Electrodynamics of neutron star magnetospheres
An example of non-neutral plasma in astrophysics

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Non Neutral Plasma Workshop, New York - 20/6/2008
What is a pulsar?
- Observations
- Theory

Electrosphere
- How to do?
- Geometry

Stability properties
- Diocotron instability
- Magnetron instability
- Non-linear evolution
  - Quasi-linear model

Conclusions
INTRODUCTION
**Discovery**

**The first pulsar**

- discovered fortuitously at Cambridge Observatory (UK) in 1967 at radio-frequencies
  - signal made of a **series of pulses** separated by a period \( P = 1.337 \text{ s} \)
  - pulse profile changes **randomly** but arrival time stable
  - duration of a pulse \( \Delta t \approx 16 \text{ ms} \)
    \[ \Rightarrow \text{size of the emitting region} : L \leq c \Delta t \approx 4800 \text{ km} \]
    \[ \Rightarrow \text{evidence for a compact object} \]

**Radio signal measured from PSR1919+21**

*(Bell & Hewish, 1968) (already 40 years ago)*

![Radio signal](image)

**Basic assumption**

Pulsar = strongly magnetised rotating neutron star
Typical neutron star parameters

Orders of magnitude

- mass $M_* = 1.4\ M_\odot$
- radius $R_* = 10\ \text{km} \ (R_\odot = 700,000\ \text{km})$
- mean density $\rho_* = 10^{17}\ \text{kg/m}^3 \ (\rho_\odot = 1.410\ \text{kg/m}^3)$
- crust temperature $T_* = 10^6\ \text{K}$
- moment of inertia $I_* = 10^{38}\ \text{kg m}^2$
- magnetic field strength at the stellar surface $B_* = 10^5...8\ \text{T} \ (B_\odot = 5 \times 10^{-3}\ \text{T})$
- induced electric field $E_* = 10^{10...13}\ \text{V/m} \Rightarrow \text{particles extracted from the surface}$
- particle density in the magnetosphere $n = 10^{17}\ \text{m}^{-3}$

Magnetic field strength estimation

Magnetic field intensity at the stellar crust estimated from dipolar magnetic radiation in vacuum, assuming a dipolar magnetic field (period $P$ slowly increases $\dot{P} = dP/dt > 0$)

$$B \propto \sqrt{P \dot{P}}$$

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>Period $P$</th>
<th>Period derivative $\dot{P}$</th>
<th>$B_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>radio</td>
<td>1 s</td>
<td>$10^{-15}$</td>
<td>$10^8\ T$</td>
</tr>
<tr>
<td>millisecond</td>
<td>1 ms</td>
<td>$10^{-18}$</td>
<td>$10^5\ T$</td>
</tr>
</tbody>
</table>
The “standard cartoon” (not a physical model)

Fundamental problem in astrophysics

- no measurement/experiment in situ possible
- only observations coming from the electromagnetic radiation emitted
  ⇒ underlying plasma processes must be studied indirectly
The space charge distribution in the magnetosphere

Basic physics
(Goldreich-Julian, 1969)

Assumptions
- aligned rotator ($\vec{\Omega}_* \parallel \vec{\mu}$)
- closed magnetosphere entirely filled with the corotating plasma
- electrostatic equilibrium: $\vec{E} + \vec{v} \wedge \vec{B} = 0$
- the corotating charge density at equilibrium $\rho = -2 \varepsilon_0 \vec{\Omega}_* \cdot \vec{B}$
- the null surface: region where the charge density vanishes ($\vec{\Omega}_* \cdot \vec{B} = 0$)
- particles follow the electric drift motion in the $\vec{E} \wedge \vec{B}$ direction
- open field lines sustain a wind

Does it work? Is this picture self consistent and stable?
- does not pulse (because aligned rotator)!
- simulations have shown that it is NOT stable!

Extracting charges from the stellar surface
⇒ non vacuum solution

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Charged wind: source of particles

Pair creation cascades (Sturrock 1970, Ruderman & Sutherland 1975)

Assumptions
- corotation impossible outside the light cylinder \( R_L = c/\Omega_\ast \)
- charged wind emanating from the polar caps
- charged particles \((e^- e^+)\) are produced by \(\gamma + B \rightarrow e^+ + e^-\) in the polar caps
- open field lines sustain a wind made of particles of both signs ⇒ increase or decrease of the total charge of the system (star+magnetosphere) ⇒ no constraint to force charge conservation

Inconsistent global picture
⇒ problem of the current closure
ELECTROSPHERE

= 

PART OF THE MAGNETOSPHERE
FILLED WITH PLASMA
Magnetospheric model

What is the structure of the magnetosphere?
How does a stable plasma distribution look like for a pulsar?
All models proposed so far are electrodynamically unstable and not self-consistent!!

Assumptions

- the neutron star = perfect spherical conductor of radius $R_*$, generating a dipolar magnetic field of strength $B_*$ and in solid body rotation with speed $\Omega_*$
- an aligned rotator, i.e., magnetic moment and spin axis are parallel
- charges extracted freely from the stellar crust whatever their nature
- magnetic field induced by the magnetospheric currents are neglected $\Rightarrow$ constant and dipolar
- any force other than electromagnetic is neglected (even the gravitational attraction, $F_g/F_{em} = 10^{-9}$!!)
- electric drift approximation $\vec{v} = \frac{\vec{E} \wedge \vec{B}}{B^2}$

Numerical simulations

- extract particles from the neutron star crust and let them fill the magnetosphere until they reach equilibrium
- stop when no more charges are left on the stellar surface
Plasma configuration

An example (Pétri, Heyvaerts & Bonazzola, A&A 2002)

3D structure of the electrosphere

Rotation rate

Surface charge

Charge density

Total charge of the system $Q_{\text{tot}} = \text{only free parameter}$
Results: magnetospheric structure

Main features

- Magnetosphere of both sign of charge
- Finite in extent
- Large gaps appear between the equatorial belt and the polar domes
- No electric current circulation in the gaps
- Differential rotation of the disk, overrotation ⇒ shearing between magnetic surfaces responsible for instabilities
- Same qualitative conclusions apply whatever the total charge \( Q_{\text{tot}} \) of the neutron star+magnetosphere

Neutron stars: a natural trap for non-neutral \( e^\pm \) plasma?

The electromagnetic field acts as a trap, confinement of the non-neutral \( e^\pm \) plasma:

- In the “radial” direction by the dipolar magnetic field
- In the “axial” direction by the quadrupolar electric field
STABILITY PROPERTIES
The question of the stability of the electrosphere

<table>
<thead>
<tr>
<th>Equilibrium &amp; stability</th>
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| Equilibrium does not imply stability.  
The previous model can be unstable to non-neutral plasma instabilities |

<table>
<thead>
<tr>
<th>Different approximations</th>
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</thead>
<tbody>
<tr>
<td>Studying the whole 3D structure to complicated $\Rightarrow$ simplifications</td>
</tr>
<tr>
<td>We restrict to a 2D configuration of the equatorial disk only</td>
</tr>
<tr>
<td>- a cylinder of infinite axial extent</td>
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<tr>
<td>- an infinitely thin disk</td>
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<th>Different analysis</th>
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<td>linear stability $\Rightarrow$ growth rate obtained from an eigenvalue problem</td>
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<tr>
<td>non-linear simulations</td>
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<td>quasi-linear model</td>
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<th>Different regimes</th>
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<tr>
<td>diocotron/magnetron</td>
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<td>non-relativistic/relativistic</td>
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The diocotron regime

Assumptions
- plasma column of infinite axial extent in z
- axisymmetric with differential azimuthal rotation $\Omega(r)$
- electric drift approximation (no inertia)
- only electrostatic perturbations (background magnetic field constant)
- non relativistic
- boundary conditions : inner and outer conducting walls

Equilibrium profile & Growth rates (Pétri A&A, 2007a)

![Equilibrium profile & Growth rates graph]
Relativistic stabilisation effects

Assumptions

- electric drift approximation (no inertia)
- full set of Maxwell equations taken into account
  \( \Rightarrow \) possibility of electromagnetic wave radiation
- boundary conditions: inner and outer conducting walls or outgoing e.m. waves
- relativistic regime

Growth rates for outer wall and outgoing waves (Pétri A&A, 2007b)

Main results

- relativistic motion stabilises the diocotron instability (all growth rates decrease)
- outer boundary conditions do not significantly affect the instability
The relativistic magnetron regime

Assumptions

- electric drift replaced by Lorentz force

\[
\frac{D\vec{p}}{Dt} = \left( \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} \right) \left( \gamma m \vec{v} \right) = q \left( \vec{E} + \vec{v} \wedge \vec{B} \right)
\]

therefore particle inertia \( m \) included

- full set of Maxwell equations taken into account

\[ \Rightarrow \] possibility of electromagnetic wave radiation

- relativistic regime

Growth rate for outer wall and outgoing waves (Pétri A&A, 2008)
Diocotron instability: non-linear evolution

**Goal:** determine the consequences of the diocotron instability

Two complementary problems
- isolated system (confinement theorem \(\Rightarrow\) negligible charge losses)
- magnetosphere fed with charges by pair creation.

**Method:** 2D numerical simulations

- **the charge conservation** in cylindrical coordinates \((r, \varphi)\) with a source term \(s\)

\[
\frac{\partial \sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \sigma v_r) + \frac{1}{r} \frac{\partial}{\partial \varphi} (\sigma v_\varphi) = s
\]  

\(\sigma\) charge density in the disk
- **the Poisson equation** with help on Green functions \(G\):

\[
\Phi_D(\vec{r}) = \iint_{\text{Disk}} G(\vec{r} | \vec{r}') \sigma(\vec{r}') dS
\]

- **the equation of motion** (electric drift)

\[
\vec{v} = \frac{\vec{E} \wedge \vec{B}}{B^2} \quad \text{(3)}
\]

\[
\vec{E} = -\vec{\nabla}(\Phi_* + \Phi_D) \quad \text{(4)}
\]
Non linear evolution : disk without any source
Non linear evolution: disk feeded with a source of charges
Diocotron instability: quasi-linear model

Assumptions

- model applies only when a few modes $m$ are excited and incoherent
- description of the mean density profile $< \sigma >(t)$ (1D problem)

Idea

- decompose all physical quantities $\psi(r, \varphi, t)$ in a mean value $< \psi >(r, t)$ and a fluctuation $\delta \psi(r, \varphi, t)$ around this average as follows
  
  $$\psi(r, \varphi, t) = < \psi >(r, t) + \delta \psi(r, \varphi, t)$$
  
- non-linearities kept in the evolution of $< \sigma >$
- non-linearities ignored for the perturbations.

Diffusion equation for $< \sigma >$

The quasi-linear model describes the behaviour of the average density by

$$\frac{\partial}{\partial t} < \sigma > = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mathcal{D}(r, t) \frac{\partial}{\partial r} \left( \frac{< \sigma >}{B_0} \right) \right) + s(r, t)$$

$\mathcal{D}(r, t)$ diffusion coefficient
Disk with an external source of charges
Conclusions of the simulations

Non linear simulations

- without charge feeding, particles tend to accumulate to form supercharges of high density and rotating at a high speed around the neutron star
- with an external source of charges, the initial extension of the disk grows slowly in time
- there is no significant current sustained at this stage

⇒ simulations on a longer timescale are necessary
⇒ quasi-linear model.

Quasi-linear model

- model in agreement with the full non linear evolution
- there exists a situation for which the diocotron instability generates a net flux of charges radially outwards

A positive equatorial current exists

It is an alternative solution to the current closure problem.
Conclusions & Perspectives

My pulsar model: a trap for non-neutral astrophysical electron-positron plasmas

Properties

- Electrosphere do exist, finite in extension and in electrostatic equilibrium.
- Non-neutral plasma instabilities (diocotron & magnetron) develop.
  ⇒ Particle diffusion across the magnetic field lines.
- Numerical simulations have shown the formation of an equatorial current carrying a net flux of charges towards the light cylinder.
Conclusions & Perspectives

Perspectives

- oblique rotator
  ⇒ 3D simulations
  ⇒ restore the pulsed emission properties
- 3D relativistic PIC simulations
- relativistic non-neutral plasma beams flowing in the polar caps
  ⇒ coherent electromagnetic radio-emission (brightness temperature $10^{25}$ K) ?
- method applicable to find other equilibrium solutions, multipolar electromagnetic fields

Pulsar as a laboratory?

- relativistic motion
- unusual confinement geometry (electric and magnetic field)
- electron-positron plasma
  ⇒ watch the sky!
Physics of pulsars: everything remains to be done!

Apart from the obvious need of
- general relativity
  - neutron star equation of state
  - gravitational waves
  - electromagnetic field enhancement by frame dragging effect
- quantum electrodynamics ($e^\pm$ pair creation)

Physics of pulsar needs two essential ingredients
- non neutral plasma physics
  - trapping of particles in special traps
  - stability properties of the plasma configuration
  - instabilities like diocotron and magnetron
- plasma distribution does not overlap with the magnetosphere
  $\Rightarrow$ large vacuum gaps and thus “electrospheric solution” and NOT magnetospheric one.

1967-2008 = 41 years after the discovery
with little (no) progress !!!
APPENDIX
Self-consistency?

- localised description at the magnetic poles
- how does the global magnetospheric structure look like to explain this model?
## The “models”

### “Standard” cartoons: corotating magnetosphere filled with plasma

- **The polar cap** (Sturrock 1971, Ruderman & Sutherland 1975)
  - Particle *acceleration* and *radiation close* to the neutron star surface (at the magnetic poles).

- **The outer gap** (Cheng et al. 1986)
  - Particle *acceleration* and *radiation* in the vicinity but *inside* the light cylinder.

- **The two-pole caustic** (Dyks & Rudak 2003)
  - Particle *acceleration* and *radiation* from the neutron star surface up to the light cylinder.

### Some alternative models

- **The electrosphere** (Krause-Polstorff & Michel 1985, Pétri et al. 2002a)
  - The magnetosphere is almost completely *empty*!
  - Electrosphere \( \equiv \) regions of the magnetosphere filled with a non-neutral plasma
  - Physics of pulsar electrosphere much more complicated and interesting than the previous cartoons

- **The striped wind** (Coroniti 1990, Michel 1994)
  - Radiation emanating from *outside* the light cylinder.
Outer gap model: high-energy emission

**Aim**

To explain the high energy component of the pulsar’s spectrum (gamma emission)

*(Cheng, Ho & Ruderman 1986)*

**Assumptions**

- The outer gaps are located between the light cylinder and the null surface.
- The photon disintegration is impossible because $B$ too weak.
- The pair cascade initiated by photon-photon interaction in the outer gaps, $\gamma + \gamma \rightarrow e^+ + e^-$.
- The curvature photons emitted tangentially to the local magnetic field lines.

**An alternative model**

The two pole caustic = slot gap from the light cylinder down to the stellar surface (Dyks & Rudak 2003).
Polar gap vs outer gap prediction

**Discrimination**
- polar cap model predicts super-exponential pair creation decay with distance because of photon splitting probability in a strong magnetic field
- outer gap predicts exponential decay
⇒ sharp or smooth cut-off at the highest energies

**High-energy cut-off**

![Graph showing high-energy cut-off](attachment:graph.png)