Chapter 6: Problems

AP E3200x

6-8. Find the dimensions of the parallelepiped of maximum volume circumscribed by **(a)** a sphere of radius R; **(b)** an ellipsoid with semiaxes a, b, c.

6-10. Find the ratio of the radius R to the height H of a right-circular cylinder of fixed volume V that minimizes the surface area A.

6-14. Find the shortest path between the (x, y, z) points $(0, -1, 0)$ and $(0, 1, 0)$ on the conical surface $z = 1 - \sqrt{x^2 + y^2}$. What is the length of the path? Note: this is the shortest mountain path around a volcano.

6-16. **(a)** What curve on the surface $z = x^{3/2}$ joining the points $(x, y, z) = (0, 0, 0)$ and $(1, 1, 1)$ has the shortest arc length? **(b)** Use a computer to produce a plot showing the surface and the shortest curve on a single plot.

6-18. A particle of mass m is constrained to move under gravity with no friction on the surface $xy = z$. What is the trajectory of the particle if it starts from rest at $(x, y, z) = (1, -1, -1)$ with the z-axis vertical?

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8. (a) $a_1 = b_1 = c_1 = \frac{2}{\sqrt{3}}R$ (b) $a_1 = a\frac{2}{\sqrt{3}}$; $b_1 = b\frac{2}{\sqrt{3}}$; $c_1 = c\frac{2}{\sqrt{3}}$

10. $R = \frac{1}{2}H$

14. length $= 2\sqrt{2}\sin\frac{\pi}{2\sqrt{2}}$

16. $y(x) = \frac{8}{13^{3/2} - 8}\left[\left(1 + \frac{9x}{4}\right)^{3/2} - 1\right]$ and $z = x^{3/2}$

18. $x = -y = \sqrt{-z}$ where $x > 0$, $y < 0$, $z < 0$. Parabolic line.