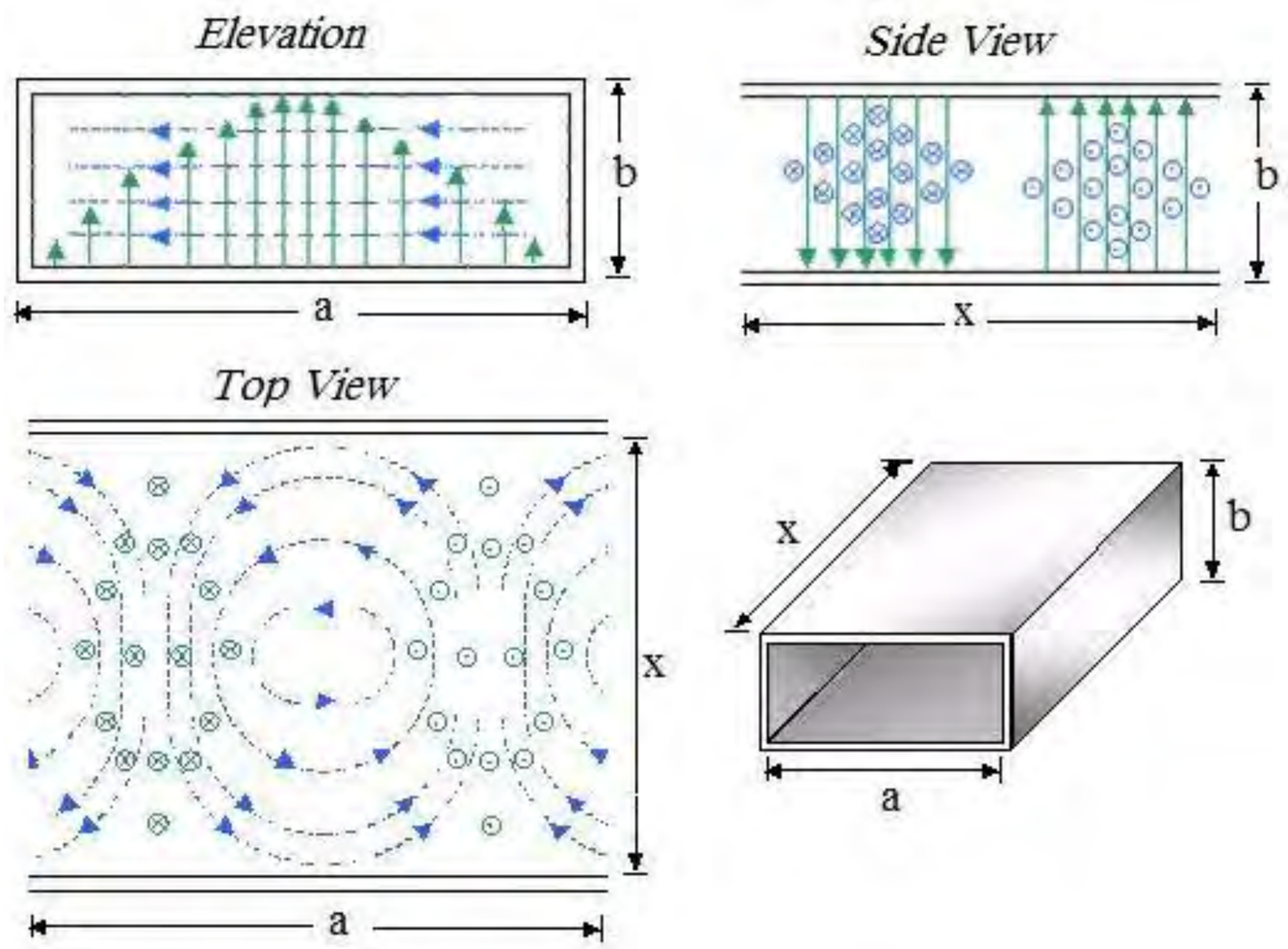
Microwave Experiment: Week 2

AP 4018
Columbia University

Objective

- Measure the wavelength of microwaves in a rectangular waveguide



Rectangular Waveguide

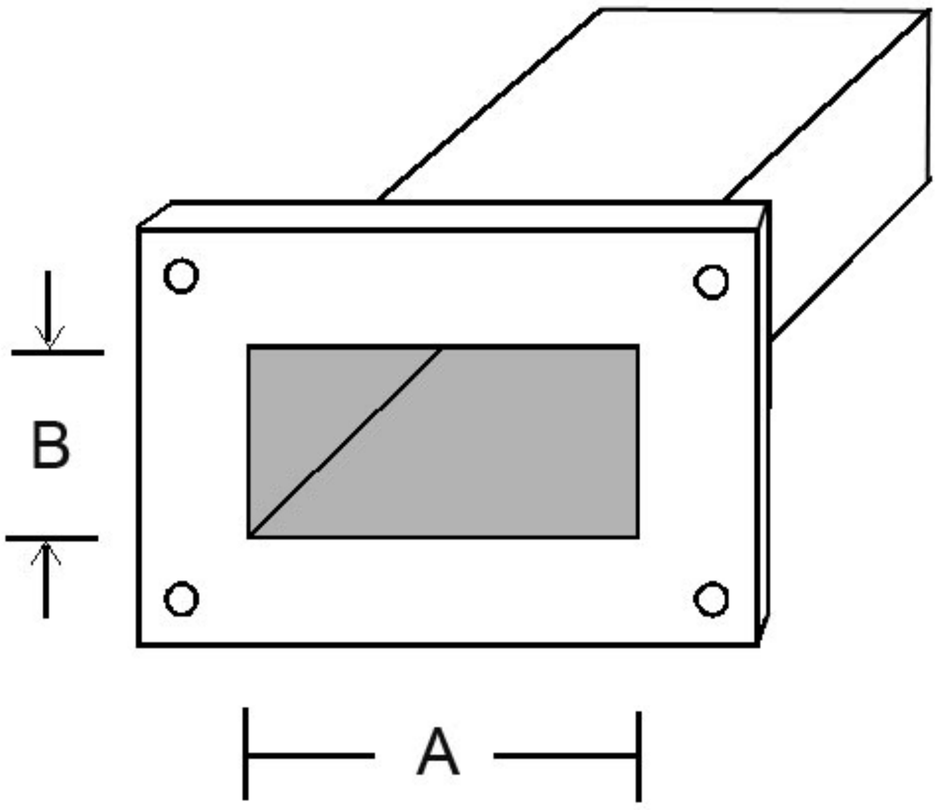
WR90 Specifications

- Recommended Frequency Band:
  8.20 to 12.40 GHz

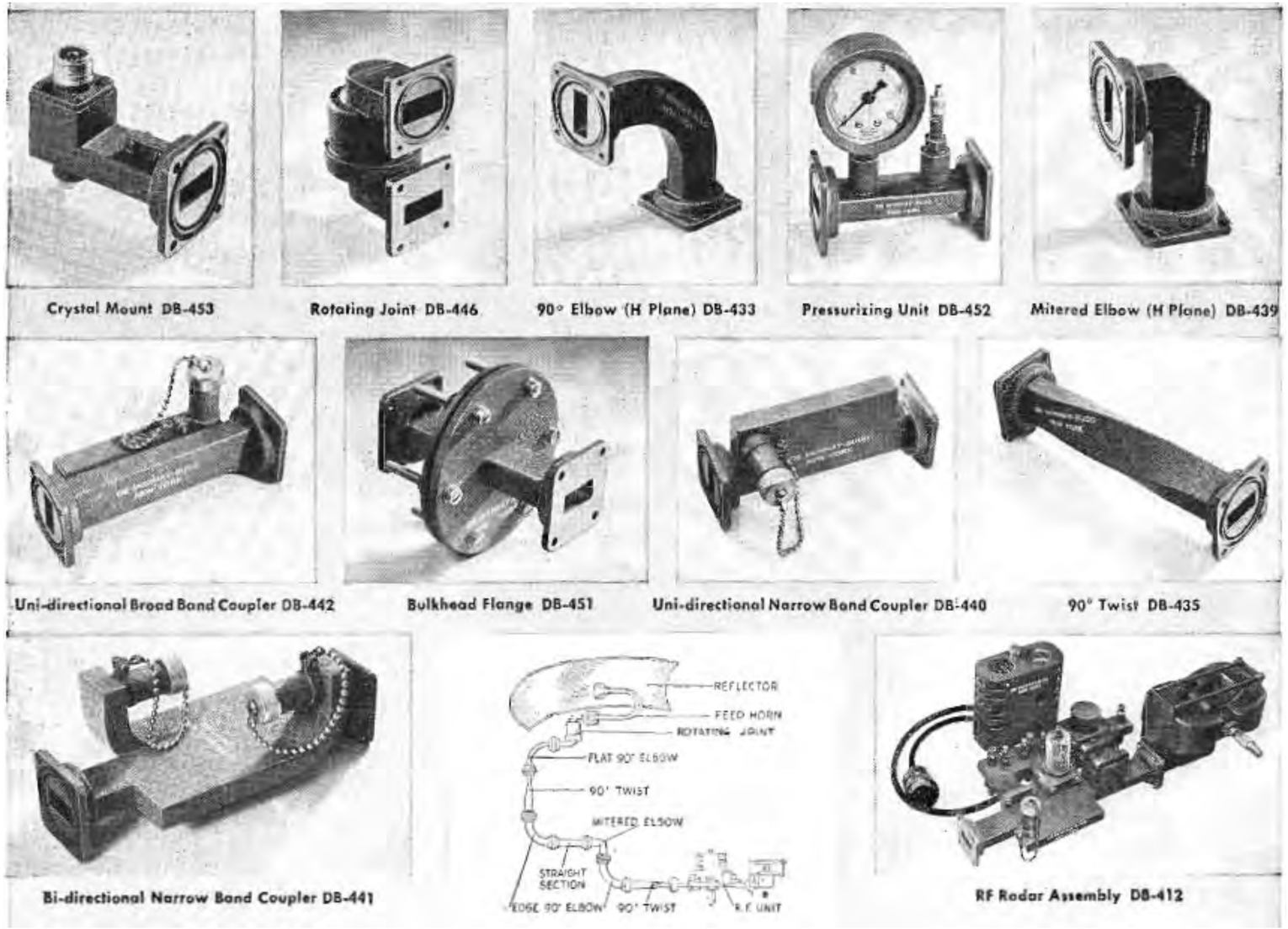
- Cutoff Frequency of Lowest Order Mode:
  6.557 GHz

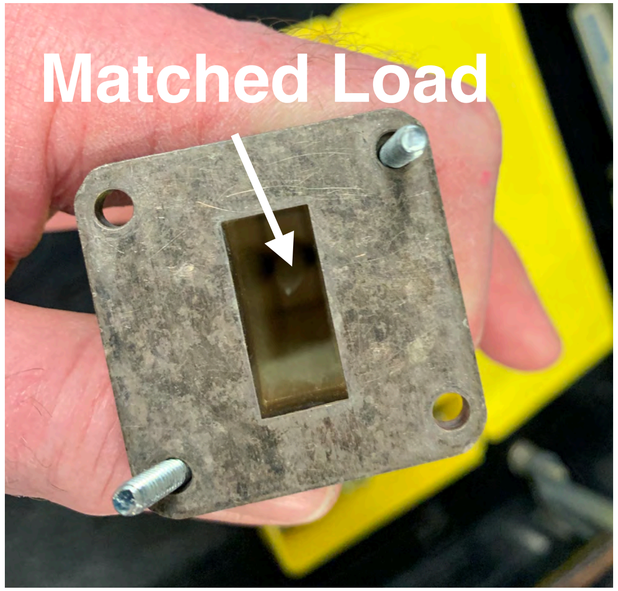
- Cutoff Frequency of Upper Mode:
  13.114 GHz

- Dimension:
  0.9 Inches [22.86 mm] x 0.4 Inches [10.16 mm]



Rectangular Waveguide: Various Components

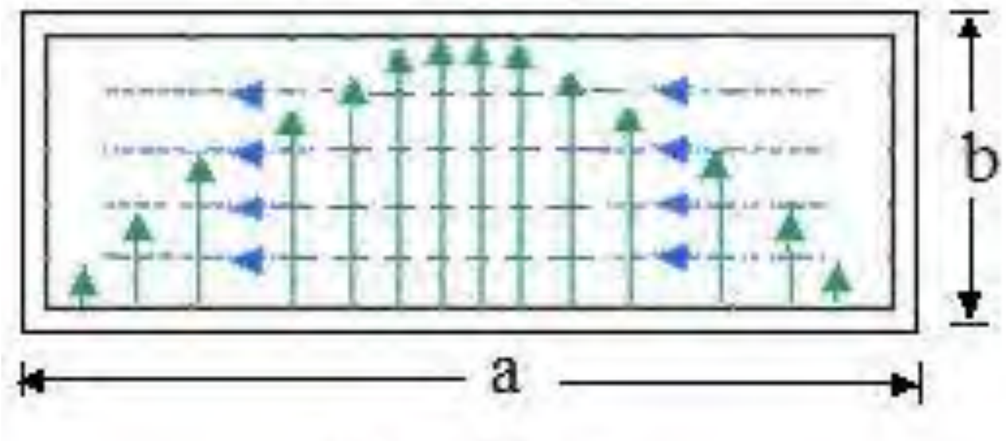




https://en.wikipedia.org/wiki/Waveguide_(radio_frequency)

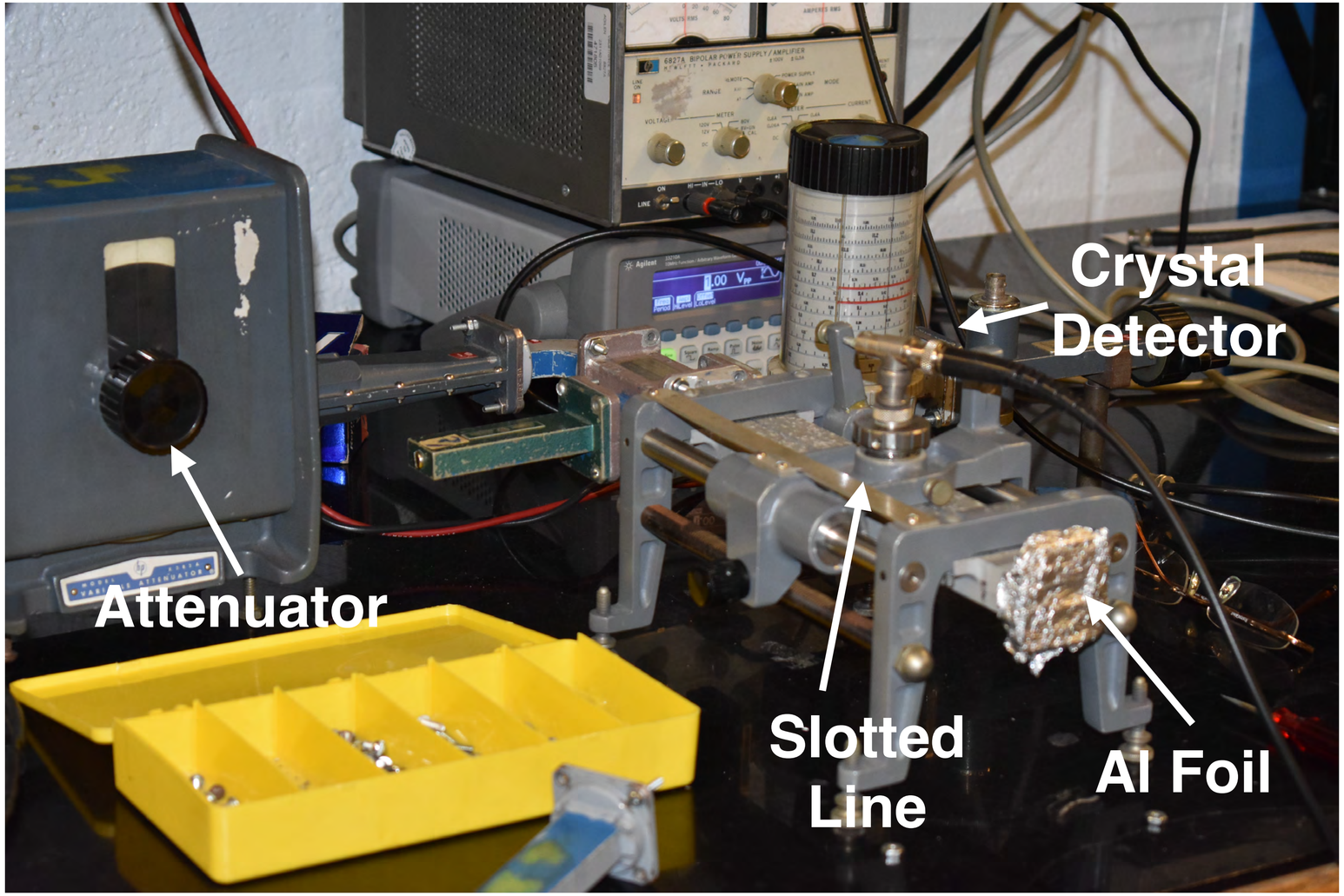
Dispersion Relation

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2$$



$$k_y = 0 \text{ and } k_x = \frac{\pi}{a}$$ (*why?*)

Total reflection makes an interference pattern in the waveguide (that is easy to detect!)



Slotted Line (Vernier Scale)

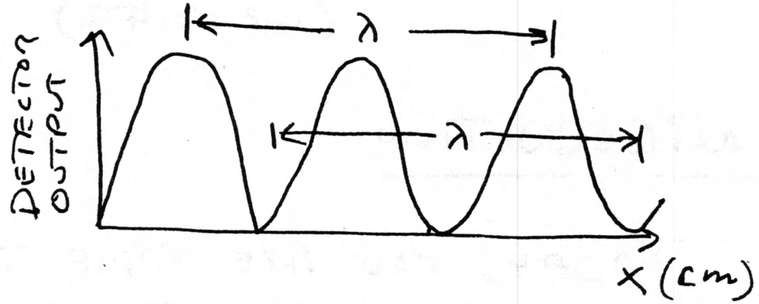
What is the Dispersion Relation?

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2$$

THE DISPERSION RELATION REFERS TO THE RELATIONSHIP BETWEEN THE WAVELENGTH OF THE LIGHT AND ITS FREQUENCY. FOR THIS LAB, WE DEAL ONLY WITH THE WAVELENGTH ALONG THE AXIS OF THE WAVE GUIDE.

Procedure - Week 2

PROCEDURE: FIRST, SHORT THE END OF THE "SLOTTED LINE" WITH ALUMINUM FOIL (OR ANY GOOD CONDUCTOR). THEN, MOVE THE DETECTOR ALONG THE SLOTTED LINE TO RECORD THE DISTANCE BETWEEN TWO MAXIMUM OR MINIMUM.



SECONDLY, USING THE TECHNIQUES YOU LEARNED LAST WEEK, ADJUST THE MICROWAVE FREQUENCY. (FOR EXAMPLE, CHANGE THE REFLECTOR VOLTAGE.) REPEAT YOUR MEASUREMENT OF THE WAVELENGTH. PLOT WAVELENGTH VS. FREQUENCY AND COMPARE WITH FORMULA PROVIDED IN NOTES.

Reflection Coefficient

THE REFLECTION COEFFICIENT DETERMINES THE STANDING WAVE PATTERN DETECTED IN YOUR SLOTTED WAVEGUIDE. IF THERE IS NO REFLECTION (A MATCHED LOAD), THEN THERE IS NO STANDING WAVE PATTERN. IF THERE IS TOTAL REFLECTION, THE DETECTOR SIGNAL MODULATED BY 100%. THE REFLECTION COEFFICIENT HAS A MAGNITUDE AND A PHASE. THE PHASE DETERMINES WHERE THE MAX + MIN ARE. TO DETERMINE THE REFLECTION COEFFICIENT YOU MUST MEASURE THE STANDING WAVE PATTERN WITH THE SLOTTED LINE.

For these experiments, we only used Al foil.
What is the reflection coefficient, Γ = ??

EQUATIONS:

E_I = FORWARD ELECTRIC FIELD

E_R = REFLECTED ELECTRIC FIELD $= \Gamma E_I$

Γ = REFLECTION COEFFICIENT (COMPLEX NUMBER)

MEASURED VALUE $= |E_{TOTAL}(z)|^2$

$$E_{TOTAL} = E_I \left(e^{ikz} + \Gamma e^{-ikz} \right)$$

$$|E_{TOTAL}| = |E_I| \left| 1 + \Gamma e^{-i2kz} \right|$$

$$\frac{E_{MAX}}{E_{MIN}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad (\text{WHERE } \Gamma = |\Gamma| e^{i\phi})$$

E_{MIN} OCCURS WHEN $2kz = \pi + \phi + 2\pi m$

E_{MAX} OCCURS WHEN $2kz = 0 + \phi + 2\pi m$

JUST MEASURE $\left|\frac{E_{MAX}}{E_{MIN}}\right|^2$ AND LOCATION OF E_{MIN}!

Summary and Question

- Measure: three frequencies: (*for example*) 9.419 GHz, 9.243 GHz, and 9.514 GHz
- Record: the positions of the *Minimum* and *Maximum* electric field intensity (*i.e.* $V_{det} \propto E^2$) measured with slotted line.
- What is the wavelength?
- Compare with expected dispersion relation.

Summary: Week 2

- Measure the wavelength of microwaves in a rectangular waveguide
- Compare wavelength to expected dispersion relationship