AP 3200x

Final Review

Chapters on the Exam

- Ch 1 (Matrices, vectors, …)
- Ch 2 (Single particle Newton's Laws)
- ➡ **Ch 3 (Oscillations)**
- Ch 5 (Gravity and potential energy)
- ➡ **Ch 6 (Calculus of Variations)**
- ➡ **Ch 7 (Hamilton's Principle, Lagrangian and Hamiltonian Dynamics)**
- ➡ **Ch 8 (Central Force Problems)**
- ➡ **Ch 9 (Systems of interacting particles, Conservation principles)**
- ➡ **Ch 10 (Rotating frames)**
- ➡ **Ch 11 (Rigid Body Motion, Inertia Tensor, Precession)**
- ➡ **Ch 12 (Coupled Oscillators, Normal Modes)**

Ch 3 Oscillations

3-12. A simple pendulum consists of a mass m suspended from a fixed point by a weightless, extensionless rod of length l. Obtain the equation of motion and, in the approximation that $\sin\theta \cong \theta$, show that the natural frequency is $\omega_0 = \sqrt{g/l}$, where g is the gravitational field strength. Discuss the motion in the event that the motion takes place in a viscous medium with retarding force $2m\sqrt{gl}\,\dot{\theta}$.

3-44. Consider a damped harmonic oscillator. After four cycles the amplitude of the oscillator has dropped to $1/e$ of its initial value. Find the ratio of the frequency of the damped oscillator to its natural frequency.

Ch 6 Calculus of Variations

6-2. Show that the shortest distance between two points on a plane is a straight line.

6-14. Find the shortest path between the (x, y, z) points $(0, -1, 0)$ and $(0, 1, 0)$ on the conical surface $z = 1 - \sqrt{x^2 + y^2}$. What is the length of the path? Note: this is the shortest mountain path around a volcano.

Ch 7 Lagrangian Dynamics

7-7. A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. If the two pendula have equal lengths and have bobs of equal mass and if both pendula are confined to move in the same plane, find Lagrange's equations of motion for the system. Do not assume small angles.

7-12. A particle of mass m rests on a smooth plane. The plane is raised to an inclination angle θ at a constant rate α ($\theta = 0$ at $t = 0$), causing the particle to move down the plane. Determine the motion of the particle.

7-15. A pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k. Find Lagrange's equations of motion.

7-23. Consider a particle of mass m moving freely in a conservative force field whose potential function is U. Find the Hamiltonian function, and show that the canonical equations of motion reduce to Newton's equations. (Use rectangular coordinates.)

Ch 8 Central Force

8-5. Two particles moving under the influence of their mutual gravitational force describe circular orbits about one another with a period τ. If they are suddenly stopped in their orbits and allowed to gravitate toward each other, show that they will collide after a time $\tau/4\sqrt{2}$.

8-14. Find the force law for a central-force field that allows a particle to move in a spiral orbit given by $r = k\theta^2$, where k is a constant.

8-34. Consider the problem of the particle moving on the surface of a cone, as discussed in Examples 7.4 and 8.7. Show that the effective potential is

$$V(r) = \frac{l^2}{2mr^2} + mgr \cot \alpha$$

(Note that here r is the radial distance in cylindrical coordinates, not spherical coordinates; see Figure 7-2.) Show that the turning points of the motion can be found from the solution of a cubic equation in r. Show further that only two of the roots are physically meaningful, so that the motion is confined to lie within two horizontal planes that cut the cone.

8-46. Two double stars of the same mass as the sun rotate about their common center of mass. Their separation is 4 light years. What is their period of revolution?

Ch 9 Dynamics of System of Particles

9-9. A projectile is fired at an angle of 45° with initial kinetic energy E_0. At the top of its trajectory, the projectile explodes with additional energy E_0 into two fragments. One fragment of mass m_1 travels straight down. What is the velocity (magnitude and direction) of the second fragment of mass m_2 and the velocity of the first? What is the ratio of m_1/m_2 when m_1 is a maximum?

9-15. A smooth rope is placed above a hole in a table (Figure 9-D). One end of the rope falls through the hole at $t = 0$, pulling steadily on the remainder of the rope. Find the velocity and acceleration of the rope as a function of the distance to the end of the rope x. Ignore all friction. The total length of the rope is L.

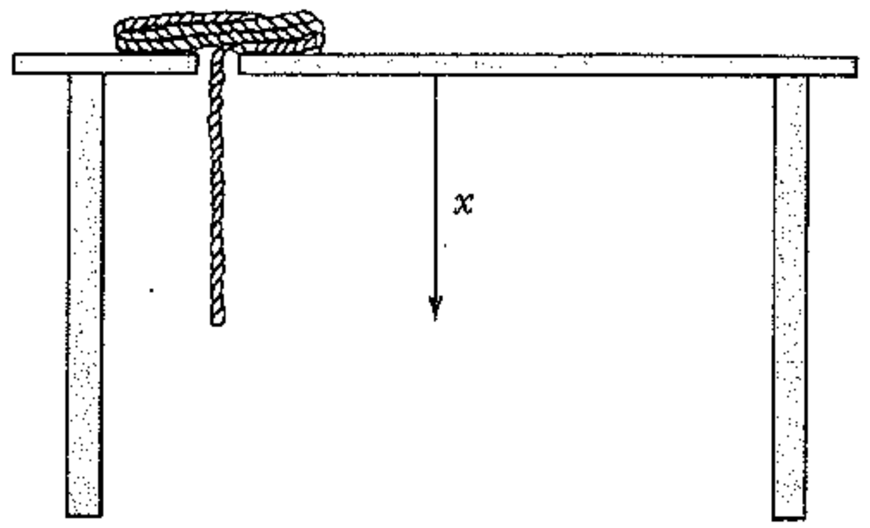

FIGURE 9-D Problem 9-15.

9-23. A particle of mass m_1 and velocity u_1 collides with a particle of mass m_2 at rest. The two particles stick together. What fraction of the original kinetic energy is lost in the collision?

9-40. A particle of mass m_1 and velocity u_1 strikes head-on a particle of mass m_2 at rest. The coefficient of restitution is ε. Particle m_2 is tied to a point a distance a away as shown in Figure 9-H. Find the velocity (magnitude and direction) of m_1 and m_2 after the collision.

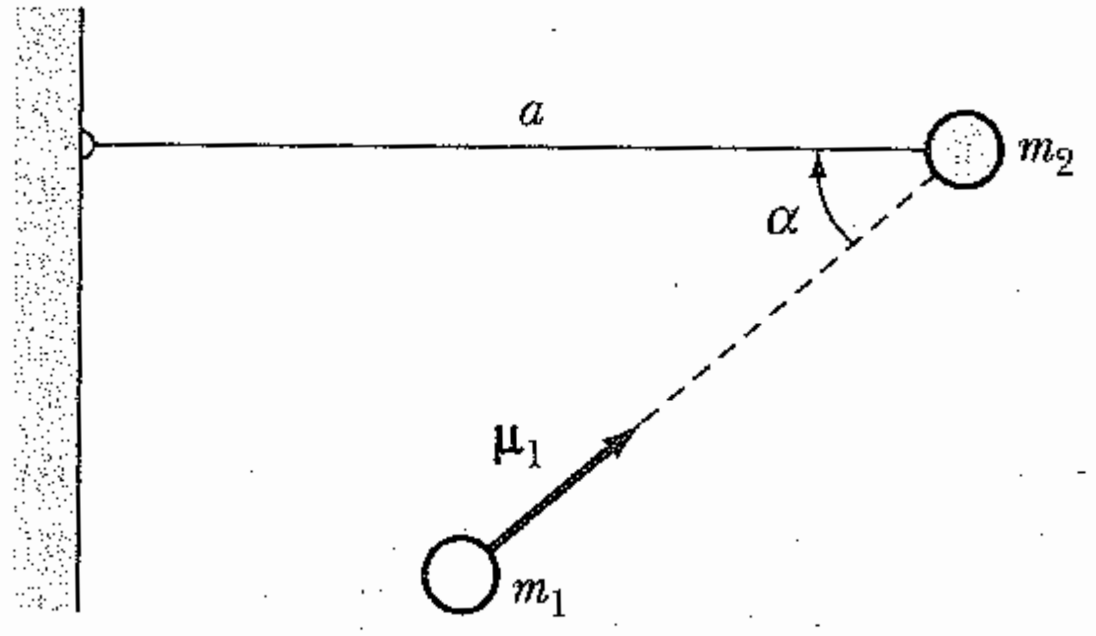

FIGURE 9-H Problem 9-40.

Ch 10 Noninertial Frames

10-8. If a particle is projected vertically upward to a height h above a point on Earth's surface at a northern latitude λ, show that it strikes the ground at a point $\frac{4}{3}\,\omega \cos \lambda \cdot \sqrt{8h^3/g}$ to the west. (Neglect air resistance, and consider only small vertical heights.)

10-12. Show that the small angular deviation ε of a plumb line from the true vertical (i.e., toward the center of Earth) at a point on Earth's surface at a latitude λ is

$$\varepsilon = \frac{R\omega^2 \sin \lambda \cos \lambda}{g_0 - R\omega^2 \cos^2 \lambda}$$

where R is the radius of Earth. What is the value (in seconds of arc) of the maximum deviation? Note that the entire denominator in the answer is actually the effective g, and g_0 denotes the pure gravitational component.

Ch 11 Rigid Body Motion

11-4. Consider a thin rod of length l and mass m pivoted about one end. Calculate the moment of inertia. Find the point at which, if all the mass were concentrated, the moment of inertia about the pivot axis would be the same as the real moment of inertia. The distance from this point to the pivot is called the **radius of gyration.**

11-7. A homogeneous disk of radius R and mass M rolls without slipping on a horizontal surface and is attracted to a point a distance d below the plane. If the force of attraction is proportional to the distance from the disk's center of mass to the force center, find the frequency of oscillations around the position of equilibrium.

11-9. A homogeneous slab of thickness a is placed atop a fixed cylinder of radius R whose axis is horizontal. Show that the condition for stable equilibrium of the slab, assuming no slipping, is $R > a/2$. What is the frequency of small oscillations? Sketch the potential energy U as a function of the angular displacement θ. Show that there is a minimum at $\theta = 0$ for $R > a/2$ but not for $R < a/2$.

11-24. Find the frequency of small oscillations for a thin homogeneous plate if the motion takes place in the plane of the plate and if the plate has the shape of an equilateral triangle and is suspended **(a)** from the midpoint of one side and **(b)** from one apex.

Ch 12 Coupled Oscillations

12-7. A particle of mass m is attached to a rigid support by a spring with force constant κ. At equilibrium, the spring hangs vertically downward. To this mass–spring combination is attached an identical oscillator, the spring of the latter being connected to the mass of the former. Calculate the characteristic frequencies for one-dimensional vertical oscillations, and compare with the frequencies when one or the other of the particles is held fixed while the other oscillates. Describe the normal modes of motion for the system.

12-8. A simple pendulum consists of a bob of mass m suspended by an inextensible (and massless) string of length l. From the bob of this pendulum is suspended a second, identical pendulum. Consider the case of small oscillations (so that $\sin\theta \cong \theta$), and calculate the characteristic frequencies. Describe also the normal modes of the system (refer to Problem 7-7).

12-19. In the problem of the three coupled pendula, consider the three coupling constants as distinct, so that the potential energy may be written as

$$U = \frac{1}{2}(\theta_1^2 + \theta_2^2 + \theta_3^2 - 2\varepsilon_{12}\theta_1\theta_2 - 2\varepsilon_{13}\theta_1\theta_3 - 2\varepsilon_{23}\theta_2\theta_3)$$

with ε_{12}, ε_{13}, ε_{23} all different. Show that no degeneracy occurs in such a system. Show also that degeneracy can occur *only* if $\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23}$.

12-23. Evaluate the total energy associated with a normal mode, and show that it is constant in time. Show this explicitly for the case of Example 12.3.