

Inverse Pinch Effect*

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In the conventional pinch effect an ionized gas is enclosed by a conducting cylinder, and a sufficiently large current passing through the gas and returning along the inner wall of the cylinder produces a magnetic field which compresses the gas into an axial filament. A device is described which produces an inverse pinch effect. Here the above conducting cylinder is replaced by an axial rod surrounded by the ionized gas. When a current passes through the gas and returns along the rod the magnetic field pushes the plasma outward, leaving a cylindrical vacuum region behind. The velocity and thickness of the expanding plasma front have been studied optically and by means of magnetic probes. Except at the highest gas densities, the velocity is in good agreement with the "snow plow" theory of Rosenbluth, and the thickness of the front is reasonably consistent with the "snow plow" model. At the higher densities it appears that diffusion of magnetic field into the plasma is significant. The advantages of the inverse pinch effect in studying plasma behavior and the idea of a magnetically stabilized inverse pinch are discussed.

INTRODUCTION

WHEN an electron current is passed through an ionized gas, the moving electrons exert a mutually attractive force.¹ In a discharge tube of the kind shown in Fig. 1 (top), a dynamic constriction will occur when a sufficiently high current is passed. This so-called "pinch effect" has been studied at many laboratories, especially with a view to constricting very hot plasmas.^{2,3}

One can analyze the pinch effect in terms of the interaction of moving electrons, but it is perhaps simpler to speak directly in terms of an azimuthal or pinch magnetic field,

$$B_{\theta} = 2I/r, \quad (1)$$

and the magnetic pressure gradient,

$$\frac{\partial P_m}{\partial r} = -\frac{B_{\theta}}{4\pi r} \frac{\partial}{\partial r}(rB_{\theta}) = \frac{\partial}{\partial t}(\rho v), \quad (2)$$

by means of which the plasma of the discharge is accelerated. The quantity I is the axial current flowing within a circle of radius r , ρ is the plasma mass density, and v is the radial velocity.

It is easy to recognize on Fig. 1 (top) that B_{θ} exerts an outward pressure on the current return conductor, as well as an inward pressure on the discharge. If the usual pinch geometry is now inverted (Fig. 1, bottom) so that the return conductor is on the inside, then the outward pressure will be on the discharge column. As a result, plasma

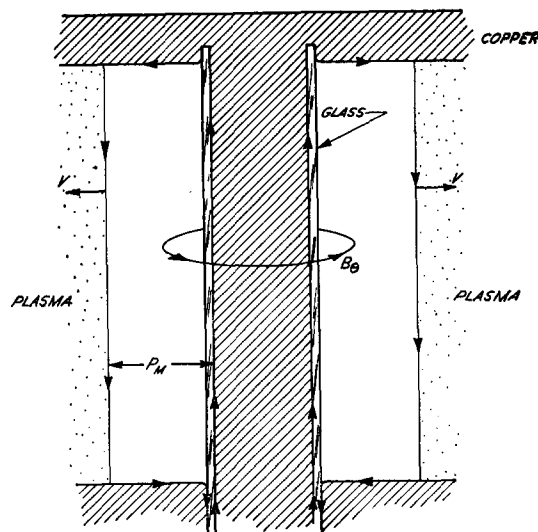
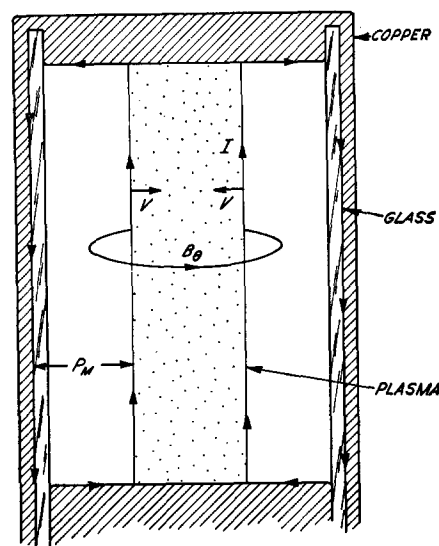


FIG. 1. The ordinary pinch effect (top) and the inverse pinch effect (bottom).

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¹ W. H. Bennett, *Phys. Rev.* **45**, 890 (1934).

² L. A. Artsimovich *et al.*, *Atomnaya Energ.* **3**, 84 (1956).

³ O. A. Anderson *et al.*, *Phys. Rev.* **110**, 1375 (1958).

is accelerated outward, and a cylindrical vacuum is left behind. The front of this vacuum column moves according to essentially the same dynamic laws as the front of the plasma column in the ordinary pinch effect.

Because the plasma of the inverse pinch is propelled toward infinity rather than toward the origin, this geometry taken by itself has little usefulness in creating a hot confined plasma. The inverse pinch experiment, however, has certain advantages from the academic point of view. Precisely because the plasma motion is not convergent, a confusing element is absent from the pinch dynamics. Furthermore, the possibility of hydromagnetic surface instabilities is precluded, because the driving magnetic field curves away from the plasma. The motion and structure of the current-carrying front can be investigated with a minimum of perturbation by means of magnetic probes pointing radially inward. Such probes are not reached by the discharge until the instant of actual measurement. The optical accessibility of the plasma front is essentially 100%, which is important for photographic and spectroscopic work. The experimental function of the inverse pinch may perhaps be characterized with the remark that it constitutes an ordinary pinch turned inside out for more efficient inspection.

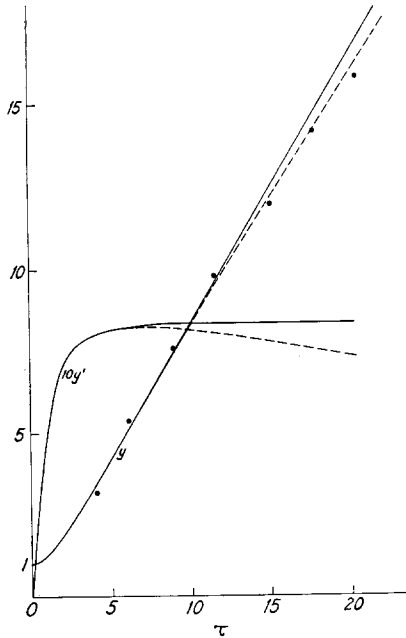


FIG. 2. Theoretical dependence of y and $dy/d\tau$ on τ (solid lines), corrected for nonlinearity of the current, with $\omega/k = 0.017$ (dotted lines). Some experimental points at 1000 μD_2 , 5 kv, are given.

THEORY OF THE MOTION

In an earlier paper on the pinch effect, the validity of Marshall Rosenbluth's "snow plow" theory of motion was considered.³ The agreement of predicted and observed pinch contraction times was shown to be satisfactory. The theory can now be tested further by applying its basic equation to the inverse pinch effect.

The "snow plow" theory assumes that a driving current I flows on the pinch surface. As the surface moves radially outward (or, in the case of the ordinary pinch, radially inward), gas particles are swept into a thin surface sheath and ionized. The accumulated mass per unit length of pinch surface is

$$M = \rho\pi(r^2 - R^2), \quad (3)$$

where ρ is the mass density of the gas, r is the radius of the surface, and R is the starting radius.

The equation of motion is

$$\frac{1}{2\pi r} \frac{d}{dt} \left(M \frac{dr}{dt} \right) = \frac{B_\theta^2}{8\pi} = \frac{I^2}{2\pi r^2}. \quad (4)$$

If one assumes for I the simple form $I = I_0\omega t$, and defines the dimensionless variables

$$y = r/R, \quad \tau = kt, \quad k = \left(\frac{I_0^2 \omega^2}{\pi \rho R^4} \right)^{1/2}, \quad (5)$$

Eq. (4) becomes

$$\frac{d}{d\tau} (1 - y^2) \frac{dy}{d\tau} = -\frac{\tau^2}{y}. \quad (6)$$

The boundary conditions are $dy/d\tau = 0$ and $y = 1$ at $\tau = 0$. The appropriate solution to Eq. (6) is given by the solid curves in Fig. 2. Evidently y assumes a nearly linear dependence on τ , as soon as y exceeds 3. It is easy to verify that $y = 2^{-1/2}\tau$ gives an analytic solution of this form when $y \gg 1$.

In general, the expression $I = I_0 \sin \omega t$ will represent the pinch current more accurately than the simple linear expression. It is easy to calculate the corresponding corrections in y and $dy/d\tau$, in the limit $\omega^2/k^2 \ll 1$, $y \gg 1$. In this case Eq. (6) becomes

$$\frac{d}{d\tau} y^2 \frac{dy}{d\tau} = \frac{\tau^2}{y} \left(1 - \frac{\omega^2 \tau^2}{6k^2} \right). \quad (7)$$

Let y_0 be the solution of Eq. (6); then we can write $y = y_0 - (\omega^2/k^2)y_1$. We know also that for $y \gg 1$, $y_0 = 2^{-1/2}\tau$. Substituting these expressions into Eq. (7), we obtain

$$2y_1 + \frac{d^2}{d\tau^2} (y_1 \tau^2) = \frac{2}{3} \tau^3. \quad (8)$$

A solution that satisfies the boundary condition $y_1, dy_1/d\tau = 0$ at $\tau = 0$ is given by $y_1 = (2^{1/3}/66)\tau^3$. Accordingly we have

$$y = y_0 - \frac{2^{1/3} \omega^2}{66 k^2} \tau^3, \quad (9)$$

$$\frac{dy}{d\tau} = \frac{dy_0}{d\tau} - \frac{2^{1/3} \omega^2}{22 k^2} \tau^2. \quad (10)$$

In the experiments, the factor ω/k is of the order of $1/10$ to $1/50$. Accordingly, the above derivation can be expected to yield accurate theoretical values in the region $\tau < 10$, where most of the velocity measurements have been made. The solutions (9) and (10) are shown by the dashed curves in Fig. 2.

EXPERIMENTS WITH A SLOW CONDENSER BANK

Because of the optical accessibility of the inverse pinch, there was first of all an interest in obtaining photographs of the expanding shock front. Experience with other devices made it doubtful that enough light for photographs would be produced at large Mach numbers. Thus an experiment was set up which utilized a slow-discharge 100- μ f condenser bank. The important dimensions of the tube were as follows: central rod, $\frac{1}{4}$ -in. diameter; inner quartz tube, $\frac{3}{8}$ -in. o.d.; outer glass, 8-in. i.d., electrode spacing, 3 in. The total initial inductance of the bank, spark gap, and device was 4×10^{-8} h; maximum total inductance was 9×10^{-8} h. The time for the system to reach peak current was about 4 μ sec.

A set of photographs made with a Kerr cell (open time 10^{-7} sec) is presented in Fig. 3. The photographs were taken with a bank voltage of 5 kv; the tube was filled with argon at an initial pressure of 700 μ . It is interesting to note that the front remains straight at all times, indicating that the electrodes cause little perturbation in the pinch dynamics. By means of such photographs a rough determination can be made of the pinch diameter as a function of time. The results are consistent with Eq. (6). To get more accurate measurements, another technique, to be described, was used.

The spectrum of the light emitted by the discharge when the tube contained deuterium at 1000- μ initial pressure was examined visually and photographically. The strongest lines were the Balmer lines of deuterium. On a plate having $D\alpha$, heavily exposed $D\delta$ was clearly visible and $D\epsilon$ was just visible. The impurity lines were faint and included

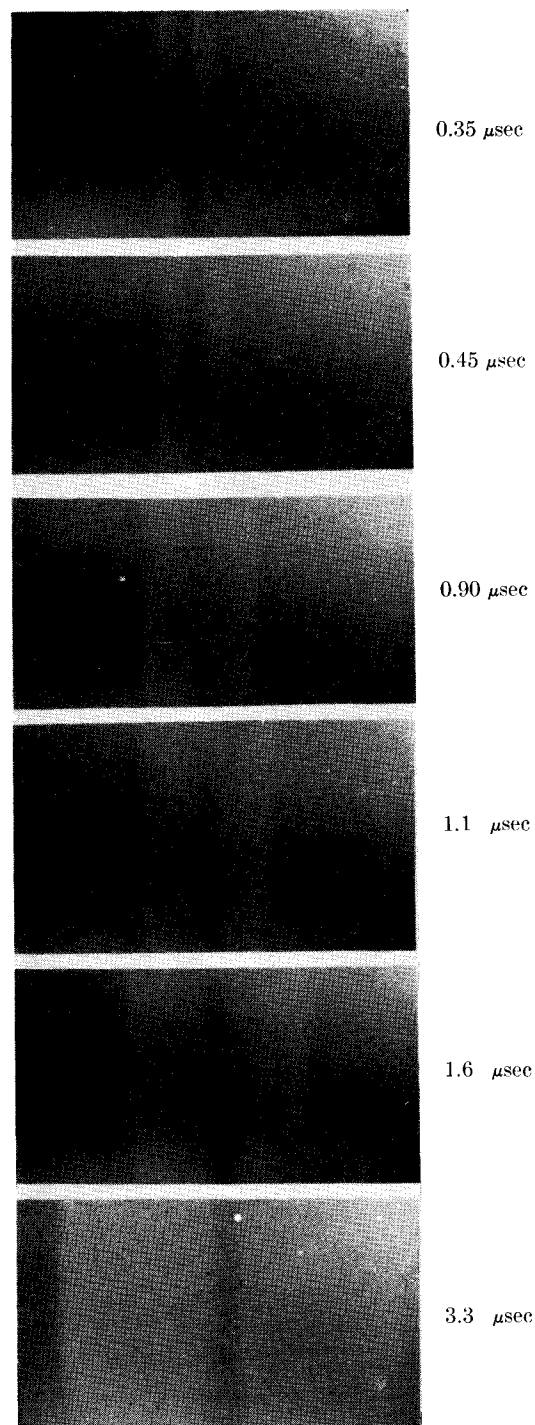


FIG. 3. Expansion of the inverse pinch in 100 μ of argon, with 5-kv driving voltage.

Cu and Al from the metal electrode plates, O and Si from the quartz and glass insulators, and C from pump oil and O-rings.

A $\frac{1}{2}$ -m-grating monochromator with a photomultiplier cell at the exit slit was used for time-resolved observations of the line $D\beta$. The chief

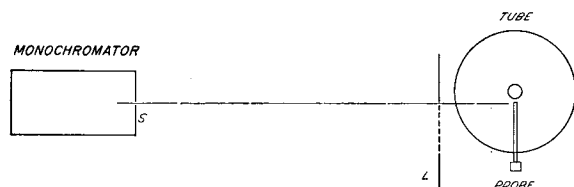


FIG. 4. Geometry employed for magnetic and optical measurements of the velocity.

interest was in determining the velocity of the expanding luminous wave front, and for this purpose the arrangement shown in Fig. 4 was used. A screen L containing a number of parallel slits of 1-mm width was placed in front of the discharge tube. The entrance slit S of the monochromator was located at a distance of $1\frac{1}{2}$ m. The geometry

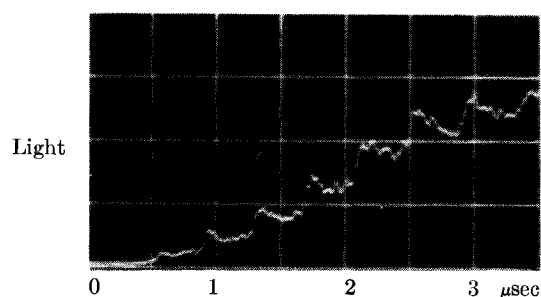


FIG. 5. Photomultiplier signal during passage of the inverse pinch front by the slit system. (1000 μ D₂, 5 kv).

was such that a line through the slit S and the first slit of L just grazed the insulating quartz tube surrounding the central copper rod. Lines through the remaining slits in L were tangent to larger cylinders, differing in their radii by 1-cm increments. In this arrangement due consideration was given

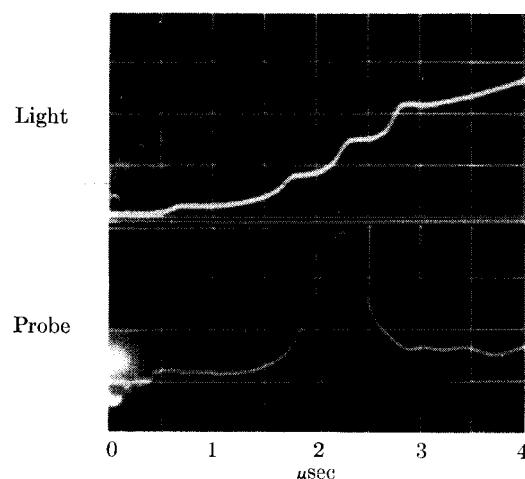


FIG. 6. Correlation of optical and magnetic probe signals ($\partial B_\theta/\partial t$) at 200 μ D₂, 5 kv.

TABLE I. Comparison between observed and calculated velocities for various condenser voltages and deuterium pressures.

kv	D ₂ (μ)	v_{calc} (cm/sec)	v_{obs} (cm/sec)
9.1	200	5.0×10^6	4.7×10^6
6.0	200	4.0×10^6	3.5×10^6
6.0	500	3.2×10^6	3.0×10^6
5.9	1000	2.7×10^6	2.7×10^6
5.0	1000	2.5×10^6	2.5×10^6

to refraction at the glass and plastic cylinders forming the outer wall of the discharge tube.

As the luminous wave front spread outward in the discharge tube, the light appeared suddenly in the successive slits of L , giving a photomultiplier response having the appearance of Fig. 5. The dependence of discharge radius on time obtained in this way has been plotted in Fig. 2 for deuterium at 1000- μ initial pressure and 5-kv bank voltage. There is good agreement with the theoretical curve as corrected to the first order for nonlinearity in the current wave form. Table I gives a comparison between some observed and calculated velocities. By v_{obs} is meant the velocity observed near $\tau = 10$, and v_{calc} is the corrected theoretical velocity of Eq. (10).

In order to relate the optical data to the electromagnetics of the pinch effect, a magnetic probe was used. This consisted of a five-turn loop of 1.5-mm diam, enclosed in a 4-mm-o.d. quartz tube. The general arrangement is indicated in Fig. 4. The probe was adjusted radially until the coil was aligned optically with one of the slits in the arrangement just described. The probe signal ($\partial B_\theta/\partial t$) was compared with the light pulse from the slit. In the typical exposure shown in Fig. 6 the slits at 2, 3, and 4 cm were open and the probe was located at the 3-cm slit. The peak probe signal, i.e., the maximum rate of change of field, very nearly coincides with the jump in light output as the pinch front passes the slit. The duration of the probe signal (at half-amplitude) is 0.2 μ sec, which at the measured shock velocity of 4.6 cm/ μ sec implies a spatial extent of 0.9 cm. The sheet-current hypothesis of the Rosenbluth theory therefore is moderately well justified in this case. The sharp localization of deuterium light confirms the "snow plow" concept that neutral gas is swept up and ionized in the pinch front.

EXPERIMENTS WITH A FAST CONDENSER BANK

The inverse pinch effect can be obtained under conditions of low gas density and small tube size

for which an ordinary pinch would not form properly. In the inverse pinch geometry it is therefore possible to produce high-velocity plasma motion with a very limited total expenditure of energy.

Some experiments with deuterium in the 10^7 cm/sec velocity range have been done in a tube substantially identical with that of the preceding section. A 0.25- μ f source capacitor was used, charged to 50 kv or 310 j. This capacitor could be discharged by means of a spark gap through a 4:1 air-core step-down transformer into the pinch tube. The design of low inductance transformers has been discussed elsewhere.⁴ Typical currents of 120-135 ka were generated, rising in 0.5 μ sec.

The pinch tube was pre-ionized by means of an azimuthal winding producing a 3000-gauss, 70-kc axial field oscillation. At the third zero of the axial field, the main pinch discharge was initiated. Satisfactory operation could be obtained down to pressures of 25 μ in deuterium.

A fast-discharge experiment lends itself well to the study of the magnetic-field configuration of the pinch. For dynamic times of the order of 0.5 μ sec, the characteristic diffusion distance of magnetic field in 10-ev plasma is only about 3 mm. In an experiment where this moderate temperature is likely to be reached, one may hope to distinguish rather clearly between dynamic and diffusive phenomena.

By integrating the signals of a magnetic probe like that of Fig. 4, the traces of Fig. 7 were obtained. At low initial densities [Fig. 7, (a)-(p)] there is a sharp magnetic front somewhat like that assumed in the Rosenbluth theory. By way of contrast, one notes that at high densities [Fig. 7, (q)-(x)] the magnetic front is concave, which is suggestive of diffusion rather than of a "snow plow" process. The mean velocity between the points $y = 1.7$ and $y = 4.8$ for 25- μ initial density is measured to be 1.15×10^7 cm/sec, in good agreement with the theoretically predicted value 1.20×10^7 cm/sec. Over the same space interval, the mean velocity of the most advanced part of the signal at 1000 μ is also found to be about 10^7 cm/sec, which is inconsistent with the predicted velocity of 5.0×10^6 cm/sec. Presumably the high rate of propagation of the magnetic field in this case is due to its diffusion through relatively cold plasma. One notes, however, that at 1000 μ the point of maximum $\partial B_\theta / \partial t$ has a mean velocity of about 5×10^6 cm/sec, in agreement with the calculated pinch velocity.

⁴ Furth, Levine, and Waniek, Rev. Sci. Instr. 28, 949 (1957).

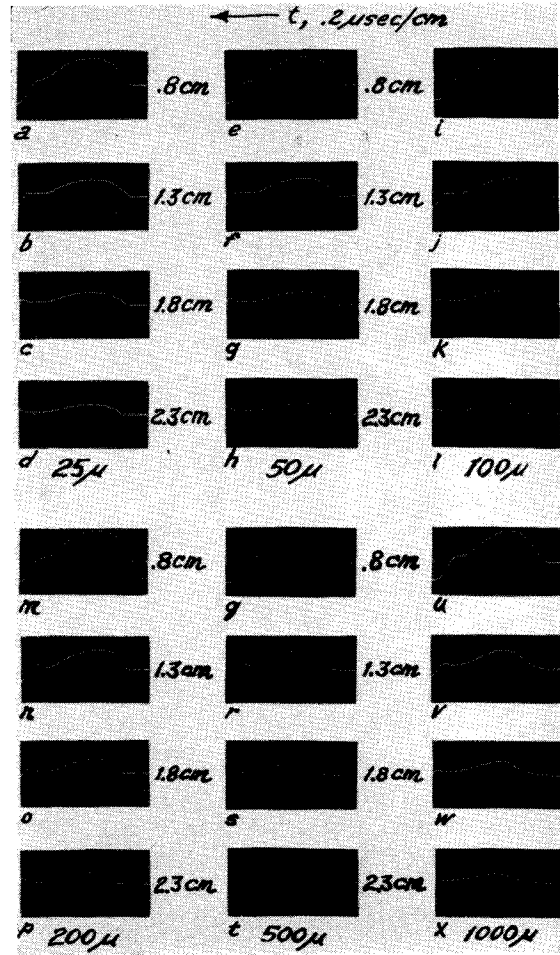


FIG. 7. Integrated magnetic probe (B_θ) signals at various densities, 19 kilogauss/division. Current is initiated at 0.17 μ sec.

Spatial distributions of rB_θ at various times are given in Fig. 8. It is clear that the magnetic pressure does not follow a step function in space even in the favorable case of 25- μ initial density. From the pressure balance requirement, it therefore follows that plasma is being accelerated not merely at the magnetic front but throughout the "vacuum" region behind the front. The time-scale is far too short to permit this broad effect to be explained in terms of plasma diffusion. One concludes that small quantities of adsorbed gas are being stripped continually from the surface of the inner insulator tube and accelerated by the magnetic field. Such effects are familiar from other pinch experiments.⁵

One of the most striking features of the spatial magnetic field distribution is the remanence and

⁵ Colgate, Ferguson, and Furth, "External conductivity theory of stabilized pinch formation," University of California Radiation Laboratory Report UCRL-5086, January, (1958).

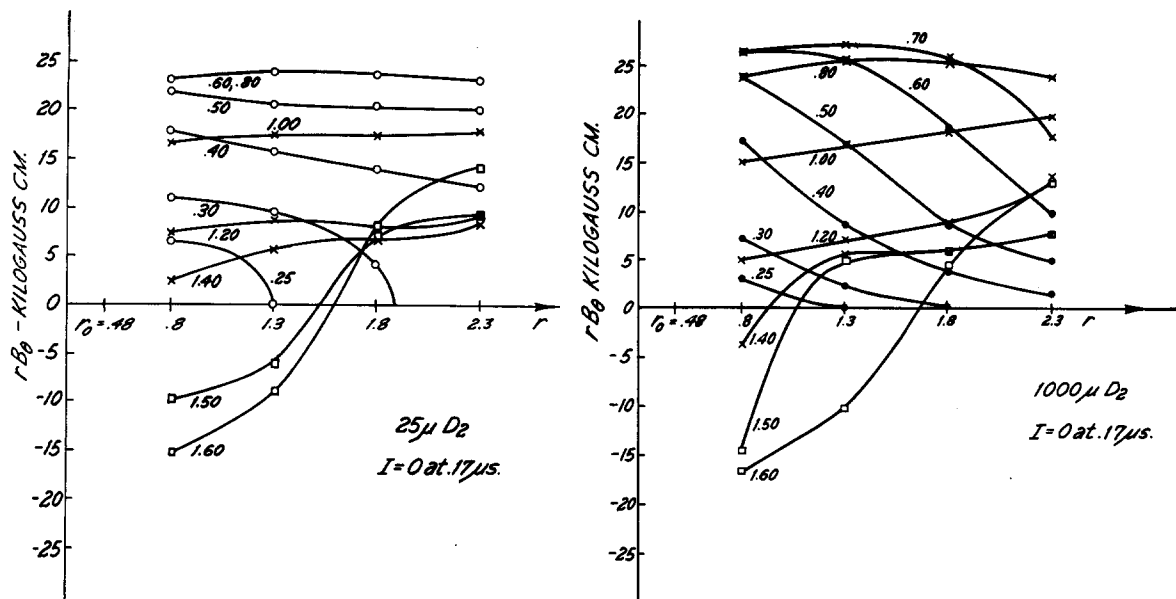


FIG. 8. Spatial distributions of rB_θ , assembled from data of Fig. 7. The time at which each curve holds is given in μsec .

recompression of the initial magnetic field in the plasma when the driving magnetic field has gone through a half-cycle and reversed its direction. Between the regions of opposite field a tubular null-field region is formed, where the current density and plasma pressure are very high. The geometry is in fact just that of the "Triax" or tubular pinch.⁶ Similar configurations have been created by S. A. Colgate with an oscillating axial magnetic field.⁷ It would seem logical that the presence of magnitude field reversal and of a null-field region plays an important role in another axial field experiment recently performed by W. C. Elmore *et al.*⁸ The reported observation of x-ray and neutron emission starting only during the second half-cycle of the current strengthens this supposition.

THE INVERSE STABILIZED PINCH

While the simple dynamic pinch has been studied at many laboratories with a view to the production

of very high plasma temperatures for very short intervals, it is recognized that the so-called "stabilized pinch"⁹ is an even more interesting subject because of its possible relevance to the problem of devising a fusion reactor. In the stabilized-pinch configuration a longitudinal magnetic field is entrapped in the plasma, and under certain conditions a state of gross hydromagnetic stability results.

The analogous inverse pinch configuration is obvious. A longitudinal field created by an external solenoid is entrapped in the plasma region. As the pinch current rises, the longitudinal magnetic field is compressed and an equilibrium configuration results. By studying the properties of this highly stable configuration, one may hope to clarify some of the puzzling fine-scale turbulence phenomena encountered in studies of the ordinary "stabilized pinch."

ACKNOWLEDGMENTS

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⁶ Anderson, Baker, Ise, Kunkel, Pyle, and Stone, *Sheet Pinch Devices, Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva* (1958), Paper No. 2349.

⁷ Colgate, Spoerlein, and Wright, *Shock Heating of Plasma, Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva* (1958), Paper No. 368.

⁸ Elmore, Little, and Quinn, *Phys. Rev. Letters* **1**, 32 (1958).

⁹ Colgate, Ferguson, and Furth, *The Stabilized Pinch, Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva* (1958), Paper No. 369.