Chapter 3: Problems

AP E3200x

3-2. Allow the motion in the preceding problem to take place in a resisting medium. After oscillating for 10 s, the maximum amplitude decreases to half the initial value. Calculate **(a)** the damping parameter β, **(b)** the frequency ν_1 (compare with the undamped frequency ν_0), and **(c)** the decrement of the motion.

3-4. Consider a simple harmonic oscillator. Calculate the *time* averages of the kinetic and potential energies over one cycle, and show that these quantities are equal. Why is this a reasonable result? Next calculate the *space* averages of the kinetic and potential energies. Discuss the results.

3-6. Two masses $m_1 = 100$ g and $m_2 = 200$ g slide freely in a horizontal frictionless track and are connected by a spring whose force constant is $k = 0.5$ N/m. Find the frequency of oscillatory motion for this system.

3-12. A simple pendulum consists of a mass m suspended from a fixed point by a weightless, extensionless rod of length l. Obtain the equation of motion and, in the approximation that $\sin\theta \cong \theta$, show that the natural frequency is $\omega_0 = \sqrt{g/l}$, where g is the gravitational field strength. Discuss the motion in the event that the motion takes place in a viscous medium with retarding force $2m\sqrt{gl}\,\dot{\theta}$.

3-14. Express the displacement $x(t)$ and the velocity $\dot{x}(t)$ for the overdamped oscillator in terms of hyperbolic functions.

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2. (a) $6.9 \times 10^{-2}\,\mathrm{s}^{-1}$ (b) $\dfrac{10}{2\pi}(1 - 2.40 \times 10^{-5})\,\mathrm{s}^{-1}$ (c) 1.0445

4. $\langle T\rangle = \langle U\rangle = \dfrac{mA^2\omega_0^2}{4}\quad \overline{U} = \dfrac{1}{2}\,\overline{T} = \dfrac{mA^2\omega_0^2}{6}$

6. $2.74\ \mathrm{rad\cdot s^{-1}}$

12. $\ddot{\theta} = -\dfrac{g}{l}\sin\theta$

14.
$$x(t) = (\cosh\beta t - \sinh\beta t)\left[(A_1 + A_2)\cosh\omega_2 t + (A_1 - A_2)\sinh\omega_2 t\right]$$
$$\dot{x}(t) = (\cosh\beta t - \sinh\beta t)\left[(A_1\omega_2 - A_1\beta)(\cosh\omega_2 t + \sinh\omega_2 t) - (A_2\beta + A_2\omega_2)(\cosh\omega_2 t - \sinh\omega_2 t)\right]$$

3-26. Figure 3-B illustrates a mass m_1 driven by a sinusoidal force whose frequency is ω. The mass m_1 is attached to a rigid support by a spring of force constant k and slides on a second mass m_2. The frictional force between m_1 and m_2 is represented by the damping parameter b_1, and the frictional force between m_2 and the support is represented by b_2. Construct the electrical analog of this system and calculate the impedance.

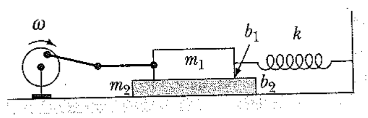

FIGURE 3-B Problem 3-26.

3-28. Obtain the Fourier expansion of the function

$$F(t) = \begin{cases} -1, & -\pi/\omega < t < 0 \\ +1, & 0 < t < \pi/\omega \end{cases}$$

in the interval $-\pi/\omega < t < \pi/\omega$. Take $\omega = 1$ rad/s. In the periodical interval, calculate and plot the sums of the first two terms, the first three terms, and the first four terms to demonstrate the convergence of the series.

3-30. Obtain the Fourier representation of the output of a full-wave rectifier. Plot the first three terms of the expansion and compare with the exact function.

3-32. Obtain the response of a linear oscillator to a step function and to an impulse function (in the limit $\tau \to 0$) for overdamping. Sketch the response functions.

26. $$\frac{R_1\left[R_2(R_2+R_1)+\omega^2L_2^2\right]+i\left[R_1\omega L_2+(\omega L_1-1/\omega C)\left((R_1+R_2)^2+\omega^2L_2^2\right)\right]}{(R_1+R_2)^2+\omega^2L_2^2}$$

28. $$F(t) = \frac{4}{\pi}\sin t + \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\sin 5t + \cdots$$

30. $$F(t) = \frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t - \cdots$$

32. (a) $$x(t) = \frac{H(0)}{\omega_0^2}\left(1 - e^{-\beta t}\cosh \omega_2 t - \frac{\beta e^{-\beta t}}{\omega_2}\sinh \omega_2 t\right)$$

(b) $$x(t) = \frac{b}{\omega_2}e^{-\beta t}\sinh \omega_2 t;\ t > 0$$

3-34. Consider an undamped linear oscillator with a natural frequency $\omega_0 = 0.5$ rad/s and the step function $a = 1$ m/s^2. Calculate and sketch the response function for an impulse forcing function acting for a time $\tau = 2\pi/\omega_0$. Give a physical interpretation of the results.

3-36. Derive an expression for the displacement of a linear oscillator analogous to Equation 3.110 but for the initial conditions $x(t_0) = x_0$ and $\dot{x}(t_0) = \dot{x}_0$.

3-38. Use Green's method to obtain the response of a damped oscillator to a forcing function of the form

$$F(t) = \begin{cases} 0 & t < 0 \\ F_0 e^{-\gamma t} \sin \omega t & t > 0 \end{cases}$$

3-40. An automobile with a mass of 1000 kg, including passengers, settles 1.0 cm closer to the road for every additional 100 kg of passengers. It is driven with a constant horizontal component of speed 20 km/h over a washboard road with sinusoidal bumps. The amplitude and wavelength of the sine curve are 5.0 cm and 20 cm, respectively. The distance between the front and back wheels is 2.4 m. Find the amplitude of oscillation of the automobile, assuming it moves vertically as an undamped driven harmonic oscillator. Neglect the mass of the wheels and springs and assume that the wheels are always in contact with the road.

3-42. An undamped driven harmonic oscillator satisfies the equation of motion $m(d^2x/dt^2 + \omega_0^2 x) = F(t)$. The driving force $F(t) = F_0 \sin(\omega t)$ is switched on at $t = 0$. **(a)** Find $x(t)$ for $t > 0$ for the initial conditions $x = 0$ and $v = 0$ at $t = 0$. **(b)** Find $x(t)$ for $\omega = \omega_0$ by taking the limit $\omega \rightarrow \omega_0$ in your result for part **(a)**. Sketch your result for $x(t)$. *Hint*: In part **(a)** look for a particular solution of the differential equation of the form $x = A \sin(\omega t)$ and determine A. Add the solution of the homogeneous equation to this to obtain the general solution of the inhomogeneous equation.

3-44. Consider a damped harmonic oscillator. After four cycles the amplitude of the oscillator has dropped to $1/e$ of its initial value. Find the ratio of the frequency of the damped oscillator to its natural frequency.

34. $x(t) = \begin{cases} 0 & t < 0 \\ 4[1 - \cos(0.5t)]\ \text{m} & 0 < t < 4\pi \\ 0 & t > 4\pi \end{cases}$

36. $x(t) = e^{-\beta(t-t_0)}\left[x_0 \cos \omega_1(t - t_0) + \left(\dfrac{\dot{x}_0}{\omega_1} + \dfrac{\beta x_0}{\omega_1} + \dfrac{b}{\omega_1}\right) \sin \omega_1(t - t_0)\right]; \quad t > t_0$

38.
$$x(t) = \frac{F_0}{m}\,\frac{\omega}{[(\beta-\gamma)^2 + (\omega+\omega_1)^2][(\beta-\gamma)^2 + (\omega-\omega_1)^2]}$$
$$\times\left[e^{-\gamma t}\left[2(\gamma-\beta)\cos\omega t + \left([\beta-\gamma]^2 + \omega_1^2 - \omega^2\right)\frac{\sin\omega t}{\omega}\right] + e^{-\beta t}\left[2(\beta-\gamma)\cos\omega_1 t + \left([\beta-\gamma]^2 + \omega^2 - \omega_1^2\right)\frac{\sin\omega_1 t}{\omega_1}\right]\right]$$

40. Amplitude $= -0.16$ mm, minus sign indicates spring is compressed

42. (a) $x(t) = \dfrac{F}{m\omega_0}\dfrac{1}{(\omega_0 + \omega)(\omega_0 - \omega)}(\omega_0 \sin \omega t - \omega \sin \omega_0 t)$. (b) $x(t) = \dfrac{F_0 t^3 \omega_0}{6m}$

44. $\dfrac{\omega_1}{\omega_0} = \dfrac{8\pi}{\sqrt{64\pi^2 + 1}}$