Overview of Intense Beam Simulation Experiments Performed Using the Paul Trap Simulator Experiment (PTSX)*

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9th International Workshop on Nonneutral Plasmas
Columbia University
June 17th, 2008

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*This work is supported by the U.S. Department of Energy.
PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

• Purpose: PTSX simulates, in a compact experiment, the transverse nonlinear dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems.

• Applications: Accelerator systems for high energy and nuclear physics applications, heavy ion fusion, spallation neutron sources, and high energy density physics.

Other Intense-Beam Studies, Paul-trap-based and otherwise:

Okamoto and Tanaka
Drewsen et al.
Kishek et al.
Scientific Motivation

As self-field effects become important, it is important to develop an understanding of:

• Beam mismatch and envelope instabilities;
• Collective wave excitations;
• Chaotic particle dynamics and production of halo particles;
• Mechanisms for emittance growth;
• Compression techniques; and
• Effects of distribution function on stability properties.
Magnetic Alternating-Gradient Transport Systems

\[ B_{q}^{\text{foc}}(x) = B'_q(z) \left( y\hat{e}_x + x\hat{e}_y \right) \]
\[ F_{\text{foc}}(x) = -\kappa_q(z) \left( x\hat{e}_x - y\hat{e}_y \right) \]

\[ \kappa_q(z) \equiv \frac{ZeB'_q(z)}{\gamma m\beta c^2} \]
Transverse Focusing Frequency and Phase Advance Characterize the Motion

In one lattice period, $S$, the smooth trajectory’s vacuum phase advance, $\sigma_v$, is 35 degrees.

Transverse focusing frequency, $\omega_q$, must be less than 180 for bounded orbits.

Space-charge decreases $\sigma_v$ to $\sigma$.

In one lattice period, $S$, the smooth trajectory’s vacuum phase advance, $\sigma_v$, is 35 degrees.
PTSX Configuration – A Cylindrical Paul Trap

\[ e \phi_{ap}(x, y, t) = \frac{1}{2} \kappa'_q(t)(x^2 - y^2) \]

\[ \kappa'_q(t) = \frac{8eV_0(t)}{m\pi r_w^2} \]

\[ \xi = \frac{1}{2\sqrt{2\pi}} \]

\[ \xi = \frac{\eta\sqrt{3} - 2\eta}{4\sqrt{3}} \]

\[ \sigma_v^{sf} = \frac{\omega_q f}{f^2} \approx \frac{V_{0\text{ max}}}{f^2} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma length</td>
<td>2 m</td>
</tr>
<tr>
<td>Maximum wall voltage</td>
<td>~ 400 V</td>
</tr>
<tr>
<td>Wall radius</td>
<td>10 cm</td>
</tr>
<tr>
<td>End electrode voltage</td>
<td>&lt; 150 V</td>
</tr>
<tr>
<td>Plasma radius</td>
<td>~ 1 cm</td>
</tr>
<tr>
<td>Frequency</td>
<td>&lt; 100 kHz</td>
</tr>
<tr>
<td>Cesium ion mass</td>
<td>133 amu</td>
</tr>
<tr>
<td>Pressure</td>
<td>5x10^{-9} Torr</td>
</tr>
<tr>
<td>Ion source grid voltages</td>
<td>&lt; 10 V</td>
</tr>
<tr>
<td>Density</td>
<td>10^5-10^6 cm^{-3}</td>
</tr>
</tbody>
</table>
Transverse Dynamics are the Same Including Self-Field Effects

\[
B^{\text{foc}}_q(x) = B'_q(z) \left( y\hat{e}_x + x\hat{e}_y \right)
\]

\[
F^{\text{foc}}_q(x) = -\kappa_q(z) \left( x\hat{e}_x - y\hat{e}_y \right)
\]

\[
\kappa_q(z) = \frac{ZeB'_q(z)}{\gamma m \beta c^2}
\]

\[
\psi = \frac{Ze}{\gamma m \beta^2 c^2} \left[ \phi(x, y, z) - \beta A_z(x, y, s) \right]
\]

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi = -\frac{2\pi K}{N} \int dx' dy' f_b
\]

\[
e\phi_{ap}(x, y, t) = \frac{1}{2} \kappa'_q(t)(x^2 - y^2)
\]

\[
\kappa'_q(t) = \frac{8eV_0(t)}{m\pi r_w^2}
\]

usual \( \phi_{\text{self}}(x, y, t) \)

Field Equations

Vlasov Equation

\[
\left\{ \frac{\partial}{\partial s} + x' \frac{\partial}{\partial x} + y' \frac{\partial}{\partial y} - \left( \kappa_q(s) x + \frac{\partial \psi}{\partial x} \right) \frac{\partial}{\partial x'} - \left( -\kappa_q(s) y + \frac{\partial \psi}{\partial y} \right) \frac{\partial}{\partial y'} \right\} f_b = 0
\]
Transverse Dynamics are the Same Including Self-Field Effects

If…

• Long coasting beams
• Beam radius << lattice period
• Motion in beam frame is nonrelativistic

Then, when in the beam frame, both systems have…

• Quadrupolar external forces
• Self-forces governed by a Poisson-like equation
• Distributions evolve according to nonlinear Vlasov-Maxwell equation

Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system in the beam frame.
Smooth-Focusing Equilibria are Parameterized by $s$

In thermal equilibrium, 

$$n(r) = n(0) \exp \left[- \frac{m \omega_q^2 r^2 + 2 q \phi^s(r)}{2 kT} \right]$$

Poisson’s equation

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \phi^s}{\partial r} = \frac{q n(r)}{\varepsilon_0}$$

becomes a nonlinear equation for $\phi^s$ that must be solved numerically.

$s = 0 \ldots 0.9$

$s = 0.99$

$s = 0.9999$

$s = 1$

$\zeta(s)$ is determined numerically.

$\zeta(0) = 1$

$\zeta(1) = 2$

Normalized intensity parameter $s$.

$s \sim 0.2$ for SNS and Tevatron injector.

$s \sim 0.99$ for HIF

$$s \equiv \frac{\omega_p^2(0)}{2 \omega_q^2} < 1$$
Global Force Balance

If $p = n kT$, then the statement of local force balance on a fluid element can be manipulated to give a global force balance equation.

$$m n \omega^2 q r = q n E - kT \frac{\partial n}{\partial r}$$

$$\int_0^\infty r^2 \, dr \left( m n \omega^2 q r = q n E - kT \frac{\partial n}{\partial r} \right)$$

$$m \omega^2 q R^2 = \frac{N q^2}{4 \pi \varepsilon_0} + 2kT$$

\[ \vec{F}_p = -m n \omega^2 q \vec{r} \]

$\vec{F}_E = n q \vec{E}$

$\vec{F}_{\text{pressure}} = -kT \vec{\nabla} n$

where $R$ is the root-mean-squared radius and $N$ is the line charge.

Note that $N$ and $R$ are measured by integrating the experimentally obtained density profiles $n(r)$.

$kT$ is inferred from this global force balance.
Electrodes, Ion Source, and Collector

Broad flexibility in applying $V(t)$ to electrodes with arbitrary function generator.

Increasing source current creates plasmas with intense space-charge.

Large dynamic range using sensitive electrometer.

Measures average $Q(r)$.
PTSX Simulates Equivalent Propagation Distances of 7.5 km

- At $f = 75$ kHz, a lifetime of 100 ms corresponds to 7,500 lattice periods.

- If $S$ is 1 m, the PTSX simulation experiment would correspond to a 7.5 km beamline.

\[ s = \frac{\omega_p^2}{2\omega_q^2} = 0.18. \]

\[ V_0 = 235 \text{ V} \]
\[ f = 75 \text{ kHz} \]
\[ \sigma_v = 49^\circ \]
Temporary Amplitude Changes Cause Mismatch Oscillations that Heat the Plasma

\[ m \omega^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_0} \]

Voltage Waveform Amplitude

5 Cycles

Experimental Data

\[ Q(r=0) \mathrm{pC} \]

Number of Cycles

WARP 2D

Exp. Data: 30 % Increase

Exp. Data: 50 % Increase
Transverse Bunch Compression by Increasing $\omega_q$

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

If line density $N$ is constant and $kT$ doesn’t change too much, then increasing $\omega_q$ decreases $R$, and the bunch is compressed.

$$\omega_q = \frac{8eV_{0\text{ max}}}{m\pi r_w^2 f\xi}$$

Either
1.) increasing $V_{0\text{ max}}$ (increasing magnetic field strength) or
2.) decreasing $f$ (increasing the magnet spacing) increases $\omega_q$
Compression is Possible but Increasing Phase Advance Degrades Transverse Confinement

\[ \omega_q \rightarrow 1.5 \omega_q \]

\[ N \sim \text{constant} \]
- \( s = 0.08, kT = 1.6 \text{ eV} \)
- \( s = 0.14, kT = 1.4 \text{ eV} \)
- \( s = 0.02, kT = 4.7 \text{ eV} \)

\[ m \omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o} \]

\[ \omega_q \propto \frac{V_{0 \text{ max}}}{f} \]

\[ \sigma_v \propto \frac{V_{0 \text{ max}}}{f^2} \]

\[ \sigma_v \quad 60\% \]
- \( \sigma_v = 50^\circ \)
- \( \sigma_v = 75^\circ \)
- \( \sigma_v = 112^\circ \)

\[ \text{Emittance increases} \]
Adiabatic Amplitude Increases Transversely
Compress the Bunch Better

20% increase in $V_{0\text{ max}}$

90% increase in $V_{0\text{ max}}$

Baseline

<table>
<thead>
<tr>
<th>$R$ (cm)</th>
<th>$kT$ (eV)</th>
<th>$s$</th>
<th>$\epsilon \sim R\sqrt{kT}$</th>
<th>$\Delta \epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.83</td>
<td>0.12</td>
<td>0.20</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>0.79</td>
<td>0.16</td>
<td>0.18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adiabatic

<table>
<thead>
<tr>
<th>$R$ (cm)</th>
<th>$kT$ (eV)</th>
<th>$s$</th>
<th>$\Delta \epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.63</td>
<td>0.26</td>
<td>0.10</td>
<td>10%</td>
</tr>
<tr>
<td>0.93</td>
<td>0.58</td>
<td>0.08</td>
<td>140%</td>
</tr>
</tbody>
</table>

Instantaneous

<table>
<thead>
<tr>
<th>$s$</th>
<th>$\Delta \epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88</td>
<td>150 V</td>
</tr>
<tr>
<td>60 kHz</td>
<td>49°</td>
</tr>
</tbody>
</table>

- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $\nu/v_0 = 0.88$
- $V_{0\text{ max}} = 150$ V
- $f = 60$ kHz
- $\sigma_v = 49°$
Less Than Four Lattice Periods are Needed to Make the Transition Adiabatic

\[ \sigma_v = 111^\circ \]
\[ \sigma_v = 81^\circ \]
\[ \sigma_v = 63^\circ \]

WARP 2D simulations are in good agreement.

- \( s = \omega_p^2 / 2 \omega_q^2 = 0.20 \)
- \( \nu / \nu_0 = 0.88 \)
- \( V_{0\text{ max}} = 150 \text{ V} \)
- \( f = 60 \text{ kHz} \)
- \( \sigma_v = 49^\circ \)
Increasing $\omega_q$ by Adiabatically
Decreasing $f$

$$V(t) = V_{0\text{ max}} \sin \phi(t)$$

$$\omega_q = \frac{8eV_{0\text{ max}}}{m\pi r_w^2 f}$$

$$\phi(t) = \frac{f_i + f_f t + f_f - f_i}{2\pi} \tau \ln \left[ \cosh \left(\frac{t-t_{1/2}}{\tau/2}\right) \right] - f_0$$

$$\dot{\phi}(t) = f_f t + \frac{f_f - f_i}{2} \left[ \tanh \left(\frac{t-t_{1/2}}{\tau/2}\right) + 1 \right]$$
Adiabatically Decreasing $f$ Compresses the Bunch

- $s = \omega_p^2/2\omega_q^2 = 0.2$.
- $\nu/\nu_0 = 0.88$
- $V_{0\text{max}} = 150$ V
- $f = 60$ kHz
- $\sigma_v = 49^\circ$

33% decrease in $f$

Good agreement with KV-equivalent beam envelope solutions.
Transverse Confinement is Lost When Single-Particle Orbits are Unstable

\[ \frac{\phi(t)}{2\pi} = f_i t + \frac{f_i - f_f}{2} t \left[ \tanh \left( \frac{t - t_{f/2}}{\tau/2} \right) + 1 \right] \]

\[ \sigma_v^{sf} = \frac{\omega_q}{f} \propto \frac{1}{f^2} \]

Measured \( \tau_c \) (dots)

Set \( \sigma_{v_{\text{max}}} = 180^\circ \) and solve for \( \tau_c \) (line)

\( f_0 = 60 \text{ kHz} \)

\( \tau_c = \tau_c \)
Good Agreement Between Data and KV-Equivalent Beam Envelope Solutions

\[ f_0 = 60 \text{ kHz} \]

\[ \tau_0 = 0 \]

\[ \tau_0 = 19.9 \]

\[ \tau = \tau_c \]

\[ \tau_0 = 26 \]
Measuring Beam Oscillations and Inferring Beam “Normalized Intensity” s

Segmented collective-mode capacitive pick-up diagnostic is sensitive to beam collective-mode oscillations.

Measured frequencies…
… determine unique “normalized intensity.”

\[
\ell = 0 \text{ body-wave breathing mode} \\
\ell = 2 \text{ surface-wave quadrupole mode}
\]

\[
\omega = \omega_q \left(4 - 3s\right)^{1/2}
\]

\[
\omega = \omega_q \left(4 - 2s\right)^{1/2}
\]
PTSX is a Compact Experiment for Studying the Propagation of Beams Over Large Distances

• PTSX is a versatile research facility in which to simulate collective processes and the transverse dynamics of intense charged particle beam propagation over large distances through an alternating-gradient magnetic quadrupole focusing system using a compact laboratory Paul trap.

• PTSX explores important beam physics issues such as:
  • Beam mismatch and envelope instabilities;
  • Collective wave excitations;
  • Chaotic particle dynamics and production of halo particles;
  • Mechanisms for emittance growth;
  • Compression techniques; and
  • Effects of distribution function on stability properties.