

- ³ R. W. Gould, Phys. Rev. **136**, A991 (1964).
⁴ A. Y. Wong, R. W. Motley, and N. D'Angelo, Phys. Rev. **133**, A436 (1964).
⁵ B. D. Fried and R. W. Gould, Phys. Fluids **4**, 139 (1961).
⁶ A. Y. Wong, Phys. Rev. Letters **14**, 252 (1965).
⁷ H. Sato, H. Ikezi, N. Takahashi, and Y. Yamashita, Phys. Rev. **183**, 278 (1969).
⁸ H. K. Andersen, N. D'Angelo, V. O. Jensen, P. Michelsen, and P. Nielsen, Phys. Fluids **11**, 1177 (1968).
⁹ I. Alexeff and W. D. Jones, Phys. Rev. Letters **15**, 286 (1965); I. Alexeff, W. D. Jones, and D. Montgomery, Phys. Fluids **11**, 167 (1967).
¹⁰ G. M. Sessler, Phys. Rev. Letters **17**, 243 (1966).
¹¹ G. M. Sessler and G. A. Pearson, Phys. Rev. **162**, 108 (1967).
¹² J. L. Hirshfield and J. H. Jacob, Phys. Fluids **11**, 411 (1958).
¹³ H. J. Doucet and D. Gresillon, Phys. Letters **28A**, 257 (1968); N. Sato, H. Ikezi, Y. Yamashita, and N. Takahashi, Phys. Rev. Letters **20**, 837 (1968).
¹⁴ H. Ikezi and N. Takahashi, Phys. Rev. Letters **20**, 140 (1968); D. R. Baker, N. R. Ahern, and A. Y. Wong, Phys. Rev. Letters **20**, 318 (1968).
¹⁵ The Debye length used in these estimates was that of the body of the plasma. In the intergrid spacing, the value is about 10 times larger due to the density decrease accompanying ion acceleration in the ~ 6 V dc sheaths.
¹⁶ While the details of this grid structure may appear to be rather specialized, its design was the simplest of any considered which would be capable of quantitative understanding. Other experimenters have launched spatial disturbances using wire probes and single grids. The former provides a strongly nonuniform excitation and is ill-suited for calculation of the ion perturbations induced, since the rf and dc electric fields near the electrode are not known. The latter suffers from the same difficulty, but additionally introduces larger direct capacitive coupling between the transmitting and receiving grids, which can mask the propagating waves.
¹⁷ E. G. Broadbent, Proc. Roy. Soc. (London) **A311**, 211 (1969).
¹⁸ J. H. Jacob and J. L. Hirshfield, in *Proceedings of the International Conference on Physics of Quiescent Plasmas*, Paris (Ecole Polytechnique, Paris, 1969), Vol. I, p. 93.
¹⁹ It should be noted that Gould³ was able to treat a symmetric source because he neglected the reflection of the electrons by the static sheaths.
²⁰ B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic, New York, 1961).
²¹ I. M. Ryshik and I. S. Gradshteyn, *Tables of Series, Products and Integrals* (VEB Deutscher Verlag der Wissenschaften, Berlin, 1957), p. 237.
²² F. F. Chen, in *Plasma Diagnostic Techniques*, edited by R. H. Huddleston and S. L. Leonard (Academic, New York, 1965), p. 178.
²³ For a grid whose intermesh dimension exceeds a Debye length, these considerations fail, but the source function is then strongly inhomogeneous in the grid plane.
²⁴ This derivation is due largely to I. B. Bernstein.

Symmetry Considerations for Grid-Launched Ion-Acoustic Waves

I. B. BERNSTEIN, J. L. HIRSHFIELD, AND J. H. JACOB

Mason Laboratory, Yale University, New Haven, Connecticut 06520

(Received 9 March 1970)

Immersion of a fine-mesh grid (or set of parallel grids) into a plasma is widely used to stimulate ion-acoustic waves. The grid structure is usually biased so as to reflect electrons and transmit ions. The effect upon the ions due to applied rf fields in the grid region is to provide some combination of velocity and density modulation. The associated applied electric field perturbations will possess a combination of odd and even spatially symmetric parts, about the plane of symmetry of the grid system. The steady-state solution of the governing differential equation in the ion-acoustic limit is shown to depend critically upon this source symmetry, and to reduce to the usual solution only for the case of odd symmetry. Experiments designed to test these symmetry predictions have been carried out using an apparatus described previously, except that a four-grid transmitter has been substituted for the three-grid one. The outer two grids are grounded and the inner two grids can be driven either in phase (odd E -field symmetry), or 180° out of phase (even E -field symmetry). For odd symmetry, the observed spatial disturbance is similar to that reported previously with clear evidence of Fresnel interference. For even symmetry, wavelike signals are not observed near the grids, indicating a strong decrease in coupling between the free-streaming source ions and an ion-acoustic wave.

I. INTRODUCTION

This paper is part of a series devoted to examination of the excitation of linearized low-frequency electrostatic disturbances in uniform plasmas without static magnetic fields. Particular emphasis has been on the difficulties in interpretation of the spatial decay observed experimentally when a steady-state signal is launched in plasma by driving an immersed electrode from an external generator. Thus, one calculation¹ has shown that the free-streaming drifts of ions or electrons which have been perturbed by a

planar source gives rise to a spatial decay even in the absence of mutual interaction between the particles. The heavy damping limit for an ion-acoustic wave is dominated by this ballistic contribution. The ballistic contribution was first evaluated by Landau in the half-space electron plasma wave problem,² but not clearly identified as such. A rough criterion was given by us that observed decays should have an imaginary-to-real wavenumber ratio k_i/k_r somewhat less than 0.4 in order for collective effects to be clearly dominant in the decay.

In a second work,³ it was pointed out that, in the near field of a wave transducer (where most experiments are performed), Fresnel interference effects can dominate the spatial decays. Therefore, even when collective effects dominate, direct inference of the Landau damping decrement from the observation may be very difficult.

The present work examines the effect of source symmetry on the generation of ion-acoustic signals. It is pointed out that for a negatively biased source with no net charge, the only boundary condition at the source plane commensurate with the nearly vanishing current in an ion-acoustic wave is that of an *odd* symmetry source electric field. Supporting experiments are also described.

A number of other authors have recently given attention to the problem of grid excitation of ion-acoustic waves. For example, Doucet and Gresillon⁴ have shown that the effective phase velocity of a linear grid-excited disturbance in a single-ended *Q* machine varies as the cube root of the frequency, as predicted theoretically for the free-streaming signal.¹ Quang⁵ has applied kinetic considerations to try to distinguish theoretically between linear and nonlinear free-streaming behavior. Andersen *et al.*⁶ have studied the transmission, reflection, and free streaming of ions modulated at a grid in a single-ended *Q* machine. Ikezi *et al.*⁷ have studied grid excitation of spatial ion-wave echoes. Joyce *et al.*⁸ have discussed the dispersion of an ion-wave packet, stimulated by a gridded transmitter in a spherical discharge tube.

Most of the above work overlooks the feature that a grid⁹ system biased so as to turn around the bulk of a given charged species by means of the electrostatic sheaths established is formally akin to a half-space problem with a boundary condition. As we shall see this introduces certain critical features not present in the familiar infinite plasma where the driver is assumed not to influence the zero-order trajectories which are infinite straight lines. We have previously shown¹⁰ that the electric field $\mathbf{E}(\mathbf{r}, \omega)$ in the plasma half-space adjacent to the grid, driven externally at radian frequency ω , cannot be expressed as the inverse Fourier transform

$$\mathbf{E}(\mathbf{r}, \omega) = \left(\frac{1}{2\pi}\right)^3 \int d^3k \cdot \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \mathbf{E}_0(\mathbf{k}, \omega) \left(\frac{1}{D(\mathbf{k}, \omega)}\right) \quad (1)$$

with $\mathbf{E}_0(\mathbf{k}, \omega)$ as the source field, and $D(\mathbf{k}, \omega)$ the usual *infinite* plasma dielectric function.¹¹ The half-space problem was solved for the electron

plasma wave case by Landau in a form similar to Eq. (1), but this was only possible because of his judicious choice of the source field \mathbf{E}_0 (i.e., that with *odd* symmetry). Landau's solution does not carry over to the case of arbitrary source symmetry, since all kernels in the resulting integral equation are not of the difference variety.

As in our previous work on this subject,^{1,3,10} the underlying philosophy is to call into question the oversimplified interpretation oftentimes attached to observed signals in wave propagation experiments, both linear and nonlinear, where all of the elements required for application of the theory developed for infinite unbounded one-dimensional plasmas may not be fulfilled.

II. THEORY

In an earlier version of this work,¹⁰ a one-dimensional theory was developed from collisionless Boltzmann equations, as applied to both ions and electrons. The source plane symmetry entered through kinetic boundary conditions on the transmitted ions, and on the specularly reflected electrons. The analysis was valid for arbitrary values of the phase velocity u of the wave compared with either the electron (v_e) or ion (v_i) thermal velocity. However, detailed considerations were supplied only in the ion-acoustic limit ($v_i \ll u \ll v_e$).

The present work is limited, at the outset, to the ion-acoustic regime since fluid motions are used. This approximation has been carefully discussed previously.¹² This paper extends our previous results to three dimensions, however, and therefore also embodies the phenomenon of Fresnel interference from finite sources.

When $u \ll v_e$ and the electron velocity distribution is Maxwellian, the electron density is given by

$$n = N_0 \exp\left(\frac{e\phi(\mathbf{r}, t)}{(kT_e)}\right) \simeq N_0 \left(1 + \frac{e\phi}{kT_e}\right),$$

where $\phi(\mathbf{r}, t) = \phi(\mathbf{r})\exp(i\omega t)$ is the ion-acoustic wave potential, and T_e is the electron temperature. The ions, on the other hand, behave as a cold charged fluid, when $u \gg v_i$. Thus, the perturbed ion density¹² is given by $N_1 = -eN_0 \nabla^2 \phi / M\omega^2$. If we designate the rf charge density¹³ associated with the grid, and any region of the plasma adjacent to the grid where linearized theory fails, by $S(\mathbf{r})\exp(i\omega t)$ then Poisson's equation tells us that

$$\begin{aligned} \nabla^2 \phi(\mathbf{r}) &= 4\pi e(n_1 - N_1) - 4\pi S(\mathbf{r}) \\ &= \left(\frac{4\pi n_0 e^2}{kT_e}\right) \phi(\mathbf{r}) + \left(\frac{4\pi n_0 e^2}{M\omega^2}\right) \nabla^2 \phi(\mathbf{r}) - 4\pi S(\mathbf{r}); \quad (2) \end{aligned}$$

BLOCK DIAGRAM OF ELECTRONICS AND PLASMA BOTTLE

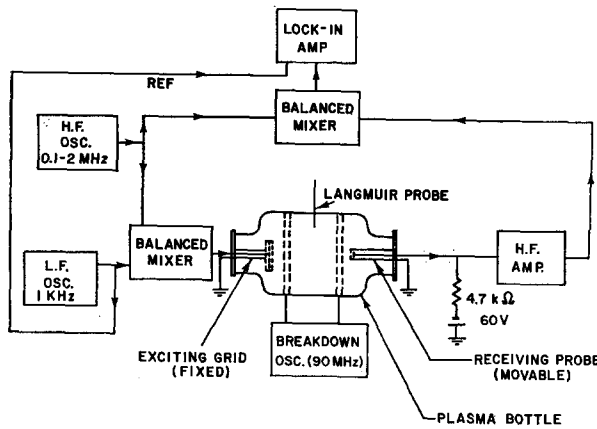


FIG. 1. Experimental arrangement.

or

$$\nabla^2 \phi(\mathbf{r}) + \kappa^2 \phi(\mathbf{r}) = 4\pi S'(\mathbf{r}), \quad (3)$$

where $\kappa^2 = k_D^2(\omega_{pi}^2/\omega^2 - 1)^{-1}$ with $k_D^2 = 4\pi n_0 e^2/kT_e$, and $\omega_{pi}^2 = 4\pi n_0 e^2/M$, and where $S'(\mathbf{r}) = (\omega_{pi}^2/\omega^2 - 1)^{-1} S(\mathbf{r})$. The solution of Eq. (3) for a localized source in an infinite medium that satisfies an *outgoing* wave condition is

$$\phi(\mathbf{r}) = - \int d^3 r' \frac{\exp(i\kappa R)}{R} S'(\mathbf{r}'), \quad (4)$$

where $R = |\mathbf{r} - \mathbf{r}'|$. Fresnel interference enters since the phase of the integrand varies as \mathbf{r}' moves over the source.

Suppose $S'(\mathbf{r}')$ is effectively a thin disk centered on the plane $z = 0$, of thickness δ .¹⁴ Then, since $R^2 = (z - z')^2 + (y - y')^2 + (x - x')^2$, we can write

$$\frac{\exp(i\kappa R)}{R} = \left(1 - z' \frac{\partial}{\partial z} + \frac{z'^2}{2} \frac{\partial^2}{\partial z^2} - \frac{z'^3}{6} \frac{\partial^3}{\partial z^3} + \dots\right) \frac{\exp(i\kappa R_0)}{R_0},$$

where $R_0^2 \equiv z^2 + (y - y')^2 + (x - x')^2$. The above series converges when $|z'(1 + \kappa^2 R_0^2)^{1/2}/R_0| < 1$ which is, in fact, in the range of interest, since $R_0(1 + \kappa^2 R_0^2)^{-1/2} \geq z \gg z' \leq \delta$. Thus, a good approximation to Eq. (4) is

$$\phi(\mathbf{r}) = - \int dx' \int dy' \int_{-\delta}^{\delta} dz' S'(\mathbf{r}') \cdot \left(1 - z' \frac{\partial}{\partial z}\right) \frac{\exp(i\kappa R_0)}{R_0} + O\left(\frac{\delta}{R_0}\right)^2. \quad (5)$$

We can see directly that, for a source function $S'(\mathbf{r}')$ which is *odd* in z' , the plasma response is lower by a factor of at least δ/R_0 , as compared with a source of the same strength *even* in z' . The present result follows either when $S'(\mathbf{r}')$ falls off in z' with a

scale length $\delta \ll R_0$, or when $\kappa R_0 \gg 1$, so that the rapid oscillations of the $\exp(i\kappa R_0)$ factor reduce the magnitude of the integral.

The physical origin of the poor coupling for $S'(\mathbf{r}')$ odd in z' emerges when one considers the constituents of the rf plasma current density $\mathbf{J}(\mathbf{r})$ accompanying an ion-acoustic wave. For an electrostatic disturbance, the electric field is given by $\mathbf{E}(\mathbf{r}) = 4\pi\mathbf{J}(\mathbf{r})/i\omega$, and must join smoothly to the source field, whose divergence is $S(\mathbf{r})$. Finite $S(z = 0)$ (odd electric field symmetry) implies $\mathbf{J}(z = 0) = 0$; vanishing $S(z = 0)$ (even electric field symmetry) implies $\mathbf{J}(z = 0) \neq 0$. However, an ion-acoustic wave is characterized by nearly perfect local charge equality (adiabatic electron gas assumption), so that $\mathbf{J}(\mathbf{r}) \simeq 0$ everywhere, with electron current $\mathbf{J}_e(\mathbf{r})$ very nearly equal and opposite to ion current $\mathbf{J}_i(\mathbf{r})$. Near the negatively biased grid $\mathbf{J}_e = 0$, since electrons are excluded altogether, so that \mathbf{J}_i must also be zero. *Ergo*, the source electric field must vanish (odd \mathbf{E} field symmetry) in order to couple significantly to an ion-acoustic wave.

Out of the ion-acoustic regime ($v_i \ll u \ll v_e$), the solution would still be of the form of Eq. (4), i.e.,

$$\phi(\mathbf{r}) = - \int d^3 r' G(\kappa, \mathbf{r}, \mathbf{r}') S'(\mathbf{r}')$$

with a Green's function different from $\exp(i\kappa R)/R$ which is invariant under the transformation $z \rightarrow -z$, $z' \rightarrow -z'$. As long as the restrictions listed above as to the ordering of δ , κ^{-1} , and R_0 were maintained, one would expect the same symmetry considerations to apply.

It is to be noted, then, that the dipole excitation taken as the source in a calculation of this problem by Gould,¹⁵ albeit well out of the ion-acoustic regime, might be expected *a priori* to yield weak coupling (as he found) since the dipole source has "bad" symmetry.

III. EXPERIMENT

The experiments to be described were carried out using the apparatus shown in Fig. 1, which has been previously described,³ except for modifications to the transmitting electrode, which will be discussed. An rf generated steady-state plasma (free of dc magnetic fields) was produced in low-pressure argon with the following characteristics:

$$n_e \simeq n_i = 5 \times 10^8 \text{ cm}^{-3}, \quad \omega_{pi}/2\pi = 740 \text{ kHz},$$

$$T_e = 0.80 \text{ eV}, \quad \lambda_D = 3 \times 10^{-2} \text{ cm}, \quad p_{\text{argon}} = 3 \text{ mTorr}.$$

In earlier experiments, the wave transducer was constructed of three parallel grids. This configuration

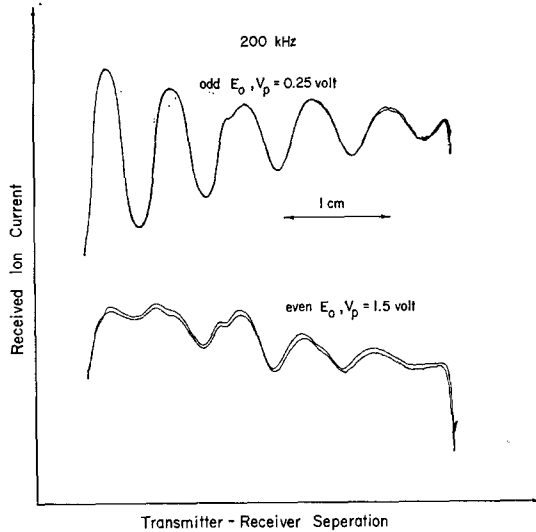


FIG. 2. Received ion current vs transmitter separation. $f = 200$ kHz. Upper trace is for odd symmetry source electric field, bottom trace for even. Two runs are superimposed.

only permitted source fields E_0 with odd symmetry. In the present work, a four-grid system was used. The grids were 4.5 cm in diameter and were positioned parallel to one another with an intergrid spacing of 7.5×10^{-2} cm. The grid mesh size was 2.5×10^{-2} cm. Due to ion acceleration in ~ 6 V sheaths surrounding the entire grid structure, the shielding length in the intergrid space was about 0.3 cm. This made it possible to neglect space-charge shielding in the transducer. The two outer grids were grounded, and the two inner ones could be driven in-phase (odd E_0 field symmetry) or 180° out-of-phase (even E_0 -field symmetry). The receiving electrode consisted of an ion current collector shielded by a grounded grid, which reflected electrons, again in a ~ 6 V negative sheath. The receiver could be moved axially in the discharge over an excursion of 5 cm, beginning about 1 cm in front of the transmitter. Raw data are shown in Figs. 2, 3, and 4 for three different generator frequencies. For odd-symmetric E_0 fields, all cases show wavelike behavior with wavelength within 5% of the ion-acoustic value $2\pi(kT_e/M_i)^{1/2}\omega^{-1}$. The wave decays show clear evidence of Fresnel interference since these measurements were taken in the near zone of the disk-shaped transducer. The waves are more heavily damped as the driving frequency approaches the ion plasma frequency.

For even-symmetric E_0 fields, a completely different situation prevails. First of all, however, a word is needed on the relative perturbation engendered by the two grid system. We have assumed that the grid fields provide velocity modulation to the ions

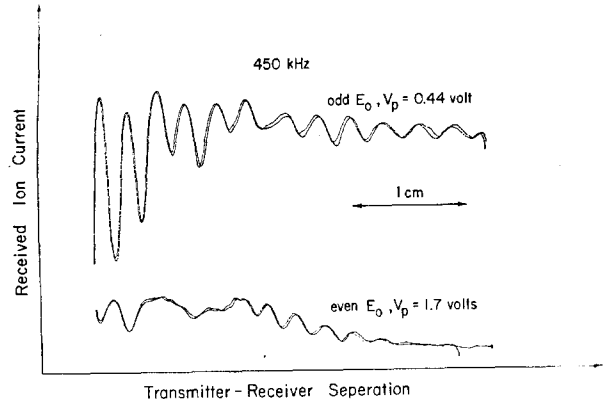


FIG. 3. Same as Fig. 2, except $f = 450$ kHz.

as they drift through the grids (nearly normal to the grids on account of the high sheath accelerations). The perturbed part of the ion velocity distribution function at the source (assuming the axial source dimension to be small compared with any other length in the problem) is obtained by integrating the collisionless Boltzmann equation along unperturbed (straight-line) orbits. For the cases of odd and even symmetry, respectively, one finds

$$f_i(v > 0, t, z \simeq 0) = \frac{eE_0}{m} \frac{\partial F_i}{\partial v} \cdot \left(\int_{t-(3l/2v)}^{t-(l/2v)} dt' \sin \omega t' - \int_{t+(l/2v)}^{t+(3l/2v)} dt' \sin \omega t' \right) \quad (\text{odd } E_0 \text{ field}), \quad (6)$$

$$f_i(v > 0, t, z \simeq 0) = \frac{eE_0}{m} \frac{\partial F_i}{\partial v} \left(\int_{t-(3l/2v)}^{t-(l/2v)} dt' \sin \omega t' - 2 \int_{t-(l/2v)}^{t+(l/2v)} dt' \sin \omega t' + \int_{t+(l/2v)}^{t+(3l/2v)} dt' \sin \omega t' \right) \quad (\text{even } E_0 \text{ field}). \quad (7)$$

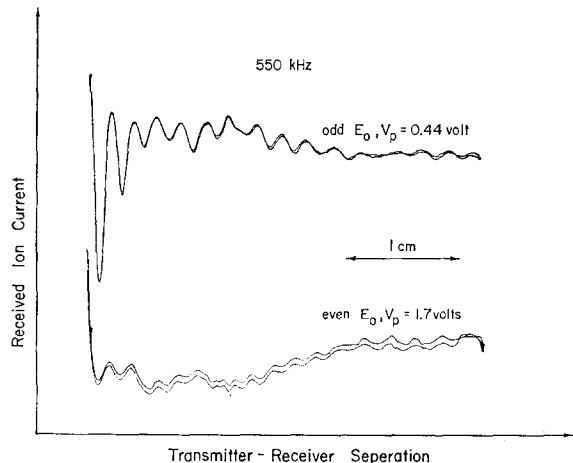


FIG. 4. Same as Fig. 2, except $f = 550$ kHz.

For the odd E_0 case, we have taken the field to be E_0 for $-3l/2 < z < -l/2$, $-E_0$ for $l/2 < z < 3l/2$, and zero elsewhere; for the even E_0 case, it is E_0 for $-3l/2 < z < -l/2$, $-2E_0$ for $-l/2 < z < l/2$, and E_0 for $l/2 < z < 3l/2$. The ratio of Eq. (6) to Eq. (7) is $2v/\omega l$, where v is the unperturbed ion velocity between the grids (corresponding to ~ 6 eV in these experiments), and where l is the spacing between adjacent grids. For the three cases shown in Figs. 2, 3, and 4, the ratios $2v/\omega l$ are 12, 5.5, and 4.5. With the applied rf voltages shown, the actual perturbation ratios were 3.1, 1.4, and 1.15, respectively. Were excitation field symmetry not a crucial point in the physics of ion-acoustic wave generation, the second trace in each of the figures would presumably have been identical to the first, except smaller by the factors 3.1, 1.4, and 1.15.

Evident in the three traces with even symmetric E_0 is a wavelike signal some distance away from the excitation grid. One possible explanation for this signal could be that it is a true ion-acoustic wave driven by free-streaming ions which have been perturbed at the source, but compensated for by electrons on skew trajectories which have bypassed the source and thus are not hindered by a boundary condition at $z = 0$, which is inconsistent with an ion-acoustic wave. Such an effect would not have been predicted by one-dimensional theory.

IV. DISCUSSION

Evidence has been presented to support the theoretical prediction that ion-acoustic wave generation depends critically on the spatial symmetry of the source fields. Ion-acoustic waves only couple to source electric fields which vanish at the source (odd E_0 field symmetry), in the limit $\omega < \omega_{pi}$ with $T_e > T_i$. The opposite symmetry should only provide a ballistic signal due to free-streaming source ions, although this has not been clearly identified in the present experiments since the smallest receiver-transmitter separation available was 1 cm.

Grid excitation in higher density plasmas, where density modulation can also be introduced by the

grid system, will produce a source field with some linear combination of even and odd symmetry. Pure density modulation corresponds to even E_0 field symmetry and should, therefore, not produce an ion acoustic wave.

Study of signal decays from excitations whose spatial symmetry properties are unknown could thus lead to errors of interpretation.

ACKNOWLEDGMENTS

The authors are indebted to D. E. Baldwin for numerous helpful conversations.

This research was supported in part by the National Science Foundation and Atomic Energy Commission Contract Number AT-30-1-3943.

¹ J. L. Hirshfield and J. H. Jacob, *Phys. Fluids* 11, 411 (1968).

² L. D. Landau, *J. Phys. USSR* 10, 24 (1946).

³ J. H. Jacob and J. L. Hirshfield, in *Proceedings of Third European Conference on Plasma Physics* (Wotters-Noordhoff, Groningen, The Netherlands, 1969), p. 97; J. L. Hirshfield, J. H. Jacob, and D. E. Baldwin, *Phys. Fluids* 14, 615 (1971).

⁴ H. J. Doucet and D. Gresillon, in *Proceedings of the International Conference on Physics of Quiescent Plasmas*, Paris (Ecole Polytechnique, Paris, 1969), Vol. I, p. 75.

⁵ D. N. Quang, in *Proceedings of the International Conference on Physics of Quiescent Plasmas*, Paris (Ecole Polytechnique, Paris, 1969), Vol. I, p. 69.

⁶ S. A. Andersen, V. O. Jensen, and P. Nielsen, *Phys. Fluids* 12, 2694 (1969).

⁷ H. Ikezi, N. Takahashi, and K. Nishikawa, *Phys. Fluids* 12, 853 (1969).

⁸ G. Joyce, K. Lonngren, I. Alexeff, and W. D. Jones, *Phys. Fluids* 12, 2592 (1969).

⁹ We take the term "grid" (in our idealized theoretical discussion) to consist of an infinite conducting plane $z = 0$, that transmits ions but is biased so as to reflect electrons (and thus to not draw a heavy plasma current), and covered by dc and rf sheaths which limit the source fields to within a few Debye lengths of the plane. As will subsequently be seen, the infinitesimally thin grid plane may be structured of layers, so that source fields with either $\mathbf{E}(z) = -\mathbf{E}(-z) \rightarrow \mathbf{E}(0) = 0$ (odd symmetry), or $\mathbf{E}(z) = \mathbf{E}(-z) \rightarrow \mathbf{E}(0) \neq 0$ (even symmetry) may be excited.

¹⁰ J. H. Jacob and J. L. Hirshfield, in *Proceedings of the International Conference on Physics of Quiescent Plasmas*, Paris (Ecole Polytechnique, Paris, 1969), Vol. I, p. 93.

¹¹ This approach is that used, for example, in Ref. 8.

¹² I. B. Bernstein and S. K. Trehan, *Nucl. Fusion* 1, 3 (1960).

¹³ We imagine that $\int d^3r S(\mathbf{r}) = 0$, i.e., the source carries no net charge, so that a "capacitive" source field extending to $\pm \infty$ is not present.

¹⁴ For an externally driven grid, δ will be of the order of the ac sheath thickness, usually several Debye lengths k_D^{-1} .

¹⁵ R. W. Gould, *Phys. Rev.* 136, A991 (1964).