feature article

The great Martian catastrophe and how Kepler fixed it

Owen Gingerich

For a few weeks every 32 years, both the Ptolemaic and Copernican predictions for the position of Mars are off by close to 5 degrees—a problem first noticed by Tycho Brahe.

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In August 1593 Tycho Brahe was in big trouble—double trouble. He had just returned to his observatory island of Hven from a frustrating trip to the royal court in Copenhagen. He boasted that Frederik II, former King of Denmark and Norway, had spent a ton of gold supporting his observatory—the finest in Europe—but Tycho wondered whether the current king would be as keen about all his wonderful instruments. What's more, Tycho, a nobleman, had fallen in love with a commoner, the Kate Middleton of his day. By Danish law, a commoner wife and children could not inherit Tycho's precious observatory. What would become of it?

Meanwhile, he was experiencing a problem with Mars. And as it would work out in the coming weeks, it was much worse than he could have imagined.

Just over a decade earlier, Tycho had hit upon a wonderful test to distinguish between the traditional geocentric Ptolemaic cosmology and the new-fangled heliocentric Copernican arrangement. Both systems predict essentially the same planetary positions. But in the Ptolemaic system, Mars always traveled in a sphere beyond the Sun, which restricted its minimum distance from Earth to be 1 astronomical unit (AU). In the Copernican system, by contrast, Mars's closest approach to Earth was half that distance, as shown in figure 1. If Tycho could measure that closest distance, he would have an *experimentum crucis*—a crucial experiment to determine which system matched reality.

He proposed using the technique known as diurnal parallax. To appreciate what he planned to do, consider a rotating Earth, which is, in fact, the case for the Copernican system. The measurement would take place when Mars was opposite the Sun in the sky, at which time Mars would necessarily be closest to Earth. First, Tycho would carefully measure the position of Mars soon after sunset, with Mars rising in the east. Later, shortly before sunrise, he would again observe Mars as it was sinking into the west. Had he done that at the terrestrial equator, Earth's rotation would

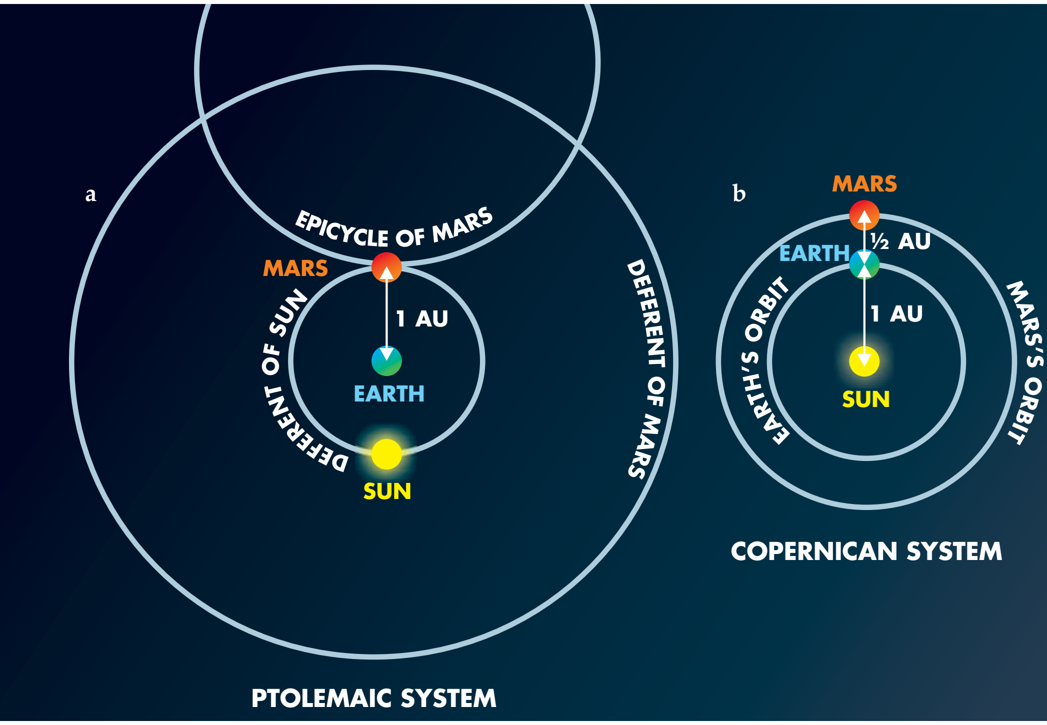

Figure 1. (a) In the Ptolemaic system, a planet moves in a circle, or epicycle, that in turn is carried along a larger circle, or deferent. Mars's epicycle is ⅔ the size of its deferent, and the minimum Earth–Mars distance, when Mars is in opposition to the Sun, is 1 AU. **(b)** In the Copernican system, Earth's orbit is ⅔ as large as Mars's orbit. The minimum distance between them—ignoring the eccentric placement of Mars's orbit—is thus ½ AU. In the 1580s Tycho Brahe hoped to distinguish between the two systems by measuring the distance to Mars. Unfortunately, the planetary system was 20 times larger than astronomers of the time realized, which made naked-eye measurements inadequate.

Figure 2. The equatorial armillary, Tycho's most impressive instrument, completed in 1585. The tilted axis running through the large (2.7-m wide) circle is parallel with Earth's axis. The circle is calibrated with 90° at the poles and 0° at the midpoint of its circumference. Two sighting arms establish the declination, the angular distance (north or south) of a star from the equator. Tycho could use one sighting arm to read the declination of a star or planet and then, in less than a minute's time, rotate the circle 180° about the tilted axis to make a second independent determination with the other arm. (Published in 1662 and based on Tycho's original version, this illustration is by Willem Blaeu, a former assistant at Tycho's Hven observatory.)

have provided a triangulation baseline equal to Earth's diameter. At his more northern latitude, the baseline was somewhat shorter, but still long enough for his purpose.

Today we know that the determination is impossible with naked-eye observations. But in the 16th century, all astronomers accepted an erroneous value for Earth's distance to the Sun—a value 20 times too small. Had the small value been correct, Tycho's amazingly precise observations would have yielded the result he was looking for. But unknowingly, he was foiled.

Making refraction tables

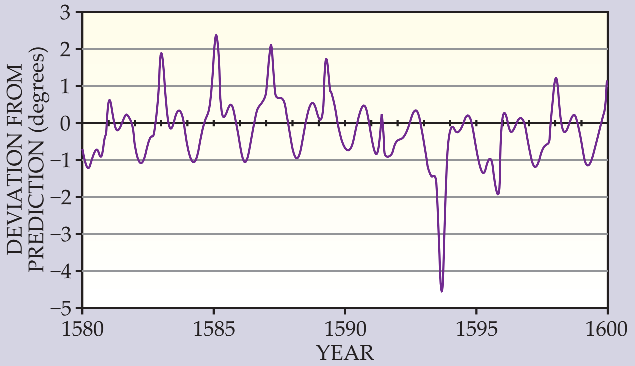

Figure 3. Errors in the predicted celestial longitudes of Mars derived from the Copernican-based *Prutenic Tables*. The errors closely repeat after 32 years. The large Martian catastrophe error of nearly 5° took place in 1593 and was repeated in 1625.

Martian oppositions come, on average, two years and seven weeks apart. Tycho tried to find the distance to Mars in December 1582 without success; his instruments were not yet good enough. Armed with an improved observatory and more stable devices, he tried again in late January 1585. But to his surprise (and annoyance) he got a negative parallax, which implied that Mars was more than infinitely far away. He knew that was absurd and soon suspected the root of the problem—what we now call differential refraction. Normally Tycho made his observations of objects high in the sky, but the diurnal parallax procedure required observing Mars when it was near the horizon, where atmospheric refraction lifts a planet or star slightly higher above the horizon. Because the comparison stars were at different altitudes than Mars, it was necessary to correct their apparent positions.[1]

So Tycho set out to establish a refraction table—two of them, in fact. Determining observationally the amount of atmospheric refraction at a given altitude requires knowing the true, or unrefracted, altitude for comparison. Tycho had two ways of deducing that unrefracted altitude—one using the Sun, the other using the stars. In the first case, he established a theory of the Sun's movement intended to give true solar positions, and he observed the midday apparent altitudes of the Sun using his large mural quadrant firmly affixed to an inside wall of his castle. But in deriving his theory for the true solar positions, he used the assumed but incorrect distance to the Sun, which led to a slightly erroneous eccentricity for the solar orbit. That, in turn, led unwittingly to a flawed refraction table.

For stars, Tycho used his new large equatorial armillary, shown in figure 2, the most impressive instrument in his observational armory. With the instrument, he directly measured the apparent declination of a star—its angular distance above the equator—at the same time that another instrument yielded the star's observed altitude, its angular distance above the horizon. Atmospheric refraction affected the declination measurement, which otherwise would have been constant. And from the varying declinations, measured as a star ascended or descended in altitude, another refraction table could be deduced.

In March 1587, with Mars again opposite the Sun in the sky, Tycho undertook another observational campaign. But which refraction table should he use? After applying the solar table, oblivious to its errors, he promptly found a parallax that matched the Copernican case—little realizing that the derived parallax was an artifact of the errors in the solar refraction table. Almost immediately he wrote to several correspondents that he had succeeded in establishing the comparatively close approach of Mars.

Ironically, however, Tycho did not adopt the Copernican cosmology. Although the Copernican cosmology nowhere

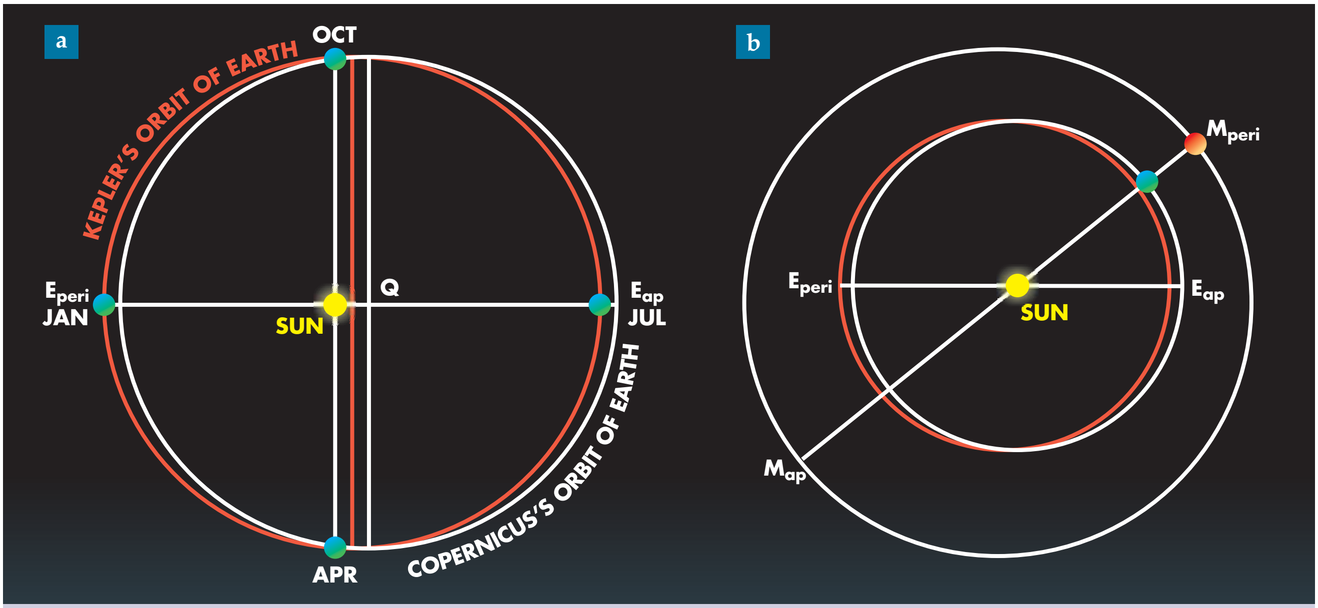

Figure 4. (a) In Nicolaus Copernicus's system, Earth (green) moves uniformly around a point Q (called an equant), the center of its eccentric orbit, from aphelion E_{ap} to perihelion E_{peri} and back. Because the observed quadrant from January to April has a shorter part of the circumference than the quadrant from April to July, the Sun appears to move more quickly during the Northern Hemisphere's winter than during its summer; hence winter is shorter than summer. But because Johannes Kepler believed that Earth should actually move faster when it is nearer the Sun, he bisected the eccentricity and thereby shifted Earth to a less eccentric orbit (red). In Kepler's system, half the Sun's apparent nonuniform motion is caused by the eccentric position of the Sun, and half is caused by the actual nonuniform motion of Earth. **(b)** The orbits of Mars (M, red) and Earth (E, green) are shown to scale, with Mars at its perihelion making its closest approach to Earth at the time of the great Martian catastrophe. When Kepler bisected the eccentricity of Earth's orbit, the orbit's center slipped along the apsidal line away from the direction of its aphelion. That repositioned Earth just enough to eliminate the major errors in the observed longitude of Mars. In both diagrams the eccentricity of Earth's orbit is greatly exaggerated for clarity.

offended the principles of mathematics, he argued, Earth, as a lazy, sluggish body, was unfit for motion. So he opted instead for a geoheliocentric alternative. In that so-called Tychonic system, Earth stands immobile in the center of the universe, while Mars circles the Sun as the Sun in turn circles Earth. In both Tychonic and Copernican systems, Mars came twice as close as in the Ptolemaic system.

Tycho was, however, a brilliant scientist unwilling to let the rather arbitrary choice of refraction table go unexamined. Using his equatorial armillary, he proceeded to derive a new refraction table for the planet Jupiter. He discovered his error and lapsed into silence concerning his measurement of the distance of Mars from Earth.

The march of the Martian oppositions around the calendar brought the next event into the short nights of summer; June 1591 was unsuitable for a fresh campaign. But in August 1593 Tycho was ready to try again. However, after his return from Copenhagen, the weather was rotten. Rain, wind, and thunder storms were recorded in his meteorological diary.

At last the skies cleared, and on 13 August he briefly resumed his examination of Mars. He tabulated the results in his log book:

Tycho Brahe's observed errors

	Longitude	Difference
Copernican tables	342° 0′	−4° 7½′
Tycho's observation	346° 7½′	
Alfonsine [Ptolemaic] tables	351° 26′	+5° 18½′

They were a catastrophic shock. The ruddy planet was nowhere near its predicted place using either cosmology. The values were off by 4 or 5 degrees, in opposite directions, and remained so for several weeks (see figure 3).

Enter Kepler

By November 1600 Tycho had solved the inheritance problem. He left Denmark for the court of Rudolf II, the Holy Roman emperor, and brought his family and his instruments. In Prague the laws were different. There his wife and children could inherit his wealth.[2]

About a year after his arrival, Tycho added a young assistant to his staff: Johannes Kepler, a Lutheran high school teacher who suddenly found himself underemployed when the Catholic Counter Reformation swept into southern Austria. In Tycho's employ, Kepler was assigned the recalcitrant problem of Mars. Did Tycho tell Kepler about the Martian catastrophe in 1593? Probably, but no one really knows. In any event, he would have found it in Tycho's log book.

Kepler soon discovered that the problem wasn't with Mars at all, but arose entirely with the orbit assigned to Earth. Contrary to today's widespread but uninformed opinion, the system proposed by Nicolaus Copernicus held that planets did *not* revolve in circular orbits *centered on the Sun*. Rather, they moved in eccentrically placed orbits—faster when closer to the Sun. Earth was the sole exception; it was thought to have an eccentric orbit and yet move at constant speed. And that, Kepler thought, had to be wrong.

In the ancient, geocentric universe, the entire heavens continually whirled about Earth, one rotation per day. The stars moved the fastest, and the planets slightly slower, down

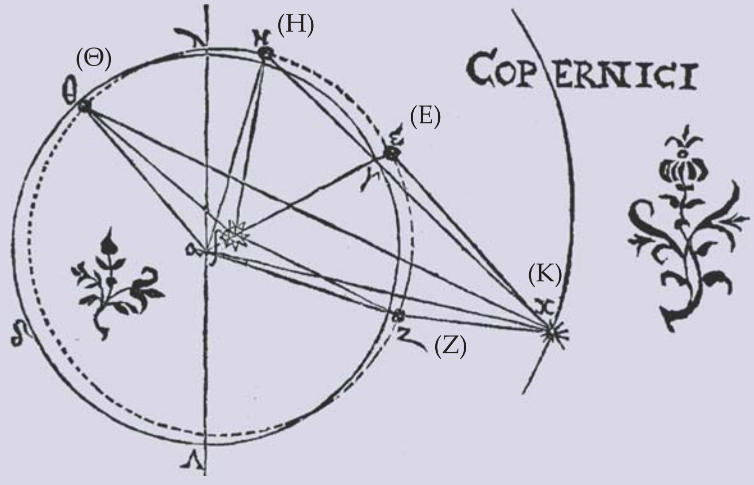

Figure 5. Johannes Kepler's triangulation diagram, adapted from his 1609 *Astronomia nova*, established the position of Earth's orbit. In the Copernican system, Mars returns to the same position K on its orbit every 687 days. Earth, starting at Θ, makes a complete counterclockwise revolution every 365 days and continues around to H after the remaining 322 days it takes Mars to complete its revolution. Similar times bring Earth to E and Z in subsequent revolutions of Mars. Kepler thus verified that Earth's orbit needed to be shifted from the solid outline to the dashed one. (Adapted from J. Kepler, *Astronomia nova*, 1609, p. 131.)

to the Moon, which lagged behind the rest. The source of the rotation, according to Aristotle, was the goodness of God, who supplied the motion from beyond the starry firmament.

In the Copernican system, by contrast, the stars provided a fixed outer framework. Hence the driving power for the planetary system had to come from the inside—from the Sun itself. And that made sense to Copernicus, who had noticed that the closer a planet was to the Sun, the faster it moved. Indeed, that aesthetic arrangement was a primary reason why the Polish pioneer had opted for a heliocentric universe. As Copernicus wrote in his *De revolutionibus orbium coelestium*, "In no other way do we find a sure bond of harmony between the movement and magnitude of the orbital circles."[3]

But why, in Copernicus's arrangement, was Earth unique in orbiting at constant speed? That was simply an artifact from his transformation of Ptolemy's epicyclic geocentric geometry into a heliocentric one. In the Copernican system, to a rough approximation, the observed position of a planet is determined by the combination of two circles—the heliocentric orbit of Earth and the heliocentric orbit of the planet. Likewise, in the Ptolemaic system, the position of a planet is roughly determined by two circles—the deferent, or carrying circle, and the epicycle. For the superior planets—Mars, Jupiter, and Saturn—the epicycle played the role of Earth's orbital motion in the Copernican system; and for simplicity Ptolemy assumed uniform circular motion in the epicycle, an assumption that turned out to be a generally acceptable approximation. So when Copernicus made the transformation, Earth's orbit, like the Ptolemaic epicycles, carried the planet in uniform circular motion.

Physics and geometry

In Kepler's day, virtually all astronomers looked to mother geometry for inspiration. Kepler was unique in seeking physical causes for celestial motions. Even his astute teacher, Michael Maestlin, urged him to forget about physical causes and to attend to geometry alone for explanation. But in Kepler's vision, Earth's constant speed in the Copernican system made no physical sense. Surely it should actually move faster in January when it was closer to the Sun. In both Ptolemaic and Copernican systems, the Sun appeared to move faster in January because of the eccentric positioning of its, or Earth's, orbital circle. Kepler called the phenomenon the optical effect. But he also wanted the total apparent motion to be a combination of the optical effect and a physical effect. If each effect played a comparable role, then the Copernican eccentricity of Earth's orbit had to be halved, or bisected, Kepler reasoned. In that case, the Sun's varying distance from Earth would be half as great as previously assumed, as shown in figure 4a.

How could Kepler measure that subtle difference between the Earth–Sun distance in January and in July? One way was to measure the apparent diameter of the Sun through the seasons. Kepler devised an instrument to do just that, but the results were not as convincing as he had hoped. A half century later, the astronomer Giovanni Domenico Cassini used a meridian projection in the Bologna cathedral in Italy to persuasively confirm the effect and establish his own brilliant reputation.[4]

Meanwhile, Kepler turned to the very precise observations that Tycho had made in his campaign to measure the distance to Mars. A famous diagram in Kepler's 1609 *Astronomia nova*, reproduced in figure 5, shows his ingenious triangulation procedure. Mars has an orbital period of 687 days, but each time Mars returned to a given position in its orbit, Earth would have fallen 43 days short of making two complete revolutions. Terrestrial observers would therefore be viewing Mars at that position from two different vantage points. By collecting from Tycho's log books observations 687 days apart, Kepler's triangulation system precisely pinpointed Earth's orbital trajectory. As he suspected, the eccentricity of Earth's orbit in the Copernican system needed to be bisected.

Whether Kepler completely realized that this move solved the Martian catastrophe is not clear. But he next turned to a more subtle problem. In tracking the heliocentric positions of Mars, Kepler found that the use of a circular orbit led to an 8′ error at the octants of its orbit. Because God had given him such a great observer in Tycho Brahe—whose typical observational errors were 2′ or less—Kepler wrote that he dared not stop with a maximum error of 8′. (When Mars was observed from Earth at close approach, the heliocentric error of 8′ became nearly a half-degree geocentric error.) Thus Kepler continued waging war on Mars, eventually coming up with the elliptical orbit, which reduced the errors by an order of magnitude.

In fact, Kepler tried a variety of oval curves that could have fit the observations equally as well as the ellipse. But he was unsatisfied with the physical basis for choosing any of them until he noticed that one focus of an approximating ellipse coincided with the Sun. That curve and focus made it easier for Kepler to conjure up a physical explanation. (*Focus*, the Latin word for "hearth," was coined by Kepler to have its current geometric sense, for the Sun is the hearth of the universe.) Kepler's erstwhile sparring partner, David Fabricius, wrote that he could equally well match the observations with a set of circles. Kepler responded petulantly, "You say a daughter was born to you of mother geometry. I saw her; she is beautiful. But she will be a most mischievous whore, who will seduce the husbands away from my many daughters born of mother physics."[5]

Kepler refused to include Fabricius's model in his *Astronomia nova*, and he broke off the long-running correspondence with Fabricius. For Kepler, a physical model was paramount in a satisfactory explanation. He subtitled his book *Aitiologitos, seu physica coelestis* ("Based on causes, or the celestial physics"). His wrestling with Mars was finished, but his path-breaking book would not be published for another four years, delayed by skirmishes with Tycho's heirs, who feared that Kepler would take all the juice out of Tycho's observations and reduce the value of the single greatest treasure in their inheritance.

Linz, August 1625

Thirty-two years after Tycho had recorded the great Martian catastrophe, the conditions were right for a repeat performance, and once again, in 1625, the Ptolemaic and Copernican tables showed a huge discrepancy. Because Mars was near the perihelion of its orbit—its closest approach to the Sun—the ellipticity of its orbit had nothing to do with the error in its longitude as observed from Earth. The entire error arose because Earth's orbit had been positioned wrong (see figure 4b). At that time Kepler was working on his great *Rudolphine Tables*, published in 1627, for predicting the positions of all the planets. With a correctly repositioned Earth, the error in Mars's predicted position disappeared. Kepler now knew that he had fixed the great Martian catastrophe.

Kepler's correction had reduced the maximum errors in the predicted positions for Mars by an order of magnitude, from nearly 5° to about 0.5°. That half-degree error came not when Mars was at perihelion but when Earth made a close approach to Mars at an octant position in its orbit. Later, after placing Earth in an elliptical orbit, Kepler reduced the maximum errors by another order of magnitude—from about 30′ to 2′.

Kepler died in 1630, before Galileo published his *Discourses and Mathematical Demonstrations Relating to Two New Sciences* (1638) with its premonition of the law of inertia. Without the concept of inertia, Kepler's physics was fatally flawed. Nevertheless, his intuition about the importance of physical causes was correct. Isaac Newton, in a 1686 letter to Edmond Halley about planetary orbits, perceptively remarked that "Kepler knew the orb to be not circular but oval & guest it to be elliptical."[6] Yet Newton was selling Kepler short, because in his insistence on an astronomy based on celestial physics, Kepler was truly paving the way for Newton himself.

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