Solved Problems from Ch. 1

AP E3200x
Mechanics
1-10. A particle moves in a plane elliptical orbit described by the position vector

\[ \mathbf{r} = 2b \sin \omega t \mathbf{i} + b \cos \omega t \mathbf{j} \, . \]

(a) Find \( \mathbf{v} \), \( \mathbf{a} \), and the particle speed.
(b) What is the angle between \( \mathbf{v} \) and \( \mathbf{a} \) at time \( t = \pi/2\omega \)?

1-12. Let \( \mathbf{a} \), \( \mathbf{b} \), \( \mathbf{c} \) be three constant vectors drawn from the origin to the points \( A \), \( B \), \( C \). What is the distance from the origin to the plane defined by the points \( A \), \( B \), \( C \)? What is the area of the triangle \( ABC \)?

1-14. Consider the following matrices:

\[ \mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \]

Find the following

(a) \( |\mathbf{A} \mathbf{B}| \)  (b) \( \mathbf{A} \mathbf{C} \)  (c) \( \mathbf{A} \mathbf{B} \mathbf{C} \)  (d) \( \mathbf{A} \mathbf{B} - \mathbf{B}' \mathbf{A}' \)

1-26. A particle moves with \( v = \text{const.} \) along the curve \( \mathbf{r} = k(1 + \cos \theta) \) (a cardiod). Find \( \mathbf{r} \cdot \mathbf{e}_r = a \cdot \mathbf{e}_r, |a|, \text{ and } \theta. \)

1-34. Evaluate the integral

\[ \int \mathbf{A} \times \mathbf{A} \, dt \]

1-36. Find the value of the integral \( \int_S \mathbf{A} \cdot d\mathbf{a} \), where \( \mathbf{A} = x \mathbf{i} - y \mathbf{j} + z \mathbf{k} \) and \( S \) is the closed surface defined by the cylinder \( z^2 = x^2 + y^2 \). The top and bottom of the cylinder are at \( z = d \) and \( 0 \), respectively.

1-38. Find the value of the integral \( \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{a} \) if the vector \( \mathbf{A} = y \mathbf{i} + z \mathbf{j} + x \mathbf{k} \) and \( S \) is the surface defined by the paraboloid \( z = 1 - x^2 - y^2 \), where \( z \geq 0 \).

1-40. The height of a hill in meters is given by \( z = 2x^2 - 3x^2 - 4y^2 - 18x + 28y + 12 \), where \( x \) is the distance east and \( y \) is the distance north of the origin. (a) Where is the top of the hill and how high is it? (b) How steep is the hill at \( x = y = 1 \), that is, what is the angle between a vector perpendicular to the hill and the \( x \) axis? (c) In which compass direction is the slope at \( x = y = 1 \) steepest?
Chapter 1

10. (a) \[ \mathbf{v} = 2b\omega \cos \omega t \mathbf{i} - b\omega \sin \omega t \mathbf{j} \quad \text{(b) 90°} \]

\[ \mathbf{a} = -\omega^2 \mathbf{r} \]

\[ |\mathbf{v}| = b\omega[3 \cos^2 \omega t + 1]^\frac{1}{2} \]

12. \[ h = \frac{|\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}|}{|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|} \]

\[ A = \frac{1}{2}|(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{b})| = \frac{1}{2}|(\mathbf{a} - \mathbf{c}) \times (\mathbf{b} - \mathbf{a})| \]

\[ = \frac{1}{2}|(\mathbf{c} - \mathbf{b}) \times (\mathbf{a} - \mathbf{c})| \]

14. (a) -104 \hspace{1cm} (b) \[ \begin{pmatrix} 9 & 7 \\ 13 & 9 \end{pmatrix} \] \hspace{1cm} (c) \[ \begin{pmatrix} -5 & -5 \\ 3 & -5 \end{pmatrix} \] \hspace{1cm} (d) \[ \begin{pmatrix} 0 & -3 & -4 \\ 3 & 0 & 6 \end{pmatrix} \]

26. \[ \mathbf{a} \cdot \mathbf{e}_r = -\frac{3}{4} \frac{v^2}{k}; \quad |\mathbf{a}| = \frac{3}{4} \frac{v^2}{k} \sqrt{\frac{2}{1 + \cos \theta}}; \quad \dot{\theta} = \frac{v}{\sqrt{2}kr} \]

34. \[ \int (\mathbf{A} \times \mathbf{A}) \, dt = (\mathbf{A} \times \mathbf{A}) + \mathbf{C}, \text{ where } \mathbf{C} \text{ is a constant vector} \]

36. \[ \pi c^2 d \]

38. \[ -\pi \]

40. (a) \( x = -2 \text{ m}, y = 3 \text{ m}, z_{\text{max}} = 72 \text{ m}; \) (c) SE