Introduction to Computational Mathematics and Physics

APAM E1601y

Prof. Michael Mauel
mauel@columbia.edu

Course Description

Introduction to computational methods in applied mathematics and physics: Students develop solutions in a small number of subject areas to acquire a first taste in the practical use of computers in solving mathematics and physics problems.

APAM1601 does not require prior programming experience (but prior computer experience and talent are *helpful*.) Topics change from year-to-year, and only a limited number of topics (typically four per term) are selected for discussion and investigation. Topics range from classical and modern physics and applied mathematics, but the course is not meant to cover these areas broadly. Instead, each topic will be self-contained and limited in scope. We try to make topics interesting and absorbing, and they will amplify and expand on a student's knowledge acquired during your first year of physics and mathematics course work. The goal of this course is to provide some depth in select topics instead of providing a general (but shallow) overview of an entire subject area.

Examples include elementary interpolation of functions, solution of nonlinear algebraic equations, curve-fitting and hypothesis testing, wave propagation, fluid motion, gravitational and celestial mechanics, chaotic dynamics. (APAM1601 is usually taught by a team of two professors, an applied physicist and an applied mathematician. This semester, the course will have a stronger computational physics emphasis because of the interests of the instructor.)

The basic requirement for this course is one year of college level Calculus and Physics; programming experience is desirable but not required. Students will gain confidence preparing, understanding, and presenting computer-assisted solutions to elementary but relevant problems.

This course uses *Mathematica* combined with detailed classnotes.

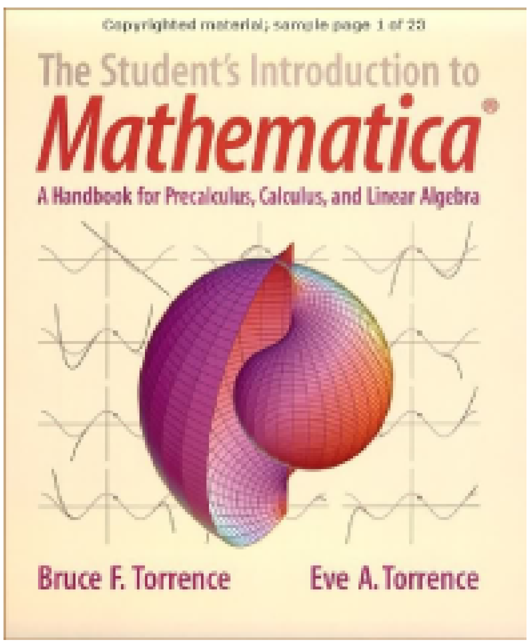
Why *Mathematica*? *Mathematica* is a very powerful system for evaluating expressions and graphing results. It's disadvantage is that it is commercial software, but it's advantage is that it is available for all major operating systems and available widely at Columbia University. The most important reason I selected *Mathematica* is that it is an interpretive system: *Mathematica* instantly responds to every expression you write. Additionally, *Mathematica* notebooks contain a perfect record of your interaction with the computer. Save your notebooks, edit them, and re-execute them. In the beginning, the syntax of *Mathematica* will seem complicated. Later, it will become second nature. Experience using *Mathematica* will be developed as the complexity of the subject material is advanced. In addition to classnotes distributed by the instructors, a textbook illustrating the use of *Mathematica* for science and engineering will be required.

(For those interested in other numerical research packages (like MATLAB, IDL, or others), please see me. I *will* accept projects in any language or system, but I will have only time in class to discuss these other packages/programming systems.)

Textbook and References

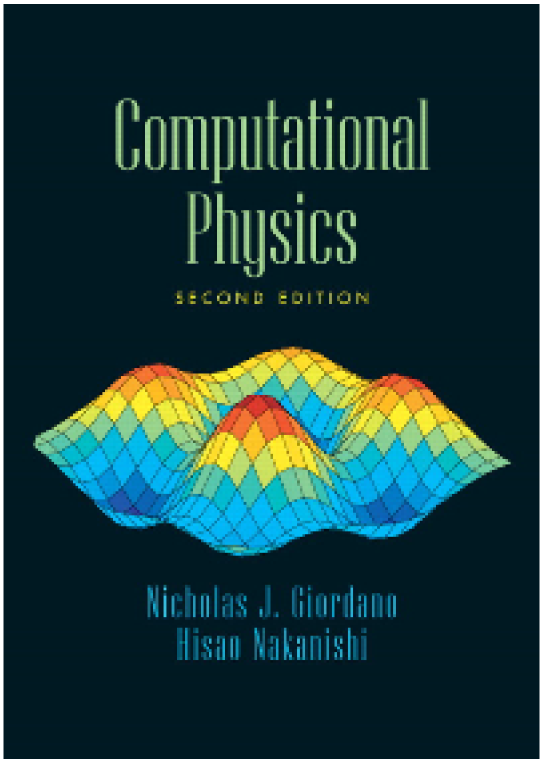
Textbooks are not required for APAM1601. *Mathematica* is fully documented in your browser. When you program with *Mathematica* **always have the help browser window open**.

For those of you not able to learn by reading a computer display, I recommend a good basic book for learning *Mathematica* by Bruce and Eve Torrence:



A Student's Introduction to Mathematica, by B. Torrence and E. Torrence, Cambridge (1999), ISBN 0-521-59461-8. (About $30.00 cheep). Now in **2nd Edition**, but new version won't be available until the end of February.

Although also not required, I will be base my lectures on a very nice textbook by Nicholas Giordano from Purdue University. This is a textbook designed for Juniors and Seniors, but some of you may want to purchase the textbook as a reference or as a source of additional and background material.



Computational Physics, by Nicholas Giordano, Prentice Hall (1997), ISBN 0-13-367723-0. (About $90.00 expensive; $47 used from Amazon.com). Also, now in **2nd Edition** (published in 2005). *Very popular and useful*.

Remember, ***you do not required to purchase either of these textbooks***. *All* lectures will be available online, and *Mathematica* is fully and conveniently documented by the "Help Browser".

Class Organization and Grades (Important)

This class combines lectures and computer demonstrations. During lectures, the fundamentals of each topics will be presented and discussed. During demonstrations, we will use *Mathematica* to explore and experiment. All of the classroom demonstrations will be available online. You should copy these notebooks to your own computer, load them into *Mathematica,* and experiment.

You will not make progress in this course without using Mathematica!

The class notebooks are available on the our class webpage :

<http://www.apam.columbia.edu/courses/ap1601y/>

Since your work can be conveniently saved as *Mathematica* notebooks, you can record and edit your work for later discussion and presentation. Remember, when you work with a Columbia University workstation, you need to use a file-transfer program (like WINSCP) to transfer your notebooks to and from your directory space on CUNIX. Setup a directory on your CUNIX account to place all of your work.

This semester, we will discuss four subjects. To help you get to know *Mathematica* and each topic, I will ask that you to prepare and submit ***simple exercises.*** **Your final grade, however, will be based entirely on your four *research projects*.** These will be *Mathematica* notebooks. I prefer you to submit each by email (with your UNI in the filename):

`<mailto:` mauel@columbia.edu>

[**Please note**: unless I tell you otherwise, please execute the menu command (from within *Mathematica*) "**Kernel>Delete All Output**" so that your *large* graphics files do not overwhelm my email box.]

After receiving your projects by email, I can run your programs and correct them if needed. If you have trouble doing this, you may give me a hardcopy. (When sending email, you will probably need to login into your `cunix` account, transfer any files from your local computer to `cunix`, and email the message from `cunix`.)

You should discuss your projects with Prof. Mauel (1) before you begin, (2) during your investigations, and (3) as you begin to finalize your project, presentation, and results. To help you develop your *Mathematica* ability, we will assign weekly exercises, and these will help you in the preparation of your projects.

Your grade will be based on entirely on the creativity, organization, and presentation of your research projects and your ability to complete the weekly exercises.

Grading will be based on primarily on your four research projects, but I will consider your exercises and your effort. There are no examinations.

■ TA: Chris Choi

This semester Chris Choi <csc2174@columbia.edu> will be available to assist students with their exercises, projects, and the various issues using *Mathematica*. *Please contact* Eric to find out the best time and place to meet with him. If you need some help and I am not available, ask Eric to schedule some tutorials and problem solving sessions.

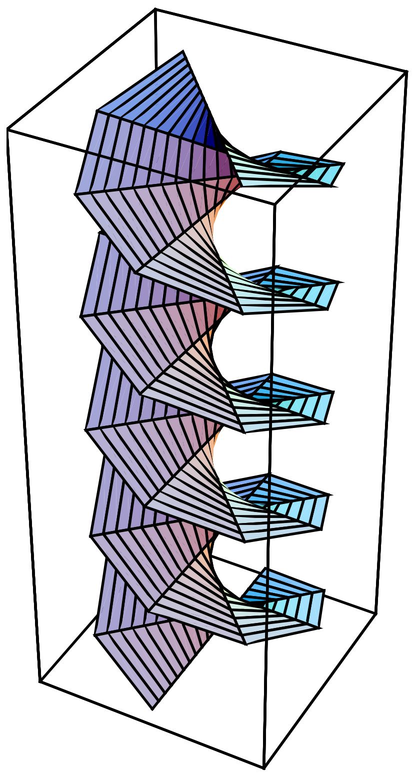
■ Class Schedule

Lectures will occur once a week at a day and time that best meets the needs of students.

We plan to cover four subjects in four three-week periods. Prior to this, we will spend two weeks getting up to speed with *Mathematica*. Although **these subjects may change**, presently, the class schedule is listed below. (Feel free to contact me with your ideas about topics. Physics and applied mathematics extends across a great many topics, and I would be very interested in preparing a three-week session on a topic of your choice!)

January 22 - January 31: *Mathematica* Basics

```
ParametricPlot3D[{u Sin[t], u Cos[t], t/3},
  {t, 0, 15}, {u, -1, 1}, Ticks → None]
```



Introduction to *Mathematica*.

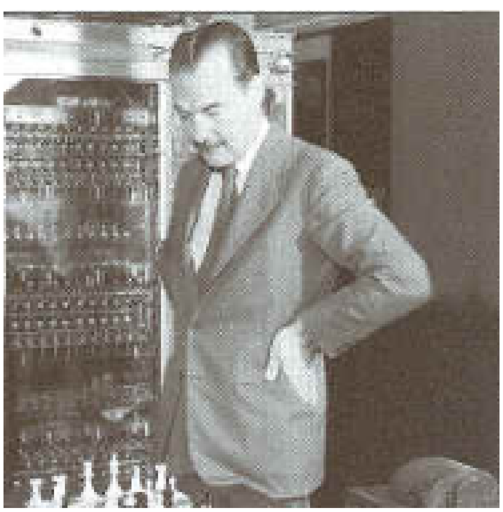
Featured mathematician: Steve Wolfram

Feb 7 - 21: ***Planetary Motion, Comets, and Asteroids (Ch. 4)***

`NDSolve[...]` and Central Force Problems.

Featured physicist: Kepler and his Laws

February 28 - March 14: Working with random numbers (Ch. 7)



Random[…], probability, statistics, and *phase-transition*.

Nicholas Metropolis (1915-1999) with the MANIAC computer.

Marshall Rosenbluth (1927-2003) the originator of the "Metropolis" algorithm for statistical physics simulations.

(With some help from Ludwig Boltzman (1844-1906)!)

Spring break: March 18-22

March 26 - April 5: *Fourier transforms and Digital Signal Processing*

Fourier[...], music, and jpeg!

(With some help from J.W. Tukey (1915-2000), who formalized the FFT!)

April 9 - 25: *Waves and Quantum Mechanics (Ch. 10)*

Time-dependent Schrödinger equation,

Solving partial differential equations.

Featured physicist: Schrödinger

(but it is the computational math that makes this topic interesting.)

About the faculty

■ Prof. Michael Mauel

http://www.columbia.edu/~mem4/
mailto: mauel@columbia.edu

My primary research interests are high temperature plasma physics and understanding how plasma dynamics impact confinement by magnetic fields and the design of thermonuclear power plants. By conducting controlled experimental studies using a variety of magnetic confinement devices, my students and I are attempting to further our fundamental understanding of the linear and nonlinear dynamics of microscopic and macroscopic plasma instabilities. The results of the experiments are applied to both man-made plasma confinement devices, such as future thermonuclear fusion plants, and to plasmas confined in planetary magnetospheres.

I am involved with two major experiments in Columbia's Plasma Physics Laboratory and with several experimental efforts at MIT, the national Plasma Physics Laboratory at nearby Princeton, and the General Atomics DIII-D tokamak located outside San Diego. At each location, large high-temperature plasma experimental devices are used to study the operational limits for fusion-relevant plasma discharges. In the high-beta tokamak experiment at Columbia (called HBT-EP), we combine detailed magnetic measurements, taken at several locations within the tokamak discharge, with sophisticated numeric techniques in order to correlate our observations with the theoretical predictions of ideal magnetohydrodynamical (MHD) equilibrium and linear stability. Direct measurements of the growth and saturation of pressure and current-driven kinks are possible, thereby testing the theories used to design much larger fusion devices. I am particularly interested in discovering the use of active and passive plasma control techniques to make the fusion reactor smaller and more economical.

My research is also directed to understand the fundamental processes involved with collisionless transport of plasma trapped within planetary magnetospheres. At Columbia, we have created the first "artificial radiation belt" by creating an energetic population of electrons trapped around a large, steady-state magnetic dipole, or terrella. A much larger experiment has been built at MIT using a strong, levitated superconducting magnet. See the Levitated Dipole Eperiment's website. A rich and evolving spectrum of intense low frequency fluctuations enable us to make controlled measurements of non-linear particle dynamics, precisely identify the onset of particle chaos, and establish basic principles of plasma transport. Someday, we may understand the physics of space and of the laboratory and be able to design and build a practical "magnetic star" on Earth.