L, and this change also increases the effective shunt resistance R_{SL}; therefore, decreasing the load will decrease the effective beam susceptance and the frequency deviation will be less. This effect becomes greater when the reflector voltage deviates from the value required for maximum output.

If most of the power is not transferred to the load, then the derivation of the maximum efficiency in (44) does not apply, and the efficiency is dependent upon the load resistance. Actually, R_L must be small compared to R_S if most of the power is to be transferred to the load, and this condition can be obtained only if the beam current available is very much larger than the starting current. The maximum efficiency is less than the theoretical value for practical values of beam current. If the beam current is seven times greater than the starting current, the maximum value of the $x/2J_1(x)$ co-ordinate in Fig. 13 will be 7.0 when R_L is infinite, corresponding to no load. The output will be zero under these conditions and the efficiency will also be zero, since, $(R_L+R_S)/R_S$ becomes infinite. As R_L is decreased corresponding to increasing the load, the output will increase.

A family of curves similar to Fig. 13 can be plotted to show the effect of power division between the resonator losses and the load. The factor $(R_L+R_S)/R_S$ in the ordinate of Fig. 13 is computed for each value of R_L considered, and the ordinates for the revised efficiency curves in Fig. 17 are directly proportional to the output efficiency. Each curve corresponds to some chosen value of beam current I_0 and πN times the efficiency is plotted as a function of R_L. The other variables in the transreduction factor are held constant. The phase angle ϕ has been assumed to be zero in this illustration, corresponding to the voltage adjustment for maximum output, therefore $\cos\phi$ is unity and has not been included in the efficiency co-ordinate.

If the beam current I_0 were equal to the starting current I_S, the transreduction factor $x/2J_1(x)$ would have a value of unity. The load would be zero, corresponding to an infinite value of R_L. When I_0 is seven times greater than I_S, the value of $x/2J_1(x)$ would be 7.0 if the load resistance R_L was infinite. The output would be zero, of course. Decreasing R_L would increase the load, and the efficiency would increase until a maximum was reached. Eventually the load would become too great and the tube would fail to oscillate when R_L was reduced until $x/2J_1(x)$ had a value of unity. Similar curves are shown for values of I_0 twelve and twenty times greater than the starting current. Note that the actual efficiency is only 90 per cent of the theoretical efficiency when the beam current is twenty times greater than the starting current.

Fig. 17 may also be used to compare the efficiencies for different values of N when the beam current remains constant. These conditions can be met by changing the reflector voltage. Consider that the curve in Fig. 17 for seven times the starting current corresponds to a value of N equal to $2\frac{3}{4}$ cycles, and the curve for 12 times the starting current corresponds to the same beam current but a value of $4\frac{3}{4}$ cycles for N. Then the actual efficiency for optimum loading would be $0.87/2.75\pi$ or 10.1 per cent for N equal to $2\frac{3}{4}$ cycles, and $1.01/4.75\pi$ or 6.8 per cent for N equal to $4\frac{3}{4}$ cycles. Although the loading required for maximum output is less for the mode with the shorter transit time, and therefore a larger proportion of the total power is dissipated in the resonator losses, the improved conversion efficiency for the shorter transit time allows the output efficiency to be greater.

Reflex-Klystron Design Considerations

Most of the previous discussion has been used to predict or explain the electrical characteristics of reflex-klystron oscillators when operating voltages, current, and loading were the only variables. It is interesting to consider the effect of varying the design of the tube itself, although it is necessary to remember that the relation between the lumped constants used in the equivalent circuit and the physical dimensions of the cavity resonator is not clearly defined. However, considering the effect of changing these constants can be quite useful in a qualitative analysis of the factors which are important in the design of klystrons.

Reference to the equivalent circuit in Fig. 8 will indicate that increasing the ratio of the small-signal beam admittance to the circuit capacitance will increase the amount of electronic tuning. This ratio may be increased by increasing the beam current I_0, increasing the transit time in the reflection space (increasing the value of N), or by decreasing the circuit capacitance. Decreasing the capacitance by increasing the resonator-gap spacing may not be satisfactory because the transit time across the gap may become excessive. This change would reduce the beam coupling coefficient, which has the same effect as reducing the beam current. Therefore we will only consider reducing the capacitance by decreasing the area of the resonator gap.

Either increasing the beam current without changing the capacitance, or reducing the area of the gap without changing the current, corresponds to increasing the current density. Therefore the problem of increasing the electronic tuning in a klystron design becomes a problem of increasing the current density. This conclusion assumes that N is already large and that additional transit time in the reflection space will not increase N appreciably.

It is equally interesting to analyze the factors affecting electronic tuning from the viewpoint that increased electron bunching permits heavier loading of the oscillator, and therefore increases the electronic tuning because the loaded Q has been reduced. Reference to Figs. 11 and 13 will emphasize the fact that the bunching parameter x has a value of 2.40 when the oscillator is adjusted for maximum output. If the beam current is increased, with no design change in the resonator, the resonator voltage E_1 will be increased and the value of the bunching parameter will increase. The magnitude of E_1 is determined by the radio-frequency current i_2 and the loaded shunt resistance R_{SL}.

$$E_1 = 2I_0R_{SL}J_1(x). \tag{57}$$

Since E_1 must be constant if x remains constant, an increase in I_0 must be accompanied by a decrease in the loaded shunt resistance R_{SL}. Therefore the increased beam current permits the oscillator to be operated with a greater load, and reducing the Q of the loaded circuit increases the electronic tuning.

The effect of decreasing the capacitance may also be related to the loaded Q of the resonator. One of the relations giving the Q of a circuit is

$$Q_L = \omega C R_{SL} = \omega C \frac{R_S R_L}{R_S + R_L}. \tag{58}$$

The unloaded Q of the circuit will be

$$Q = \omega C R_S, \tag{59}$$

therefore (58) may be rewritten

$$Q_L = Q \frac{R_L}{R_S + R_L}. \tag{60}$$

Decreasing the circuit capacitance by reducing the resonator-gap area without changing the gap spacing does not change the unloaded Q appreciably, but does increase the shunt resistance R_S. This change will not affect the loaded shunt resistance R_{SL}, since R_L is usually much smaller than R_S; therefore, the oscillator will operate with the same degree of bunching if the beam current and the load resistance R_L are unchanged However, (60) indicates that the loaded Q will decrease when the shunt resistance is increased, and the electronic tuning will be increased.

Note that the changes discussed in all of the preceding paragraphs correspond to increasing the curent density in the electron beam. The various explanations of the electronic tuning are merely different ways of looking at the problem.

The design of an efficient, high-power reflex-klystron oscillator would require a different approach. The important design factor would be the transit time in the reflection space, therefore N must be small. As pointed out in the discussion of (44), the analysis is not valid for small values of N, but the trend is indicated correctly. Decreasing N increases the starting current, and if the beam current is already as large as permitted by a practical design, then the load required for optimum output cannot be very great and the electronic tuning will be small. It is also apparent that the theoretical efficiency will not be attained if a large part of the total power goes into the resonator losses. In spite of this factor, however, the efficiency will be greater than that of a reflex klystron designed for a larger value of N. If it were possible to increase the beam current sufficiently so that most of the power could be transferred to the load, then the klystron would have as much electronic tuning as a design with a larger value of N and smaller beam current.

Acknowledgment

The presentation of reflex-oscillator characteristics with universal curves, and the form of these curves, were suggested in an unpublished report by D. R. Hamilton. Credit is also due G. E. Hackley for some of the experimental work which verified the theory.

Appendix I

Glossary of Symbols

E_0 = beam voltage or acceleration voltage.
E_r = reflector voltage (voltage between cathode and reflector electrode).
E_g = control-grid voltage.
E = instantaneous value of radio-frequency voltage across resonator gap.
E_1 = peak value of radio-frequency voltage across resonator gap.
I_0 = average beam current (value of direct current).
I_{start} = minimum value of beam current required to maintain oscillation
I_2 = instantaneous value of bunched beam current.
i_2 = fundamental component of radio-frequency current in the bunched beam.
ϕ = phase angle of bunched beam current.
v = velocity of an electron.
v_0 = average velocity of an electron (corresponds to E_0).
s = distance measured from resonator gap.
s_0 = spacing between resonator gap and reflector electrode.
F = retarding force due to reflecting field.
a = deceleration caused by retarding force F.
e = charge of an electron.
m = mass of an electron.
t = time.
t_1 = departure time when an electron leaves the resonator gap.
t_2 = arrival time when an electron returns to the resonator.
T = transit time in the reflection field.
T_0 = transit time in the reflection field of an electron with average velocity v_0.
N = number of oscillation cycles during transit of the reflection space.
f = frequency of oscillation.
ω = $2\pi f$.
ω_0 = 2π times the resonant frequency of the cavity.
x = bunching parameter equal to $\beta\pi N\dfrac{E_1}{E_0}$.
J_n = Bessel function of first kind and nth order.
J_1 = Bessel function of first kind and first order.
d = spacing of resonator gap.
δ = transit angle across the resonator gap.
β = moudlation coefficient equal to $\dfrac{\sin \delta/2}{\delta/2}$.
R_S = shunt resistance of the cavity resonator.
R_L = equivalent load resistance.
R_{SL} = loaded shunt resistance of the cavity resonator.
L = equivalent inductance of the cavity resonator.
C = equivalent capacitance of resonator gap.
Q = unloaded Q of the cavity resonator.
Q_L = loaded Q of the cavity resonator.
Y_s = total admittance of the cavity resonator.
Y_2 = equivalent admittance due to the bunched beam.
P_2 = power delivered to the resonator and load.
P_L = power delivered to the load.
Eff. = efficiency (ratio of radio-frequency output power to beam-power input).
$x/2J_1(x)$ = transreduction factor (magnitude of the ratio of small-signal transadmittance to large-signal transadmittance).
$xJ_1(x)$ = universal efficiency parameter for reflex klystrons.
$2Q_L\Delta f/f$ = frequency deviation from resonant frequency of the cavity.
$2Q_L(2\Delta f/f)_0$ = bandwidth between zero-power-output points.
$2Q_L(2\Delta f/f)_{1/2}$ = bandwidth between half-power points (frequently called electronic-tuning bandwidth).