Solved Problems from Ch. 1

AP E3200x
Mechanics

1-10. A particle moves in a plane elliptical orbit described by the position vector

$$\mathbf{r} = 2b \sin \omega t\, \mathbf{i} + b \cos \omega t\, \mathbf{j}$$

(a) Find **v**, **a**, and the particle speed.
(b) What is the angle between **v** and **a** at time $t = \pi/2\omega$?

1-12. Let **a**, **b**, **c** be three constant vectors drawn from the origin to the points *A*, *B*, *C*. What is the distance from the origin to the plane defined by the points *A*, *B*, *C*? What is the area of the triangle *ABC*?

1-14. Consider the following matrices:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix}$$

Find the following
(a) $|\mathbf{AB}|$ **(b)** $\mathbf{AC}$ **(c)** $\mathbf{ABC}$ **(d)** $\mathbf{AB} - \mathbf{B}^t\mathbf{A}^t$

1-26. A particle moves with $v = $ const. along the curve $r = k(1 + \cos\theta)$ (a *cardioid*). Find $\ddot{\mathbf{r}} \cdot \mathbf{e}_r = \mathbf{a} \cdot \mathbf{e}_r$, $|\mathbf{a}|$, and $\dot{\theta}$.

1-34. Evaluate the integral

$$\int \mathrm{A} \times \ddot{\mathrm{A}} dt$$

1-36. Find the value of the integral $\int_S \mathbf{A} \cdot d\mathbf{a}$, where $\mathbf{A} = x\mathbf{i} - y\mathbf{j} + z\mathbf{k}$ and S is the closed surface defined by the cylinder $c^2 = x^2 + y^2$. The top and bottom of the cylinder are at $z = d$ and 0, respectively.

1-38. Find the value of the integral $\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{a}$ if the vector $\mathbf{A} = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ and S is the surface defined by the paraboloid $z = 1 - x^2 - y^2$, where $z \geq 0$.

1-40. The height of a hill in meters is given by $z = 2xy - 3x^2 - 4y^2 - 18x + 28y + 12$, where x is the distance east and y is the distance north of the origin. **(a)** Where is the top of the hill and how high is it? **(b)** How steep is the hill at $x = y = 1$, that is, what is the angle between a vector perpendicular to the hill and the z axis? **(c)** In which compass direction is the slope at $x = y = 1$ steepest?

Chapter 1

10. (a) $\mathbf{v} = 2b\omega \cos \omega t\,\mathbf{i} - b\omega \sin \omega t\,\mathbf{j}$ (b) 90°

$\mathbf{a} = -\omega^2\mathbf{r}$

$|\mathbf{v}| = b\omega[3 \cos^2 \omega t + 1]^{\frac{1}{2}}$

12. $h = \dfrac{|\mathbf{a}\cdot\mathbf{b} \times \mathbf{c}|}{|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|}$

$$A = \frac{1}{2}|(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{b})| = \frac{1}{2}|(\mathbf{a} - \mathbf{c}) \times (\mathbf{b} - \mathbf{a})|$$

$$= \frac{1}{2}|(\mathbf{c} - \mathbf{b}) \times (\mathbf{a} - \mathbf{c})|$$

14. (a) -104 (b) $\begin{pmatrix} 9 & 7 \\ 13 & 9 \\ 5 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} -5 & -5 \\ 3 & -5 \\ 25 & 14 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -3 & -4 \\ 3 & 0 & 6 \\ 4 & -6 & 0 \end{pmatrix}$

26. $\mathbf{a}\cdot\mathbf{e}_r = -\dfrac{3}{4}\dfrac{v^2}{k}$; $|\mathbf{a}| = \dfrac{3}{4}\dfrac{v^2}{k}\cdot\sqrt{\dfrac{2}{1 + \cos\theta}}$; $\dot{\theta} = \dfrac{v}{\sqrt{2kr}}$

34. $\int(\mathbf{A} \times \ddot{\mathbf{A}})\,dt = (\mathbf{A} \times \dot{\mathbf{A}}) + \mathbf{C}$, where $\mathbf{C}$ is a constant vector

36. $\pi c^2 d$

38. $-\pi$

40. (a) $x = -2$ m, $y = 3$ m, $z_{\max} = 72$ m; (c) SE