Solved Problems from Ch. 2

AP E3200x
Mechanics

2-2. A particle of mass m is constrained to move on the surface of a sphere of radius R by an applied force $\mathbf{F}(\theta, \phi)$. Write the equation of motion.

2-4. A clown is juggling four balls simultaneously. Students use a video tape to determine that it takes the clown 0.9 s to cycle each ball through his hands (including catching, transferring, and throwing) and to be ready to catch the next ball. What is the minimum vertical speed the clown must throw up each ball?

2-6. In the blizzard of '88, a rancher was forced to drop hay bales from an airplane to feed her cattle. The plane flew horizontally at 160 km/hr and dropped the bales from a height of 80 m above the flat range. **(a)** She wanted the bales of hay to land 30 m behind the cattle so as to not hit them. Where should she push the bales out of the airplane? **(b)** To not hit the cattle, what is the largest time error she could make while pushing the bales out of the airplane? Ignore air resistance.

2-14. A projectile is fired with initial speed v_0 at an elevation angle of α up a hill of slope $\beta (\alpha > \beta)$.
(a) How far up the hill will the projectile land?
(b) At what angle α will the range be a maximum?
(c) What is the maximum range?

2-16. A particle is projected with an initial velocity v_0 up a slope that makes an angle α with the horizontal. Assume frictionless motion and find the time required for the particle to return to its starting position. Find the time for $v_0 = 2.4$ m/s and $\alpha = 26°$.

Chapter 2

2. $F_\theta = mR(\ddot{\theta} - \dot{\phi}^2 \sin\theta \cos\theta)$

$F_\phi = mR(2\dot{\theta}\,\dot{\phi} \cos\theta + \ddot{\phi} \sin\theta)$

4. $13.2 \text{ m} \cdot \text{s}^{-1}$

6. (a) 210 m behind (b) can be no more than 0.68 s late

14. (a) $d = \dfrac{2v_0^2 \cos\alpha \sin(\alpha - \beta)}{g \cos^2\beta}$ (b) $\dfrac{\pi}{4} + \dfrac{\beta}{2}$ (c) $d_{\text{max}} = \dfrac{v_0^2}{g(1 + \sin\beta)}$

16. $\dfrac{2v_0}{g \sin\alpha}$

2-18. Include air resistance proportional to the square of the ball's speed in the previous problem. Let the drag coefficient be $c_W = 0.5$, the softball radius be 5 cm and the mass be 200 g. **(a)** Find the initial speed of the softball needed now to clear the fence. **(b)** For this speed, find the initial elevation angle that allows the ball to most easily clear the fence. By how much does the ball now vertically clear the fence?

2-20. A gun fires a projectile of mass 10 kg of the type to which the curves of Figure 2-3 apply. The muzzle velocity is 140 m/s. Through what angle must the barrel be elevated to hit a target on the same horizontal plane as the gun and 1000 m away? Compare the results with those for the case of no retardation.

2-22. The motion of a charged particle in an electromagnetic field can be obtained from the **Lorentz equation*** for the force on a particle in such a field. If the electric field vector is **E** and the magnetic field vector is **B**, the force on a particle of mass m that carries a charge q and has a velocity **v** is given by

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

where we assume that $v \ll c$(speed of light).

(a) If there is no electric field and if the particle enters the magnetic field in a direction perpendicular to the lines of magnetic flux, show that the trajectory is a circle with radius

$$r = \frac{mv}{qB} = \frac{v}{\omega_c}$$

where $\omega_c \equiv qB/m$ is the *cyclotron frequency.*

(b) Choose the z-axis to lie in the direction of **B** and let the plane containing **E** and **B** be the yz–plane. Thus

$$\mathbf{B} = B\mathbf{k}, \quad \mathbf{E} = E_y\mathbf{j} + E_z\mathbf{k}$$

Show that the z component of the motion is given by

$$z(t) = z_0 + \dot{z}_0 t + \frac{qE_z}{2m}t^2$$

where

$$z(0) \equiv z_0 \quad \text{and} \quad \dot{z}(0) \equiv \dot{z}_0$$

(c) Continue the calculation and obtain expressions for $\dot{x}(t)$ and $\dot{y}(t)$. Show that the time averages of these velocity components are

$$\langle \dot{x} \rangle = \frac{E_y}{B}, \quad \langle \dot{y} \rangle = 0$$

(Show that the motion is periodic and then average over one complete period.)

(d) Integrate the velocity equations found in (c) and show (with the initial conditions $x(0) = -A/\omega_c$, $\dot{x}(0) = E_y/B$, $y(0) = 0$, $\dot{y}(0) = A$) *that*

$$x(t) = \frac{-A}{\omega_c}\cos \omega_c t + \frac{E_y}{B}t, \quad y(t) = \frac{A}{\omega_c}\sin \omega_c t$$

These are the parametric equations of a trochoid. Sketch the projection of the trajectory on the xy–plane for the cases (i) $A > |E_y/B|$, (ii) $A < |E_y/B|$, and (iii) $A = |E_y/B|$. (The last case yields a cycloid.)

18. (a) $35.2\ \mathrm{m \cdot s^{-1}}$ (b) $40.7°$; 1.1 m

20. $17.4°$

22. (c) $\dot{x}(t) = C_1 \cos \omega_c t + C_2 \sin \omega_c t + \dfrac{E_y}{B}$

$\dot{y}(t) = -C_1 \sin \omega_c t + C_2 \cos \omega_c t$

2-24. A skier weighing 90 kg starts from rest down a hill inclined at 17°. He skis 100 m down the hill and then coasts for 70 m along level snow until he stops. Find the coefficient of kinetic friction between the skis and the snow. What velocity does the skier have at the bottom of the hill?

2-26. A child slides a block of mass 2 kg along a slick kitchen floor. If the initial speed is 4 m/s and the block hits a spring with spring constant 6 N/m, what is the maximum compression of the spring? What is the result if the block slides across 2 m of a rough floor that has $\mu_k = 0.2$?

2-28. A superball of mass M and a marble of mass m are dropped from a height h with the marble just on top of the superball. A superball has a coefficient of restitution of nearly 1 (i.e., its collision is essentially elastic). Ignore the sizes of the superball and marble. The superball collides with the floor, rebounds, and smacks the marble, which moves back up. How high does the marble go if all the motion is vertical? How high does the superball go?

2-30. A student drops a water-filled balloon from the roof of the tallest building in town trying to hit her roommate on the ground (who is too quick). The first student ducks back but hears the water splash 4.021 s after dropping the balloon. If the speed of sound is 331 m/s, find the height of the building, neglecting air resistance.

2-32. Two blocks of unequal mass are connected by a string over a smooth pulley (Figure 2-B). If the coefficient of kinetic friction is μ_k, what angle θ of the incline allows the masses to move at a constant speed?

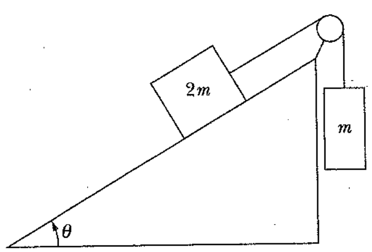

FIGURE 2-B Problem 2-32.

24. $\mu_k = 0.18$; $v_B = 15.6$ m/s

26. 2.3 m; 1.1 m

28. $h_{\text{marble}} = h\left(\dfrac{3 - a}{1 + a}\right)^2$; $h_{\text{superball}} = h\left(\dfrac{1 - 3a}{1 + a}\right)^2$ where $a = m/M$

30. 71 m

32. $\sin\theta_0 = \dfrac{1 \pm \mu_k\sqrt{3 + 4\mu_k^2}}{2(1 + \mu_k^2)}$

2-34. A particle is released from rest ($y = 0$) and falls under the influence of gravity and air resistance. Find the relationship between v and the distance of falling y when the air resistance is equal to **(a)** αv and **(b)** βv^2.

2-36. A gun is located on a bluff of height h overlooking a river valley. If the muzzle velocity is v_0, find the expression for the range as a function of the elevation angle of the gun. Solve numerically for the maximum range out into the valley for a given h and v_0.

2-38. The speed of a particle of mass m varies with the distance x as $v(x) = \alpha x^{-n}$. Assume $v(x = 0) = 0$ at $t = 0$. **(a)** Find the force $F(x)$ responsible. **(b)** Determine $x(t)$ and **(c)** $F(t)$.

2-40. A particle moves in a two-dimensional orbit defined by

$$x(t) = A(2\alpha t - \sin \alpha t)$$
$$y(t) = A(1 - \cos \alpha t)$$

(a) Find the tangential acceleration a_t and normal acceleration a_n as a function of time where the tangential and normal components are taken with respect to the velocity.

(b) Determine at what times in the orbit a_n has a maximum.

2-42. A solid cube of uniform density and sides of b is in equilibrium on top of a cylinder of radius R (Figure 2-C). The planes of four sides of the cube are parallel to the axis of the cylinder. The contact between cube and sphere is perfectly rough. Under what conditions is the equilibrium stable or not stable?

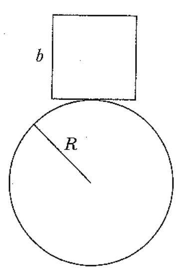

FIGURE 2-C Problem 2-42.

34. (a) $y = -\dfrac{m}{\alpha}\left[v + \dfrac{mg}{\alpha}\ln\left(1 - \dfrac{\alpha v}{mg}\right)\right]$ (b) $y = -\dfrac{m}{2\beta}\ln\left(1 - \dfrac{\beta v^2}{mg}\right)$

36. $R = \dfrac{v_0^2}{g}\cos\theta\left(\sin\theta + \sqrt{\sin^2\theta + \dfrac{2gh}{v_0^2}}\right)$

38. (a) $F(x) = -mn\alpha^2 x^{-(2n+1)}$

(b) $x(t) = [(n+1)\alpha t]^{\frac{1}{n+1}}$

(c) $F(t) = -mn\alpha^2[(n+1)\alpha t]^{-(2n+1)/(n+1)}$

40. (a) $a_t = \dfrac{2A\alpha^2 \sin \alpha t}{\sqrt{5 - 4\cos \alpha t}}$; $a_n = \dfrac{A\alpha^2|2\cos\alpha t - 1|}{\sqrt{5 - 4\cos\alpha t}}$

(b) $\dfrac{n\pi}{\alpha}$ where $n =$ integer

42. Stable if $R > b/2$; unstable if $R \leq b/2$

2-48. Two gravitationally bound stars with equal masses m, separated by a distance d, revolve about their center of mass in circular orbits. Show that the period τ is proportional to $d^{3/2}$ (Kepler's Third Law) and find the proportionality constant.

2-50. According to special relativity, a particle of rest mass m_0 accelerated in one dimension by a force F obeys the equation of motion $dp/dt = F$. Here $p = m_0v/(1 - v^2/c^2)^{1/2}$ is the relativistic momentum, which reduces to m_0v for $v^2/c^2 \ll 1$. **(a)** For the case of constant F and initial conditions $x(0) = 0 = v(0)$, find $x(t)$ and $v(t)$. **(b)** Sketch your result for $v(t)$. **(c)** Suppose that $F/m_0 = 10\ \mathrm{m/s^2}$ ($\approx g$ on Earth). How much time is required for the particle to reach half the speed of light and of 99% the speed of light?

2-52. A particle of mass m moving in one dimension has potential energy $U(x) = U_0[2(x/a)^2 - (x/a)^4]$, where U_0 and a are positive constants. **(a)** Find the force $F(x)$, which acts on the particle. **(b)** Sketch $U(x)$. Find the positions of stable and unstable equilibrium. **(c)** What is the angular frequency ω of oscillations about the point of stable equilibrium? **(d)** What is the minimum speed the particle must have at the origin to escape to infinity? **(e)** At $t = 0$ the particle is at the origin and its velocity is positive and equal in magnitude to the escape speed of part **(d)**. Find $x(t)$ and sketch the result.

2-54. A potato of mass 0.5 kg moves under Earth's gravity with an air resistive force of $-kmv$. **(a)** Find the terminal velocity if the potato is released from rest and $k = 0.01\ \mathrm{s^{-1}}$. **(b)** Find the maximum height of the potato if it has the same value of k, but it is initially shot directly upward with a student-made potato gun with an initial velocity of 120 m/s.

48. $\tau = \pi d^{3/2}\sqrt{\dfrac{2}{mG}}$

50. (a) $x(t) = \dfrac{c^2}{F}\left(\sqrt{m_0^2 + \dfrac{F^2t^2}{c^2}} - m_0\right)$, $v(t) = \dfrac{Ft}{\sqrt{m_0^2 + \dfrac{F^2t^2}{c^2}}}$

(c) $t(v = c/2) = 0.55$ yr; $t(v = 0.99c) = 6.67$ yr.

52. (a) $F(x) = -\dfrac{4U_0x}{a^2}\left(1 - \dfrac{x^2}{a^2}\right)$, (c) $\omega = \sqrt{\dfrac{4U_0}{ma^2}}$, (d) $v_{\min} = \sqrt{\dfrac{2U_0}{m}}$

(e) $x(t) = \dfrac{a[\exp(t\sqrt{8U_0/ma^2}) - 1]}{[\exp(t\sqrt{8U_0/ma^2}) + 1]}$

54. (a) $v = g/k = 1000\ \mathrm{m/s}$, (b) $\textit{height} = \dfrac{v_0}{k} + \dfrac{g}{k^2}\ln\left(\dfrac{g}{g + kv_0}\right) = 680\ \mathrm{m}$