The Turbulent Structure of a Plasma Confined by a Magnetic Dipole

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Understanding Turbulence is a Grand Challenge

- Ubiquitous in natural and laboratory plasmas.
- Turbulence drives transport in fusion devices.
- The magnetic dipole allows local and global measurements of turbulence.
- Visualizing the structure of turbulence is the first step in understanding.

Key Result

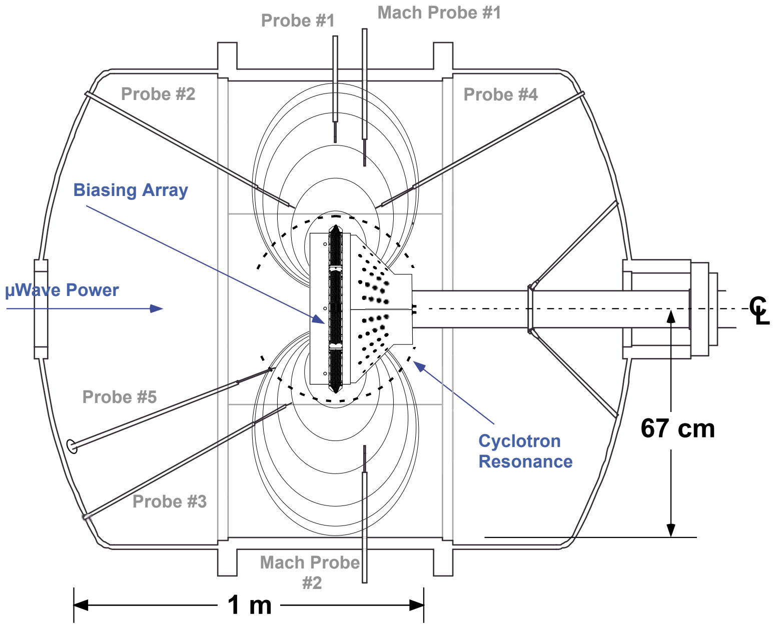
Fully developed turbulence in a dipole plasma is not described by microturbulence, but by the complex temporal dynamics of a small number of long wavelength modes.

Outline

- Making turbulent plasmas in CTX.
- Characterization of the turbulent structure.
  - Single and two-point correlation.
  - Global imaging of turbulent structure.
  - Bi-orthogonal decomposition measures the dominant mode structure of the turbulence.
- Why is there no microturbluence? Possible reason: Bispectral analysis shows small scales are damped, and transfer power with an inverse cascade to large scales.

We report the first global statistical study of the turbulent structure of plasma confined by a dipole magnetic field. During steady state turbulence, detailed measurements of the correlation between spatially separated diagnostics reveal the correlation time and length of fluctuations perpendicular and parallel to the magnetic field in the bulk plasma. The fluctuations exhibit a power-law spectrum, dominated by low frequency, long wavelength modes in the azimuthal direction, which extend to the system size. The structure of the turbulence is radially broad, with a zero parallel wavenumber ($k_{||} \approx 0$), indicating interchange-like modes. Measurement of the field-aligned current at 96 points at the magnetic pole confirm the mode structure found in the bulk plasma. Fourier-based methods are used to measure the nonlinear power transfer between modes, and indicate that power is transferred from the high frequency, short wavelength modes to the low frequency, long wavelength modes.

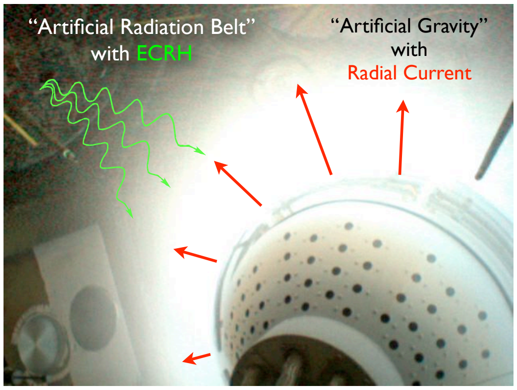
Dipole Geometry Provides Easy Access for Plasma Turbulence Studies



CTX is Well Suited to Look at the Structure of Instabilities

- Previous research with lower density plasmas allowed investigation of individual instabilities:
    - Hot electron interchange (HEI) mode driven by hot electron pressure. Exhibits frequency sweeping[*].
    - Centrifugal instability driven by strong rotation[†].

➡ This research examines higher density plasmas in a steady turbulent state.

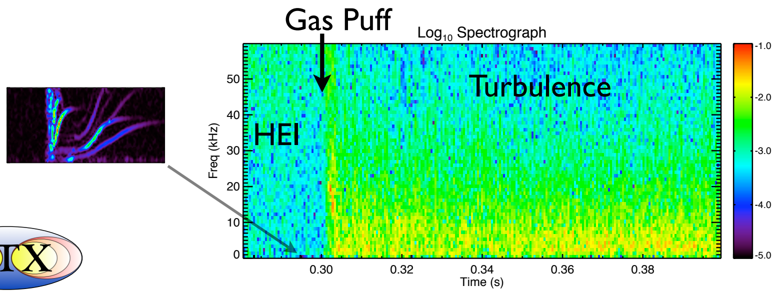


[*] Maslovsky, Levitt, Mauel PRL **90** , 2003.

[†] Levitt, Maslovsky, Mauel PRL **94** , 2005.

Increasing Plasma Density Causes Dramatic Transition to Steady Turbulence

- Low density plasmas have quasi-periodic instability bursts with nonlinear evolution.
- The transition to a turbulent regime is marked by a large amplitude coherent mode.
- Steady turbulence remains for duration of discharge.



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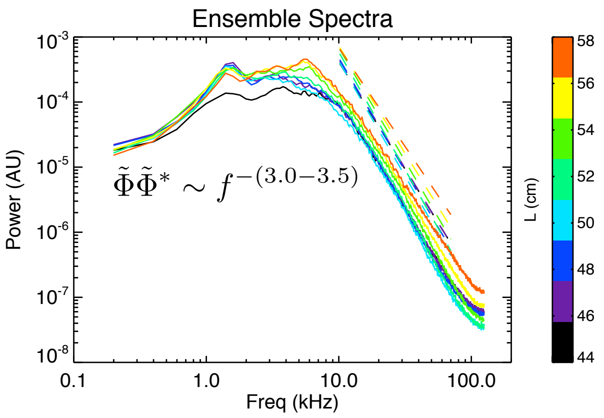
Single-Point Measurements Show Power-Law Spectrum Across the Plasma Radius

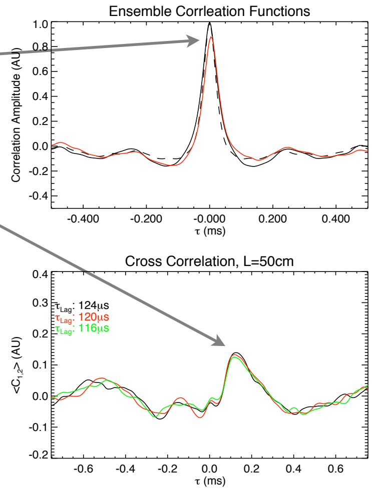
- Measurements taken at 2 cm increments across the plasma.
- The power-law spectrum for f>10 kHz is approximately uniform.
- Typical of turbulence



Two Probes Are Correlated at Short Scales and Decorrelate by 45 cm

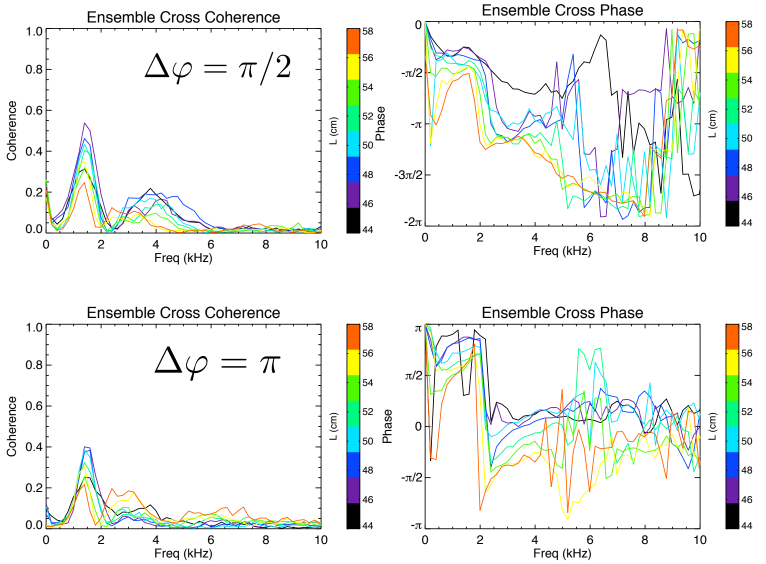
$$C_{1,2}(\tau) = \frac{\int S_1(t)S_2(t-\tau)dt}{\sqrt{\int S_1^2(t)dt \int S_2^2(t)dt}}$$

- Probes separated by $\Delta\varphi=9^\circ$ (~8 cm) show almost perfect correlation, ~90%, with a small positive lag time.
- Observation of long-wavelength correlation For probes separated by $\Delta\varphi=90^\circ$.
- Correlation amplitude decreases with probe separation ($\Delta\varphi=9^\circ,90^\circ,180^\circ$).
- Decay $Ae^{-x/\lambda}$ where λ~45 cm ($\Delta\varphi$~50°). This is 14% of the device circumference.



Ensemble Fourier Statistics For Two Probes Reveals Azimuthal Mode Structure

- Two probes separated azimuthally by 90° and 180°.
- Hundreds of measured realizations.
- An m=1 mode exists at ~1-2 kHz.
- An m=2 mode exists at ~3-4 kHz.



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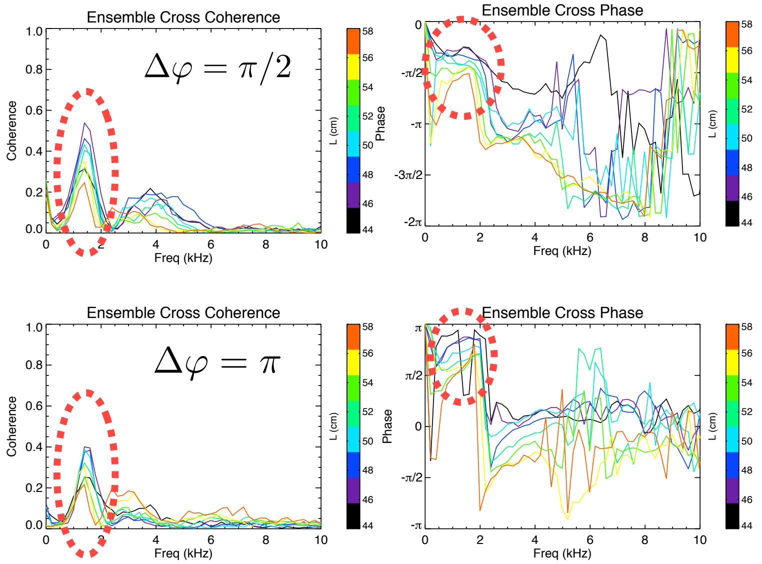

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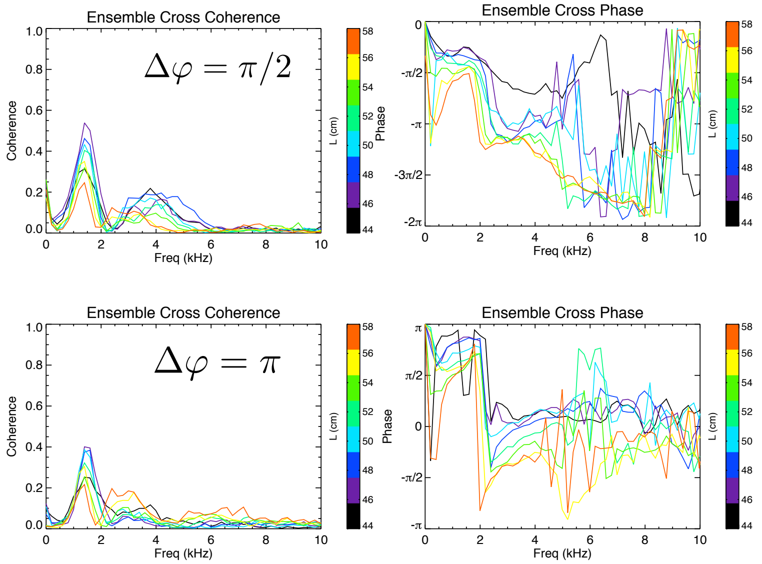

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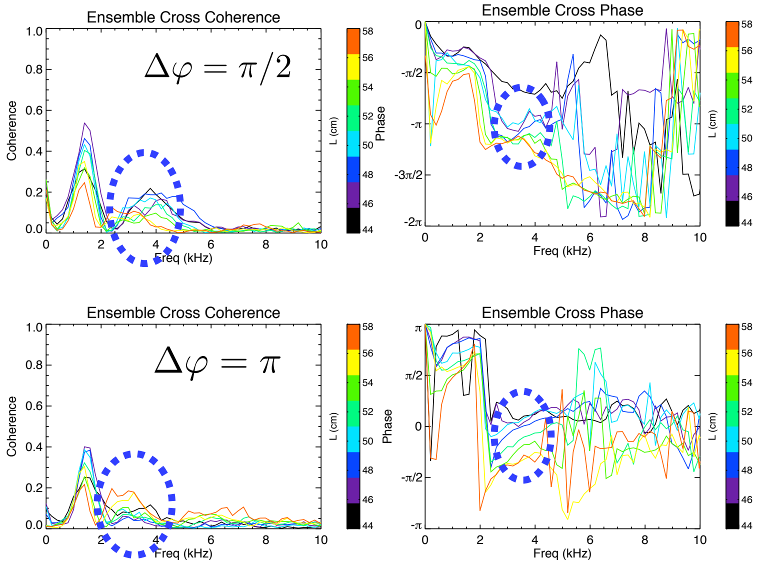

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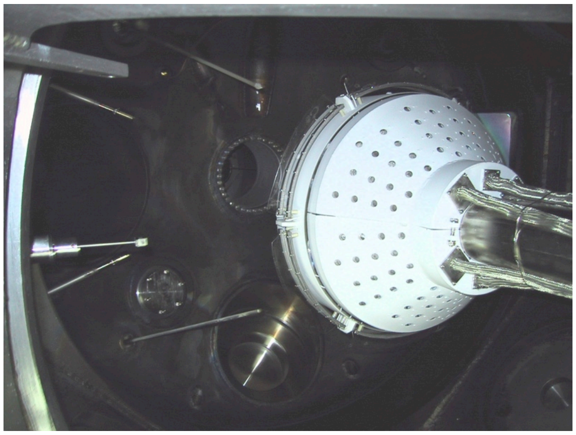
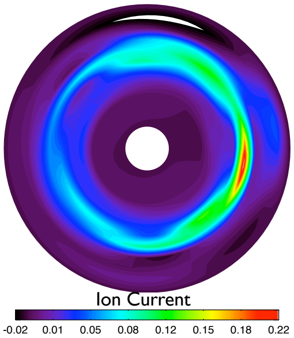
Simultaneous Global Turbulence Measurements Using the Polar Imager

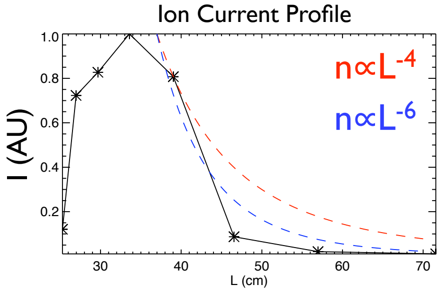
- 96 Gridded particle detectors digitized at high speed.
- Azimuthal spatial resolution $\Delta\varphi = 15^{\circ}$.



Profile of Polar Ion Current Shows Strong Central Peaking

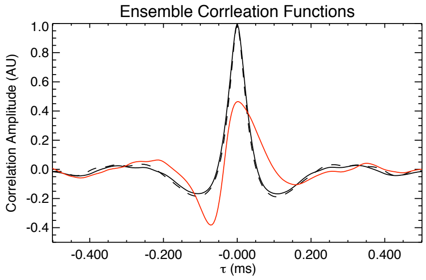
- Grids can be biased negatively to collect ion current.
- Provides a global image of plasma in time.

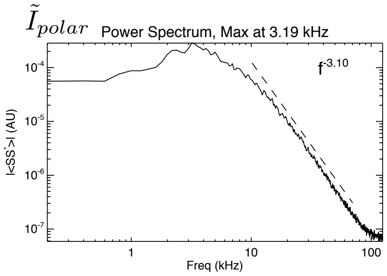


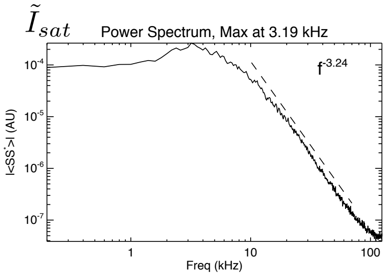


Individual Detectors are Correlated with Field-Aligned Probes

- The cross-correlation lag time is zero for a probe and detector on the same field line.
- Same spectral characteristics.

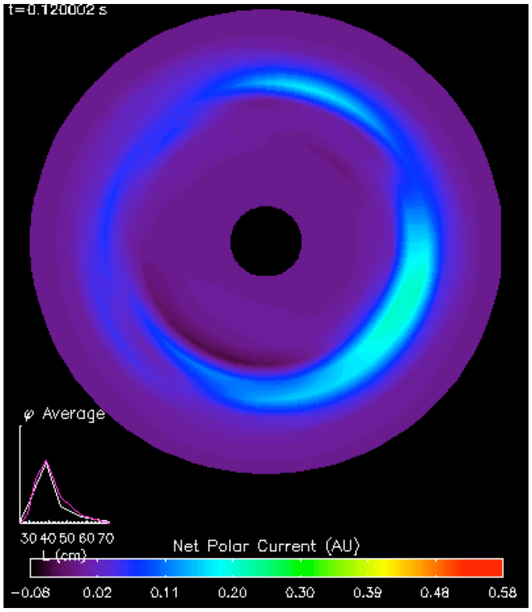




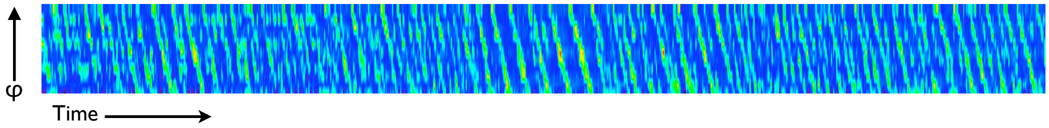


High Speed Imaging of Turbulence at 500,000 Frames Per Second

- Detectors biased to collect ion current.
- Density fluctuations rotate in E×B direction.
- Turbulence clearly visualized.



The Dominant Azimuthal Mode Structure Fluctuates



- 11 points around the circumference at the same radius.
- The azimuthal mode number amplitude can be calculated for every time point.

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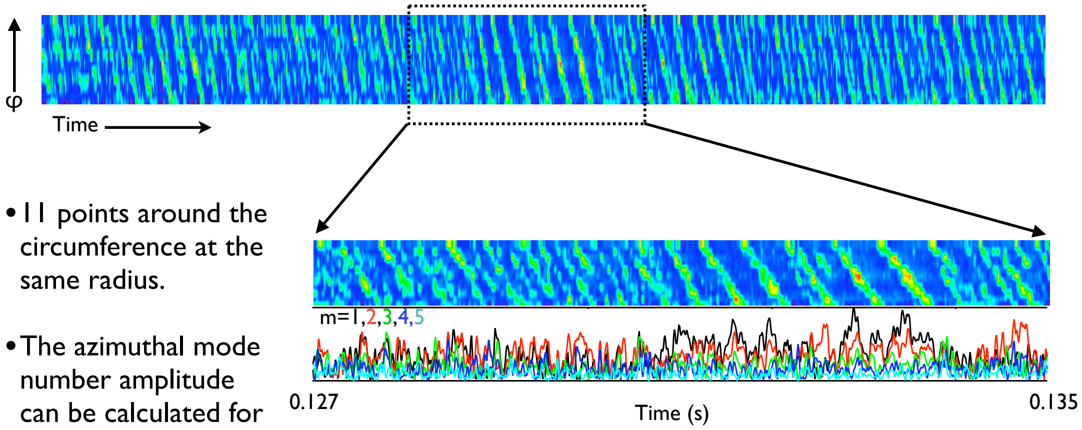

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The Bi-orthogonal Decomposition* Measures the Dominant Mode Structure

- The spatio-temporal measurement of density $n(x_i,t_j)$ is decomposed with the Singular Value Decomposition into orthogonal spatial and orthogonal temporal modes.
- There is no pre-defined basis. The basis functions are extracted from the data.
- The power in each mode is measured by a singular value.

*de Wit et. al. Phys. Plasmas **1** (10), 1994.

The Bi-orthogonal Decomposition* For Multiple Space-Time Points

- The values $(Y)_{i,j}$ are polar current (plasma density) at ‘M’ space points and ‘N’ temporal points.
- The singular value decomposition is used to calculate the values A_k, and the spatial φ_k and temporal ψ_k functions.
- Modes are orthogonal, but not pre-defined basis functions.

$$(Y)_{i,j} = \sum_{k=1}^{K} A_k \varphi_k(x_j)\psi_k(t_i)$$

$$\sum_{i=1}^{N} \psi_k(t_i)\psi_l(t_i) = \sum_{j=1}^{M} \varphi_k(x_j)\varphi_l(x_j) = \delta_{kl}$$

Eigenequations

$$S_x\varphi_k = A_k^2\varphi_k, \quad S_x = Y^TY$$

$$S_t\psi_k = A_k^2\psi_k, \quad S_t = YY^T$$

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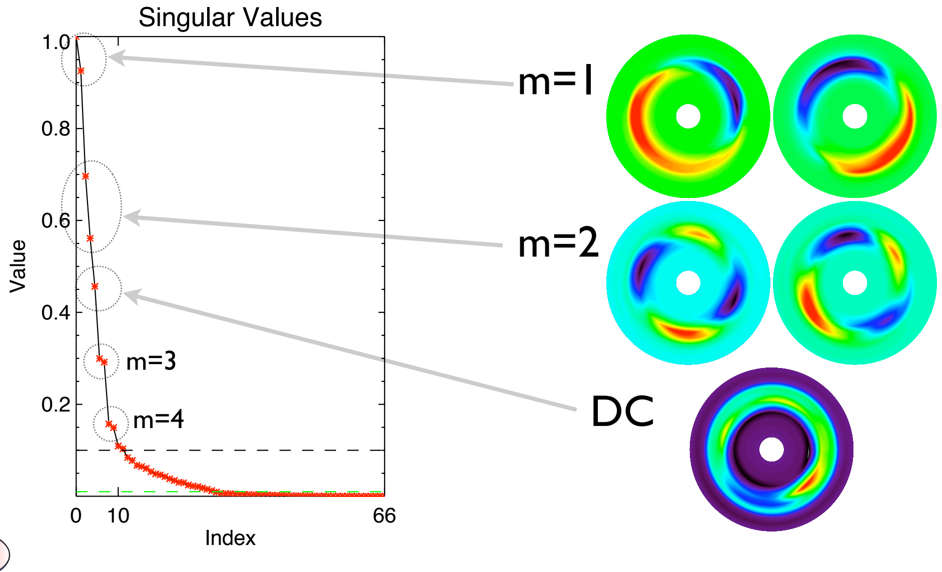
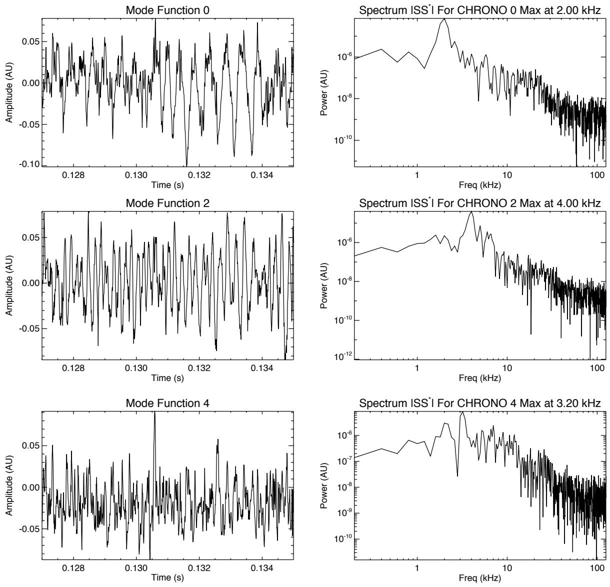
The Spatial Structure of Turbulence is Characterized by **Simple**, Long Wavelength Modes



Temporal Modes Show **Complex** Variation

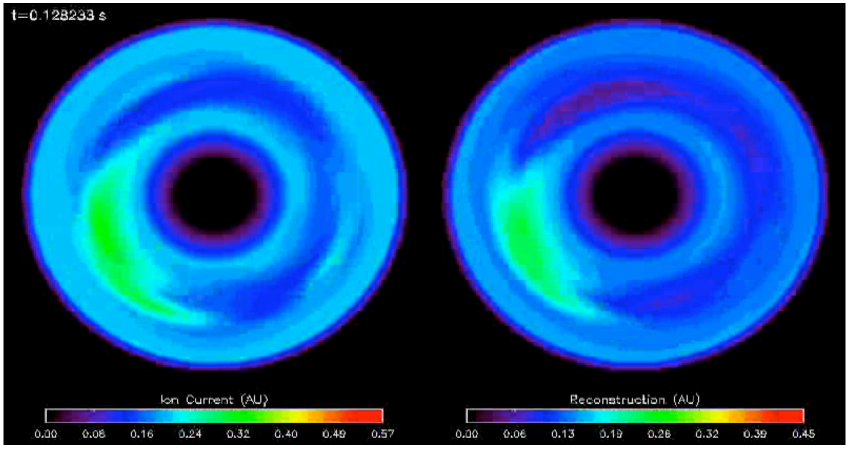
- For each mode, Sin and Cos phases indicate non-steady rotation.
- Fourier analysis shows the spectrum to be complex with a power law |SS*| like $f^{-2.5}$.



A Limited Number of Modes Reconstructs Original Data

Original Data

Reconstruction

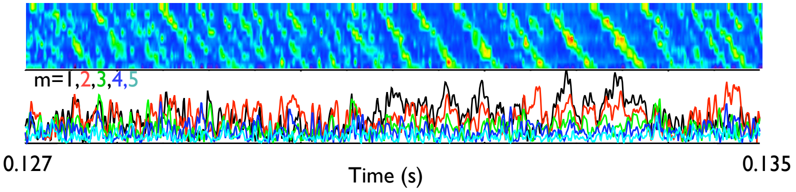


Using m=0,1,2,3,4

Global Reconstruction Reproduces Local Measurements

- Fluctuations are re-created using only a few modes.
- The local power spectra are reproduced.

Original Data



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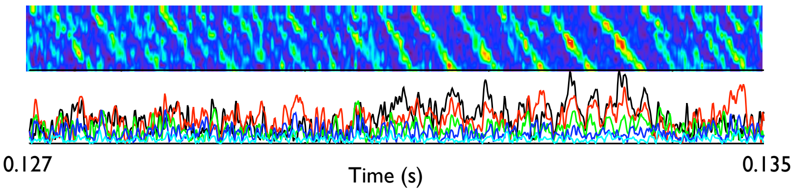

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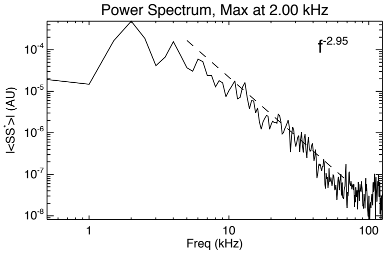

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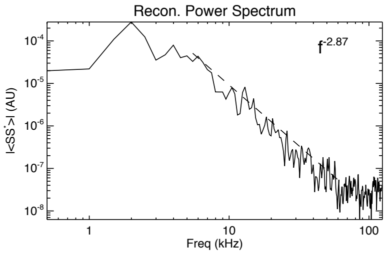

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Auto and Cross Bispectral Analysis Measures Linear and Nonlinear Mode Dynamics

- A method developed by Ritz, Kim, et al, and most recently used in the heliac by Xia and Shats.
- Solves for the complex linear dispersion ω_k and growth γ_k.
- Solves for the three-wave coupling coefficient, and determines direction of spectral energy flow.

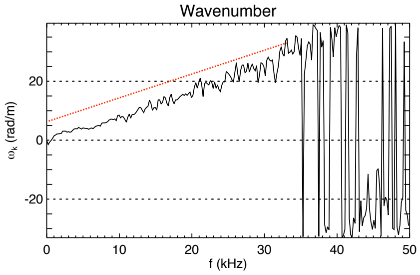
*Xia and Shats PRL **91** (15), 2003.

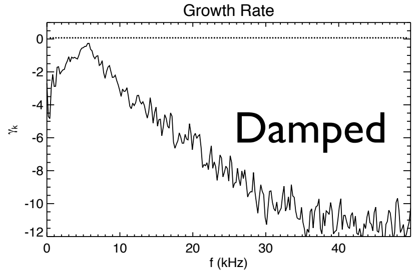
*Kim et. al. Phys. Plasmas **3** (11), 1996.

*Ritz et. al. Phys. Fluids B **1** (1), 1989.

Auto and Cross Bispectral Analysis Measures Linear and Nonlinear Mode Dynamics

- The dispersion is linear through 35 kHz, and agrees with rotation measurements of ~2 kHz.
- High frequency modes are damped.
- Marginally damped modes exist near 1-8 kHz.
- Energy is transferred via three-wave coupling from high frequency to low frequency, or small to large scale.





Damped

*Xia and Shats PRL **91** (15), 2003.
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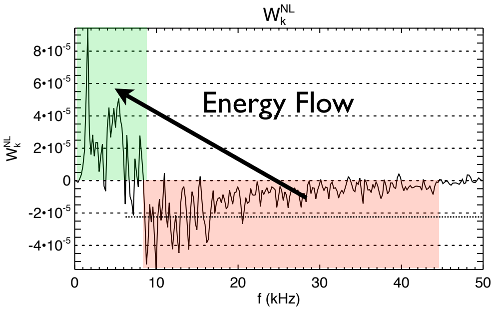

Inverse Cascade

*Xia and Shats PRL **91** (15), 2003.
*Kim et. al. Phys. Plasmas **3** (11), 1996.
*Ritz et. al. Phys. Fluids B **1** (1), 1989.

Summary

- Following a gas puff, a dipole confined plasma enters a steady-state turbulent regime.
- Locally, dipole turbulence exhibits a power-law spectrum.
- Two-point correlation and ensemble Fourier statistics reveal the correlation length to be less than the system size, and the azimuthal mode structure to be dominated by long wavelength modes.
- Simultaneous, high speed imaging of the plasma records the global dynamics of the structure of turbulence.
- Bi-orthogonal decomposition shows that the turbulence can be well represented by a limited number of **simple spatial modes**, having **complex temporal variation.**
- The observed structure of turbulence is consistent with bispectral analysis showing linear damping of high frequency modes, and a nonlinear inverse energy cascade to low frequency modes.

Thank You