CTX

Global and Local Characterization of Turbulent and Chaotic Structures in a Dipole-Confined Plasma

B. A. Grierson
Columbia University Plasma Physics Laboratory
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DEPARTMENT OF APPLIED PHYSICS AND APPLIED MATHEMATICS
COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK
FU FOUNDATION SCHOOL OF ENGINEERING AND APPLIED SCIENCE

Abstract

When the plasma density increases sufficiently, plasma confined by a strong dipole magnetic field exhibit a dramatic transition to a confined state with complex turbulent behaviors. Recent experiments using the Collisionless Terrella Experiment (CTX) used statistical tools and fast imaging to understand this turbulent state with respect to both local and global paradigms. Locally, multi-point and multiple-time autocorrelation and bispectral analyses are computed and used to estimate the linear dispersion and nonlinear structure coupling of a broad band of interacting fluctuations. Globally, the whole-plasma dynamics is observed using a unique high-speed imaging diagnostic that views the time-varying spatial structure of the polar current density. The bi-orthogonal decomposition for multiple space-time points is used to decompose the measured plasma dynamics into spatial and temporal mode functions.

The dominant spatial modes are found to be long wavelength and radially broad; however, the amplitudes of these global modes are chaotic and impulsive. In all cases, the fluctuations appear to be interchange-like and consistent with a model for two-dimensional electrostatic interchange mixing. To the best of our knowledge, this is the first time when both local and global dynamics of turbulent structures have been simultaneously measured and compared in hot magnetized plasma.

Our measurements are sufficient to compare and contrast two competing paradigms of plasma turbulence: (i) a nonlinear mode-mode structure coupling and cascade derived from a statistical treatment of measurements from closely-spaced probes, and (ii) a chaotic evolution of a few dominant and relatively long-wavelength modes that generate an equivalent local spectrum due to the impulsive amplitudes and time-varying frequencies of the global modes.

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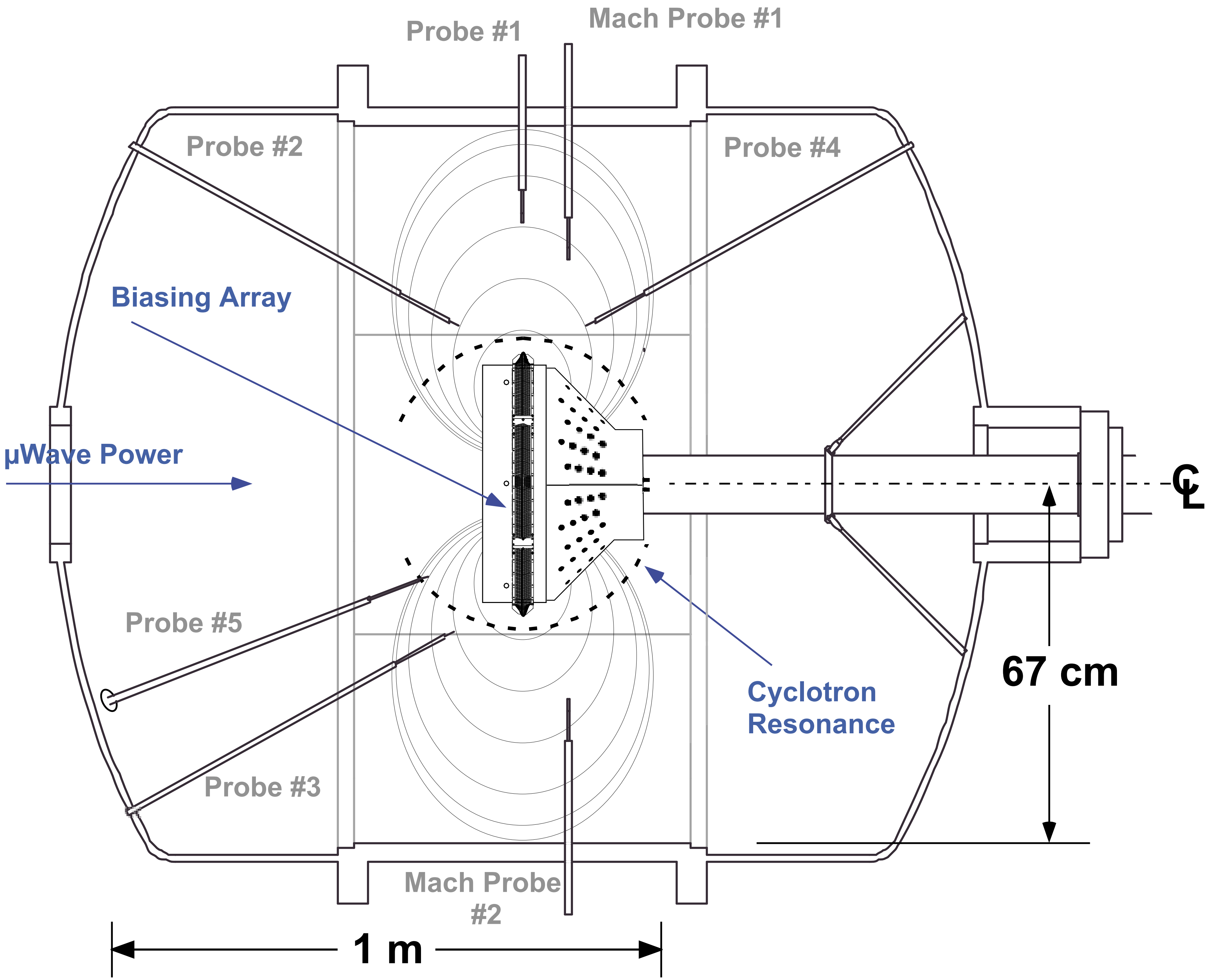
New Results

- The turbulence in CTX is dominated by large-scale, large amplitude, convective interchange motion. **Not micro-turbulence**.
- Small scale fluctuations are damped, and give their spectral power to large structures extending to the system size, consistent with the inverse energy, forward enstrophy cascade in 2D fluids and plasmas.
- The dynamics of the dominant modes in the system can be best described by a low-dimensional model, consisting of the chaotically varying amplitude of a limited number of simple global modes.

The Magnetic Dipole Provides a Geometry for 2D Plasma Turbulence Studies



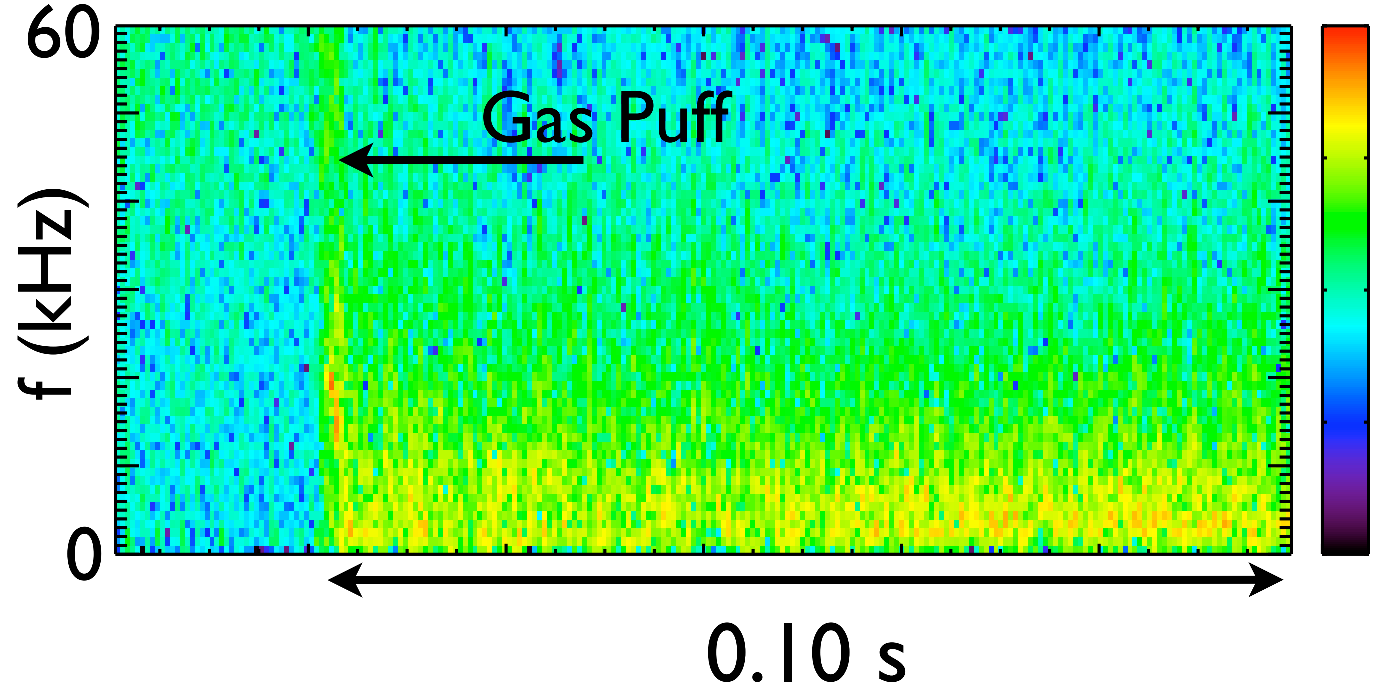
- Strong confining dipole magnetic field.
- No magnetic shear allows large, fluting modes.
- Long discharges for turbulence statistics.
- ECRH Hydrogen plasma.
- Fluctuations have $\omega \ll \Omega_{ci}$, with $k_{\parallel} \approx 0$
- Multiple movable probes for local measurements.
- Unique global imaging diagnostic.

Generating Turbulence in CTX

Increasing Plasma Density Causes Dramatic Transition to Steady Turbulence

- Low density plasmas in CTX have quasi-periodic instability bursts*, NOT steady turbulence.
- The transition to a turbulent regime, triggered by gas injection, is marked by a large amplitude coherent mode.
- Steady turbulence, with a high fluctuation level, remains for duration of discharge.

Spectrograph



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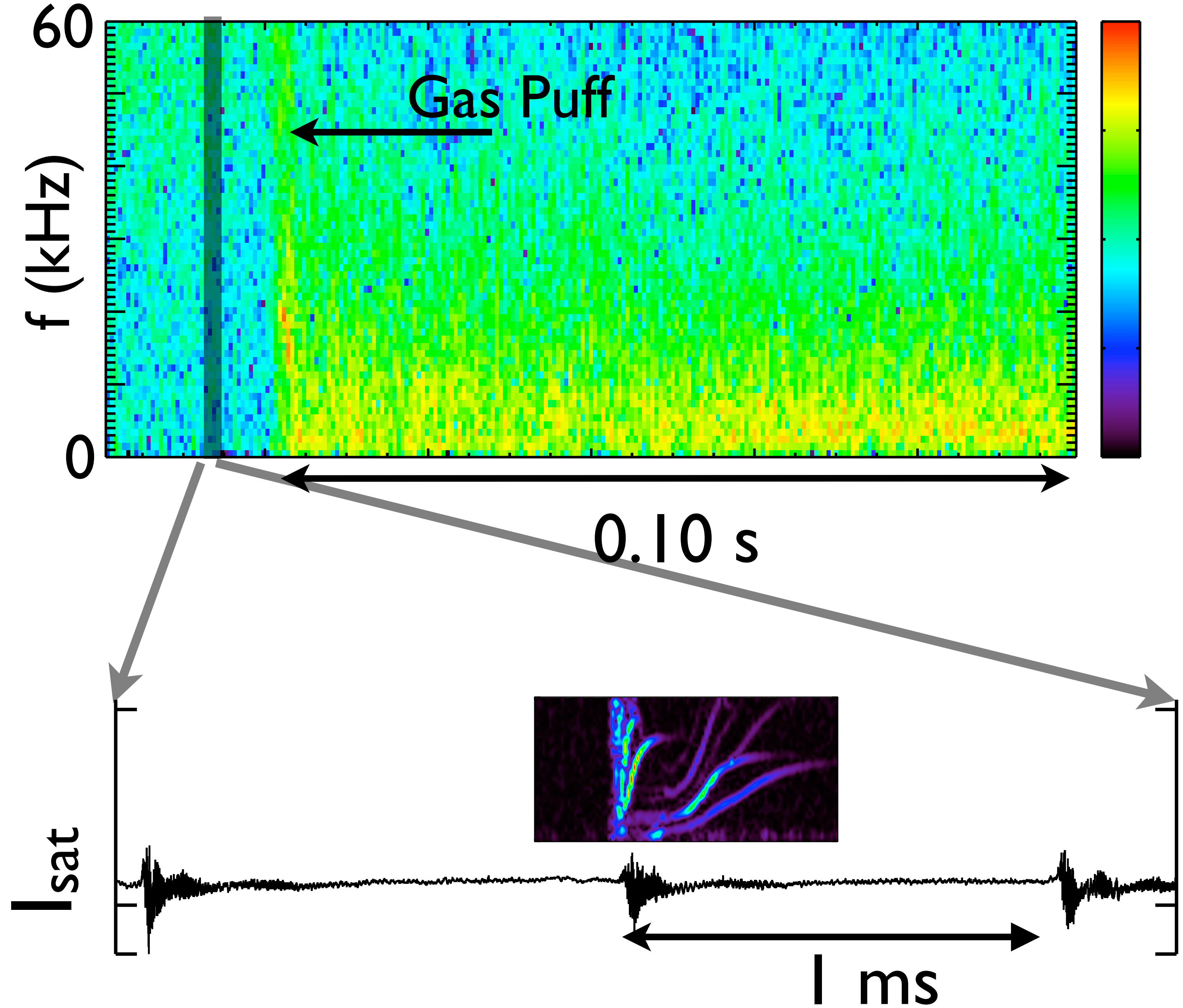

*Levitt, *et. al.* Phys. Plasmas **9** (6) 2002
Maslovsky, *et. al.* PRL **90** (18) 2003

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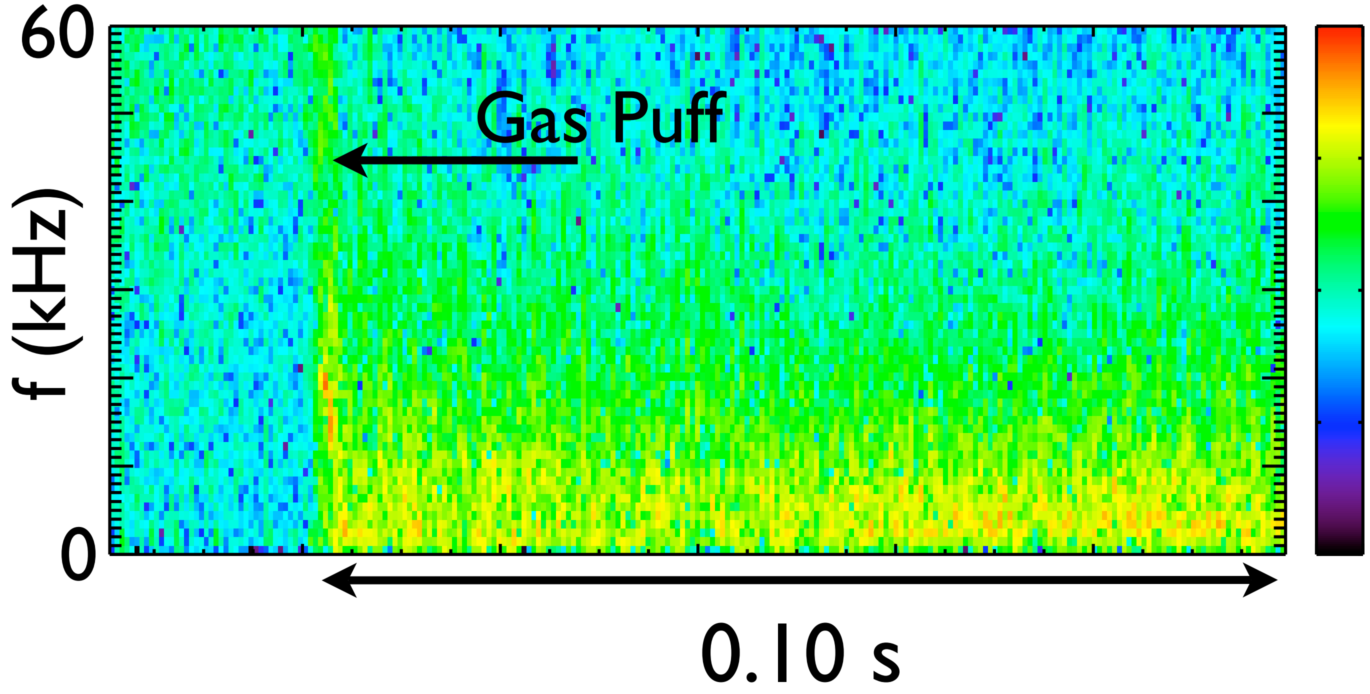

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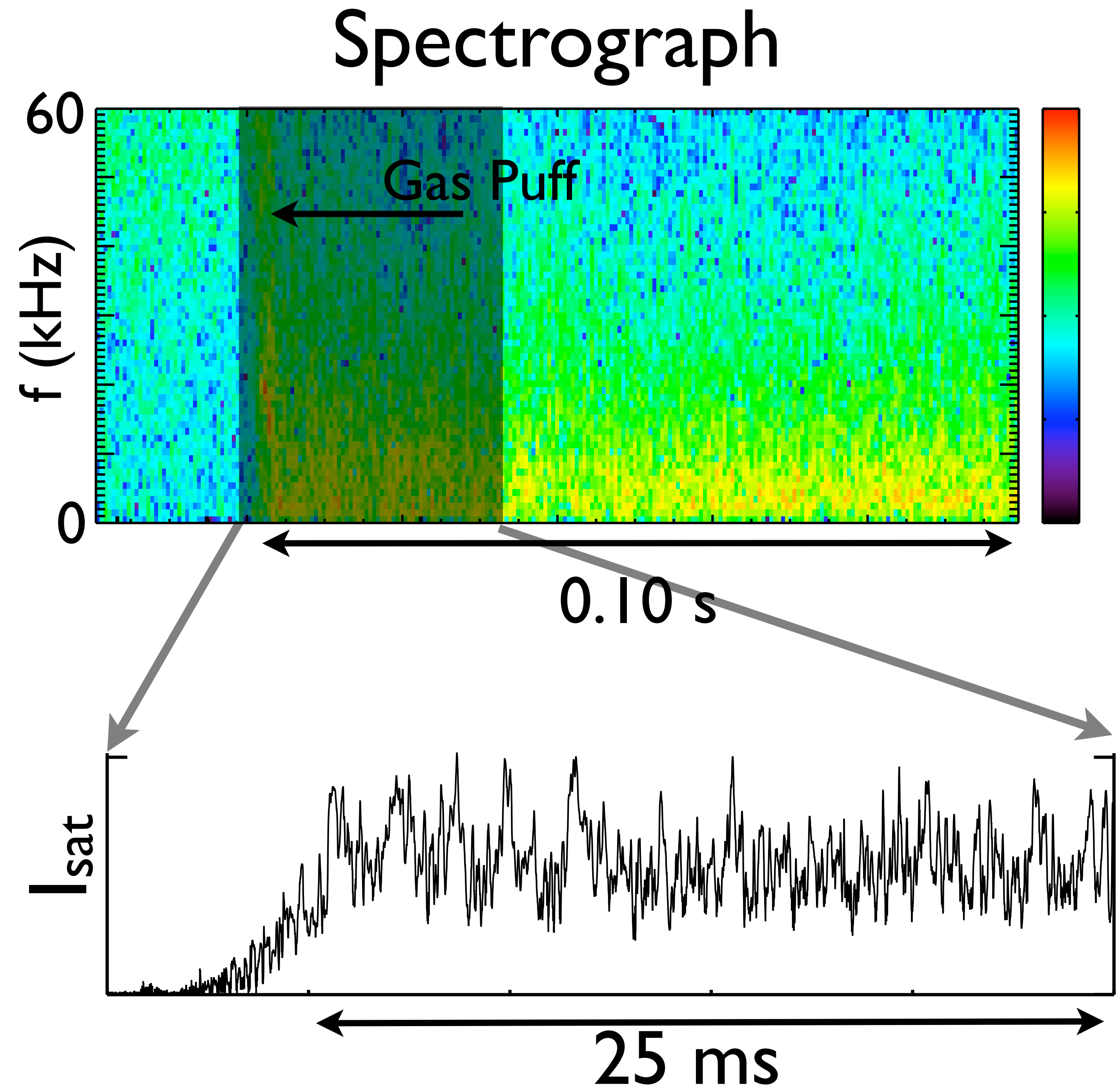

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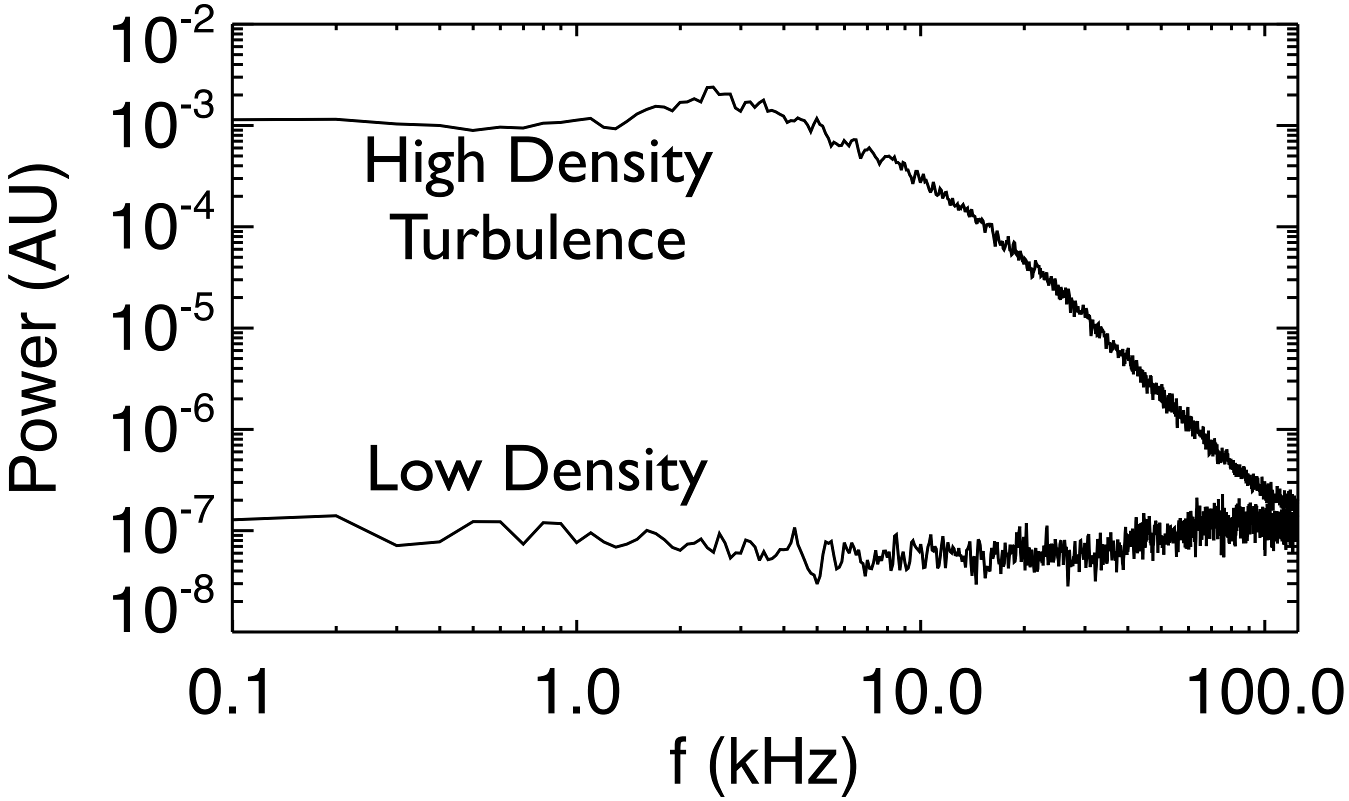
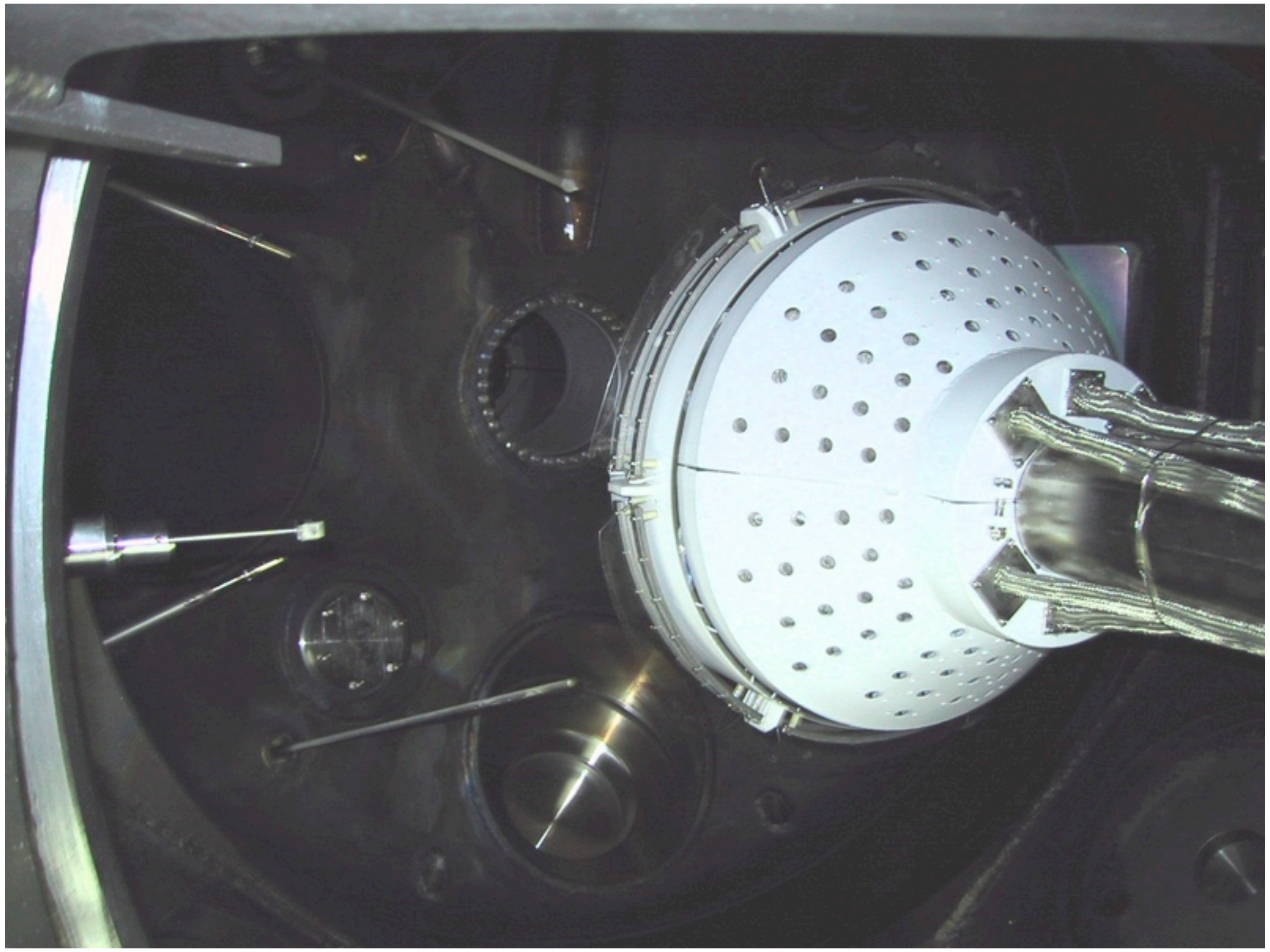


How do we view this turbulence?

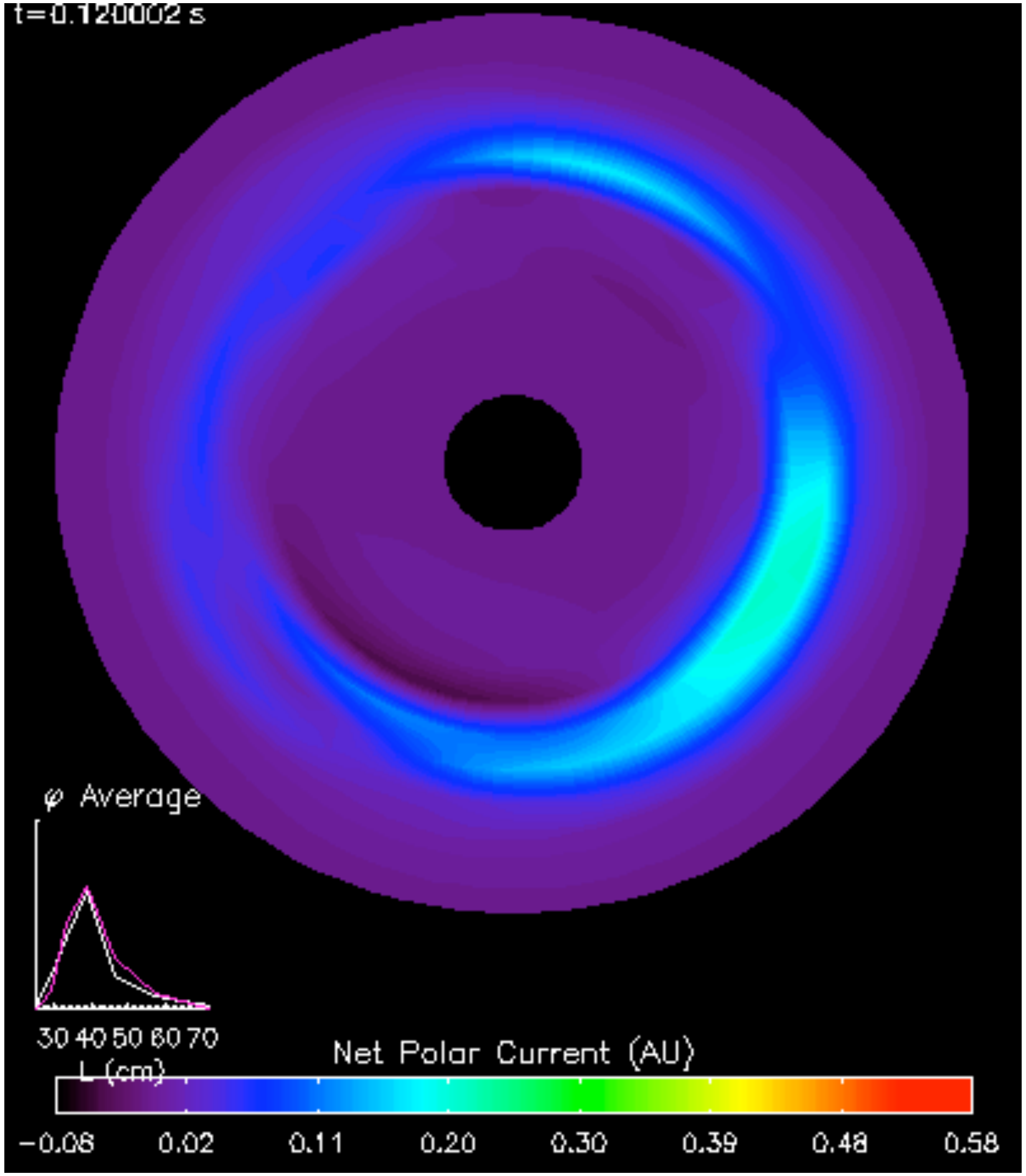
The Polar Imager Measures the Turbulence at Multiple Points Simultaneously

- 96 Gridded particle detectors digitized at high speed.
- Azimuthal spatial resolution of $\Delta\varphi=15°$ resolves structures well within the correlation length.
- Effectively detects the "auroral" currents.

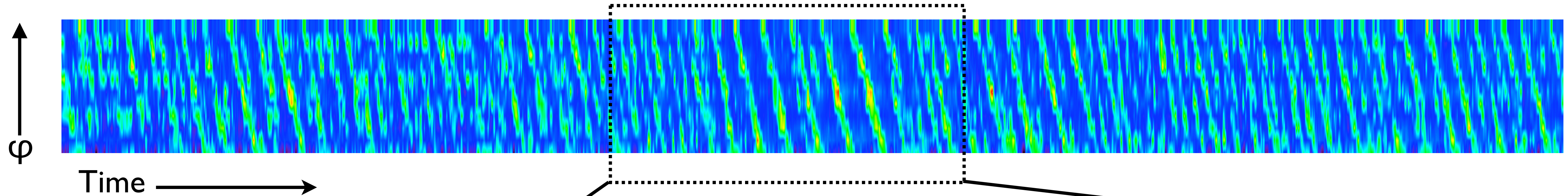


High Speed Imaging of Turbulence at 500,000 Frames Per Second

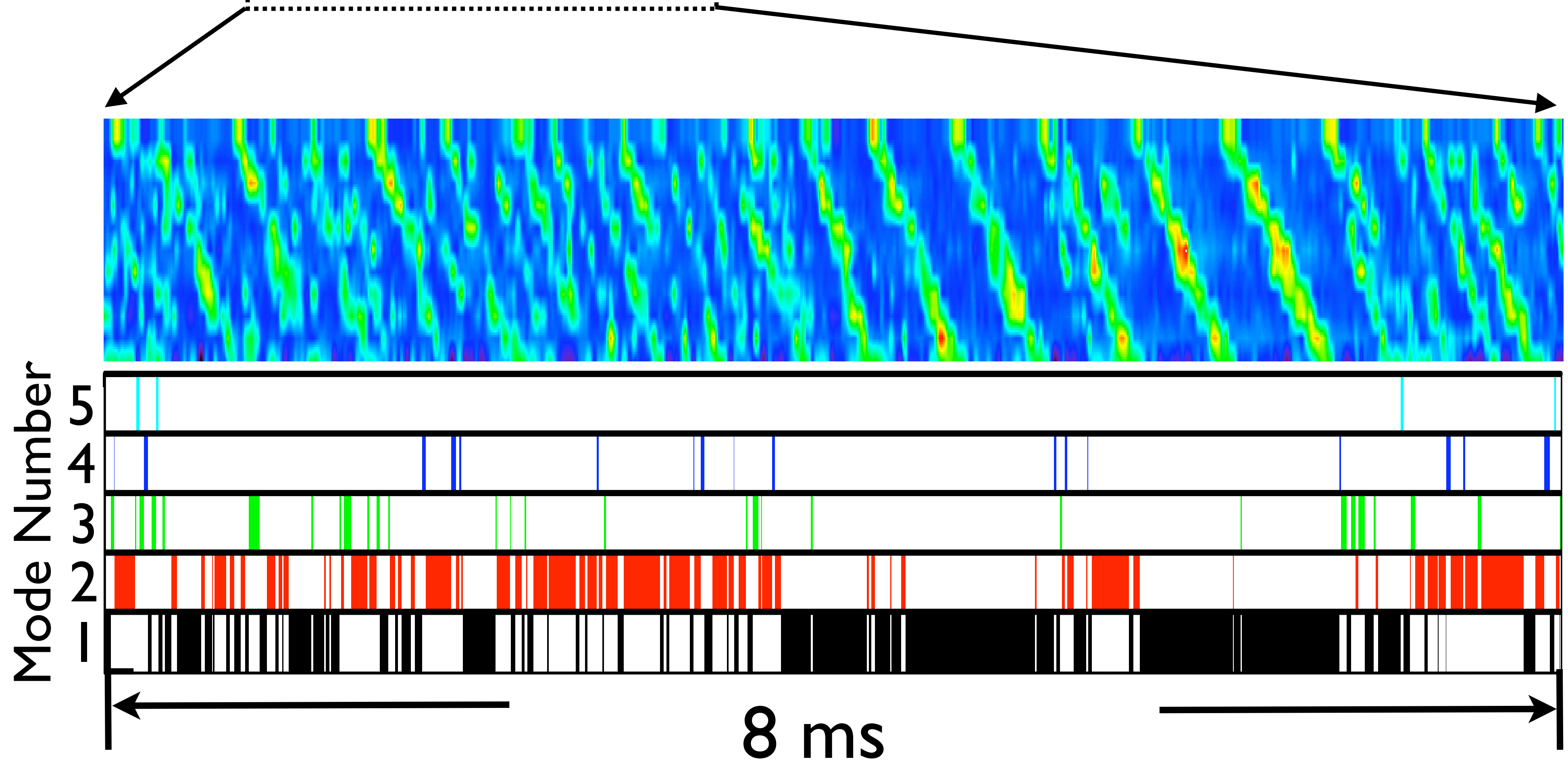
- Detectors biased to collect ion current.
- Density fluctuations rotate in $E \times B$ direction.
- Turbulence clearly visualized.
- Provides global spatio-temporal measurements for more sophisticated analysis.



The Dominant Azimuthal Mode Structure Changes in Time



- 11 points around the circumference at the same radius.
- Dominated by low order azimuthal modes.
- The dominant azimuthal mode number calculated for every time point.



The Bi-Orthogonal Decomposition* of Polar Imager Data Measures the Global Mode Structure

- The spatio-temporal measurement of density is decomposed with the Singular Value Decomposition into orthogonal spatial and orthogonal temporal modes.

$$n(\mathbf{x}, t) = \sum_k \sigma_k X_k(\mathbf{x}) T_k(t)$$

- There is no pre-defined basis. The basis functions are extracted from the data.
- The amplitude of each mode is measured by a singular value.

*de Wit et. al. Phys. Plasmas **1** (10), 1994.

The Bi-orthogonal Decomposition* For Multiple Space-Time Points

- The values $(Y)_{i,j}$ are polar current (plasma density) at 'M' space points and 'N' temporal points.
- The singular value decomposition is used to calculate the values A_k, and the spatial φ_k and temporal ψ_k functions.
- Modes are orthogonal, but not pre-defined basis functions.

$$(Y)_{i,j} = \sum_{k=1}^{K} A_k \varphi_k(x_j)\psi_k(t_i)$$

$$\sum_{i=1}^{N} \psi_k(t_i)\psi_l(t_i) = \sum_{j=1}^{M} \varphi_k(x_j)\varphi_l(x_j) = \delta_{kl}$$

Eigenequations

$$S_x\varphi_k = A_k^2\varphi_k, \quad S_x = Y^T Y$$

$$S_t\psi_k = A_k^2\psi_k, \quad S_t = Y Y^T$$

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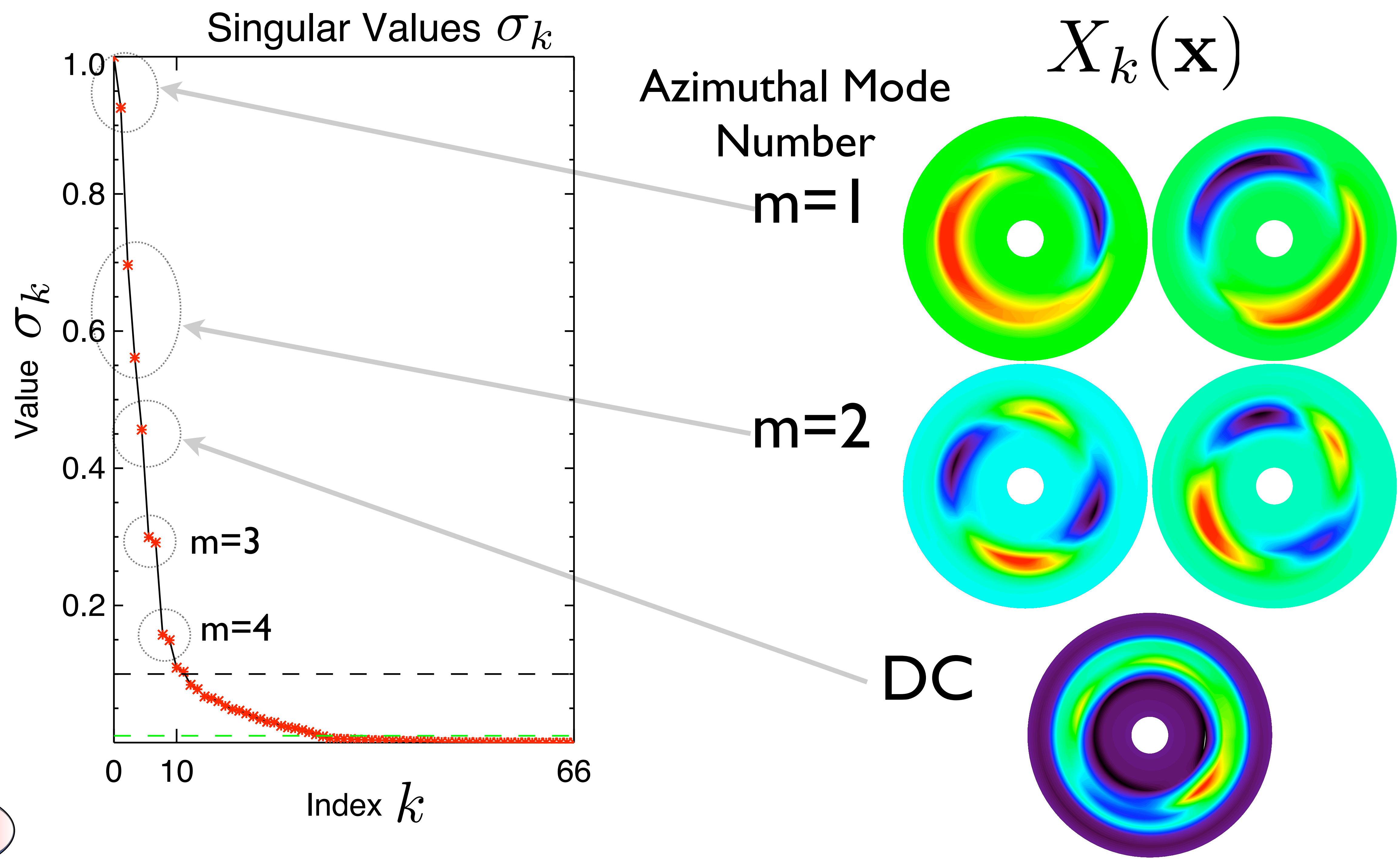
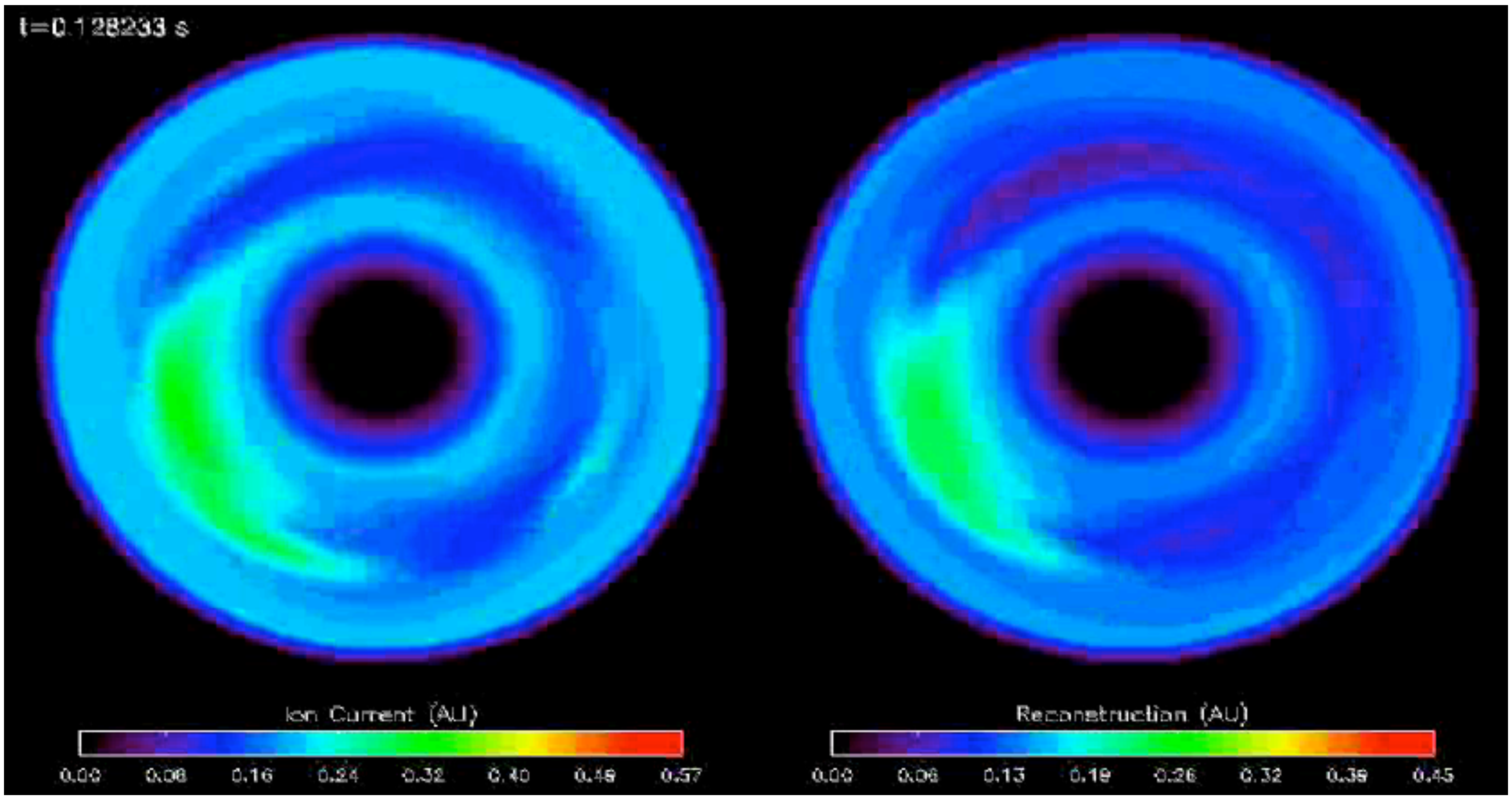
*de Wit et. al. Phys. Plasmas **1** (10), 1994.

The Spatial Structure of Turbulence is Dominated by **Simple, Rotating**, Long Wavelength Modes



A Limited Number of Modes Represents Original Data

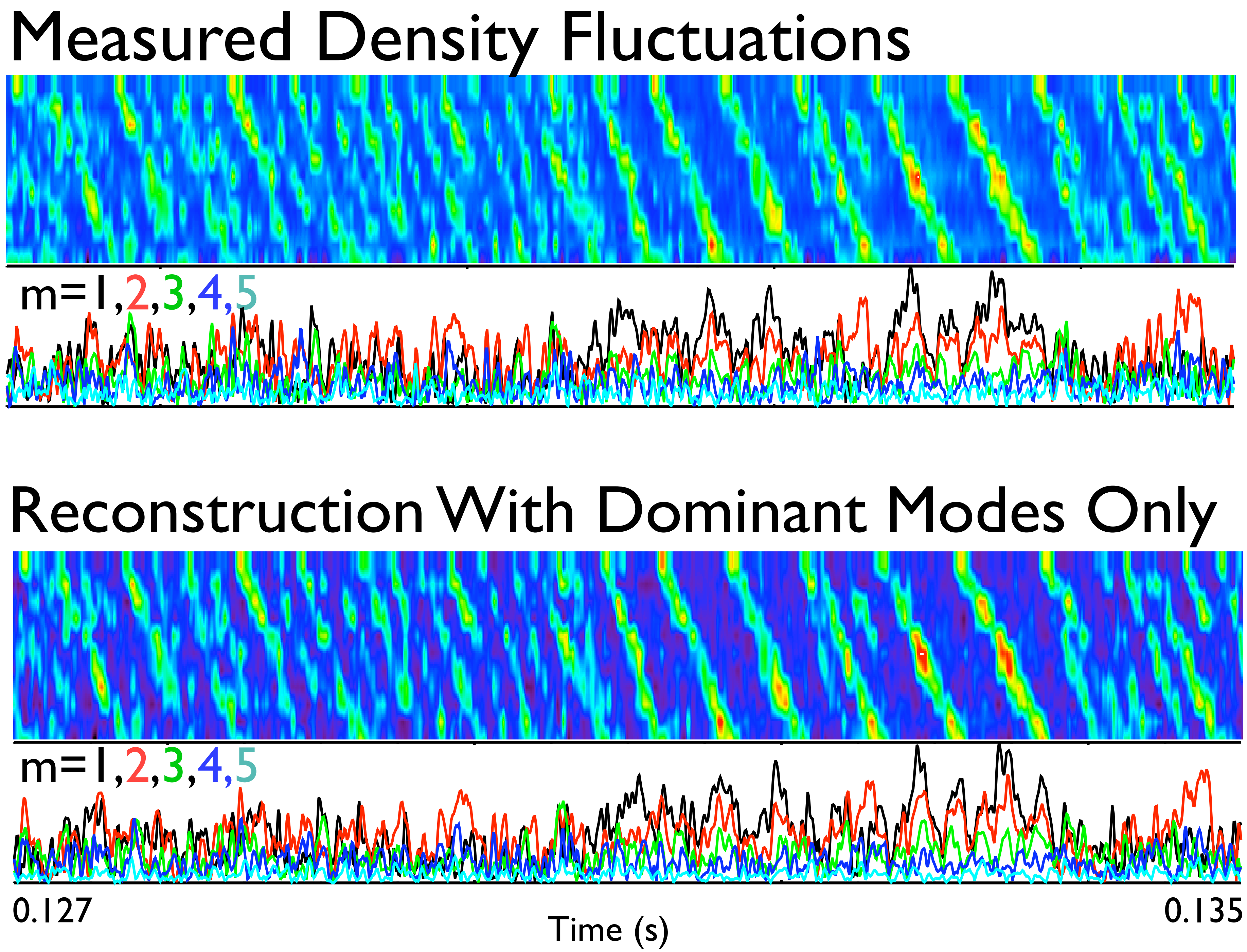
Measured Density | Dominant Modes Only



Using Azimuthal Mode Numbers m=0,1,2,3,4

Only a Few Modes Are Required to Reproduce Local Measurements

- Density fluctuations are recreated using modes m=0, ..., 4.
- The azimuthal mode number spectra are reproduced.
- The single-point frequency spectra are also reproduced.



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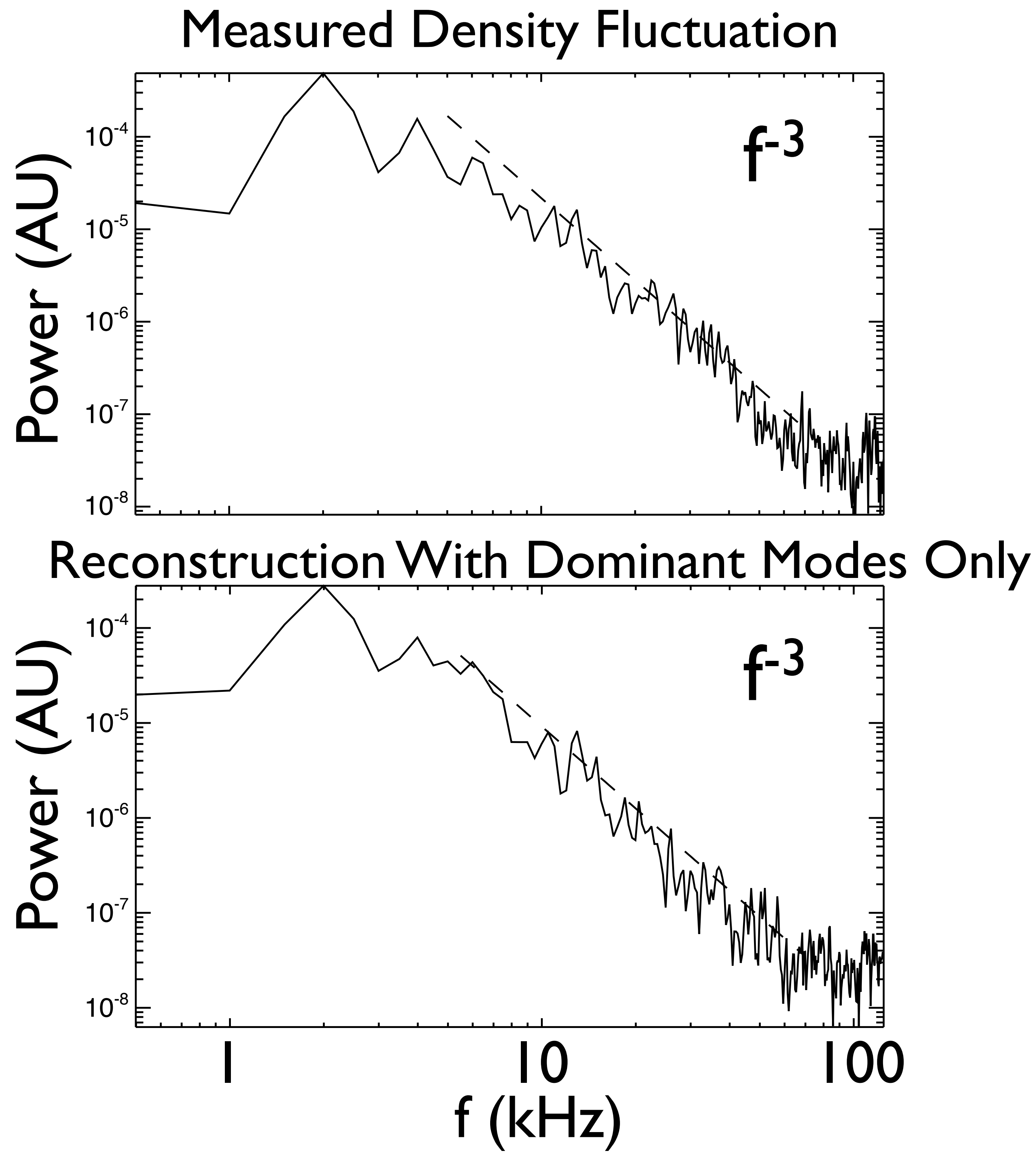

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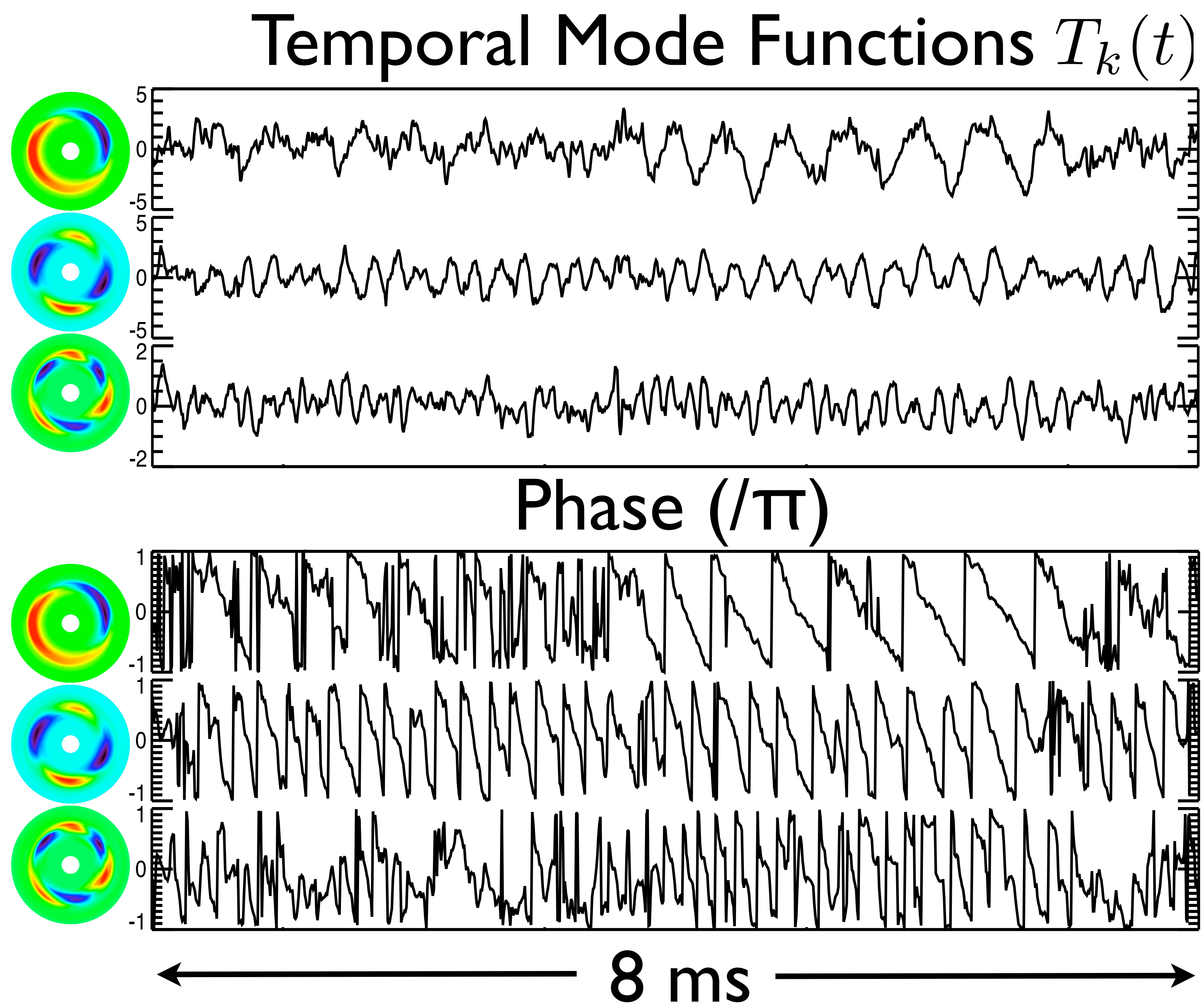
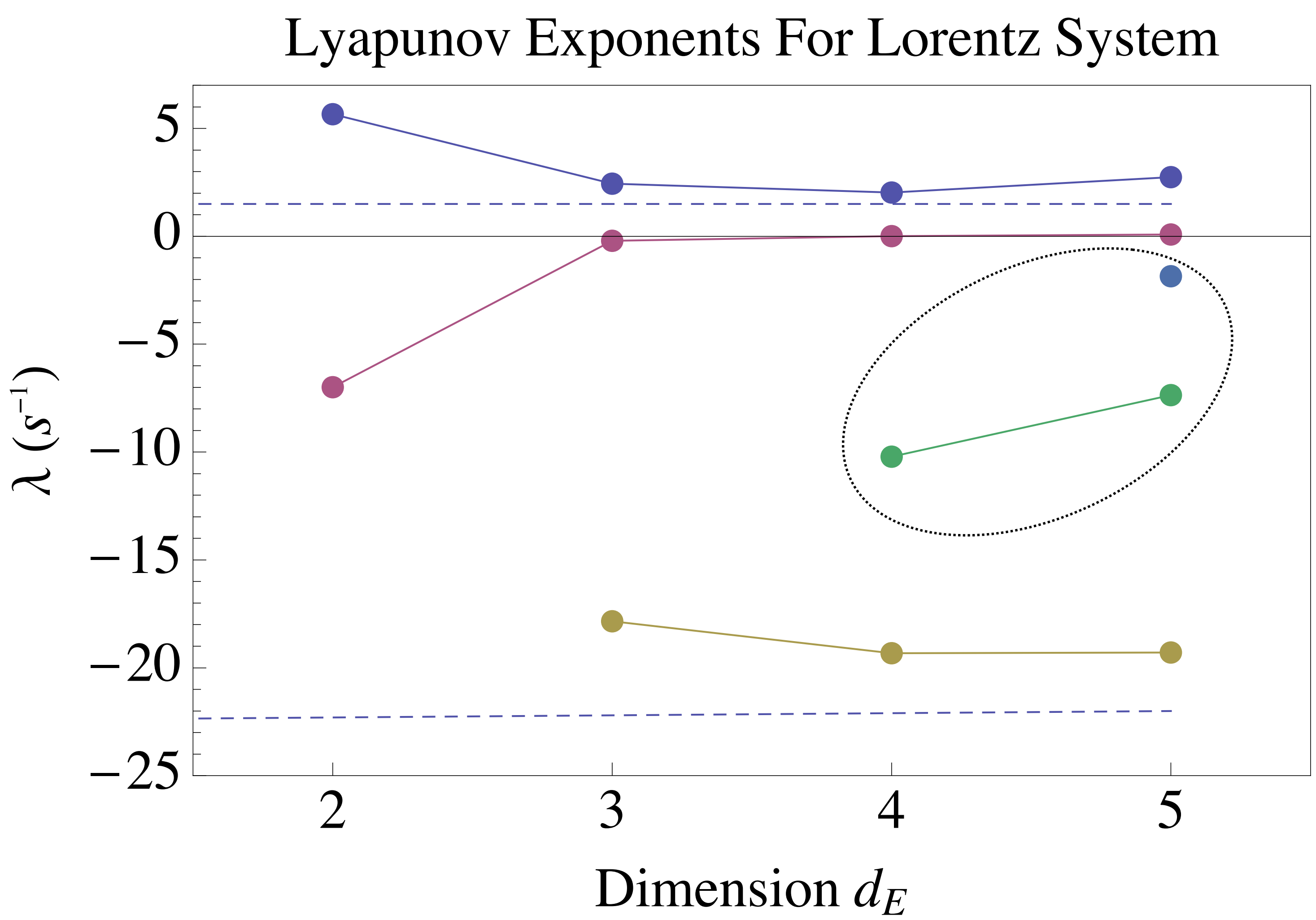
The Amplitude and Phase of the Dominant Modes Vary Irregularly in Time

- The temporal mode amplitude are irregular.
- For each mode, sin and cos phases indicate non-steady rotation.
- Modes vary like the chaotic Lorentz ODE system.



Lyapunov Exponent Analysis Shows That Density Fluctuations are Chaotic

- The Lorentz ODE system is chaotic, indicated by a positive Lyapunov Exponent.
- Density fluctuations in CTX are chaotic, with Lyapunov time of 50μs, agreeing with the auto-correlation time.
- Lyapunov analysis† also indicates low dimensionality to the system, consistent with bi-orthogonal decomposition results.
- One possible model is the plasma equivalent of Rayleigh-Bernard convection.



*Abarbanel, *RMP* **65** 1993
†Eckmann, *et. al.*, Phys. Rev. A **34** #6 1986
Rypdal, Garcia, *PoP,* **14** 2007
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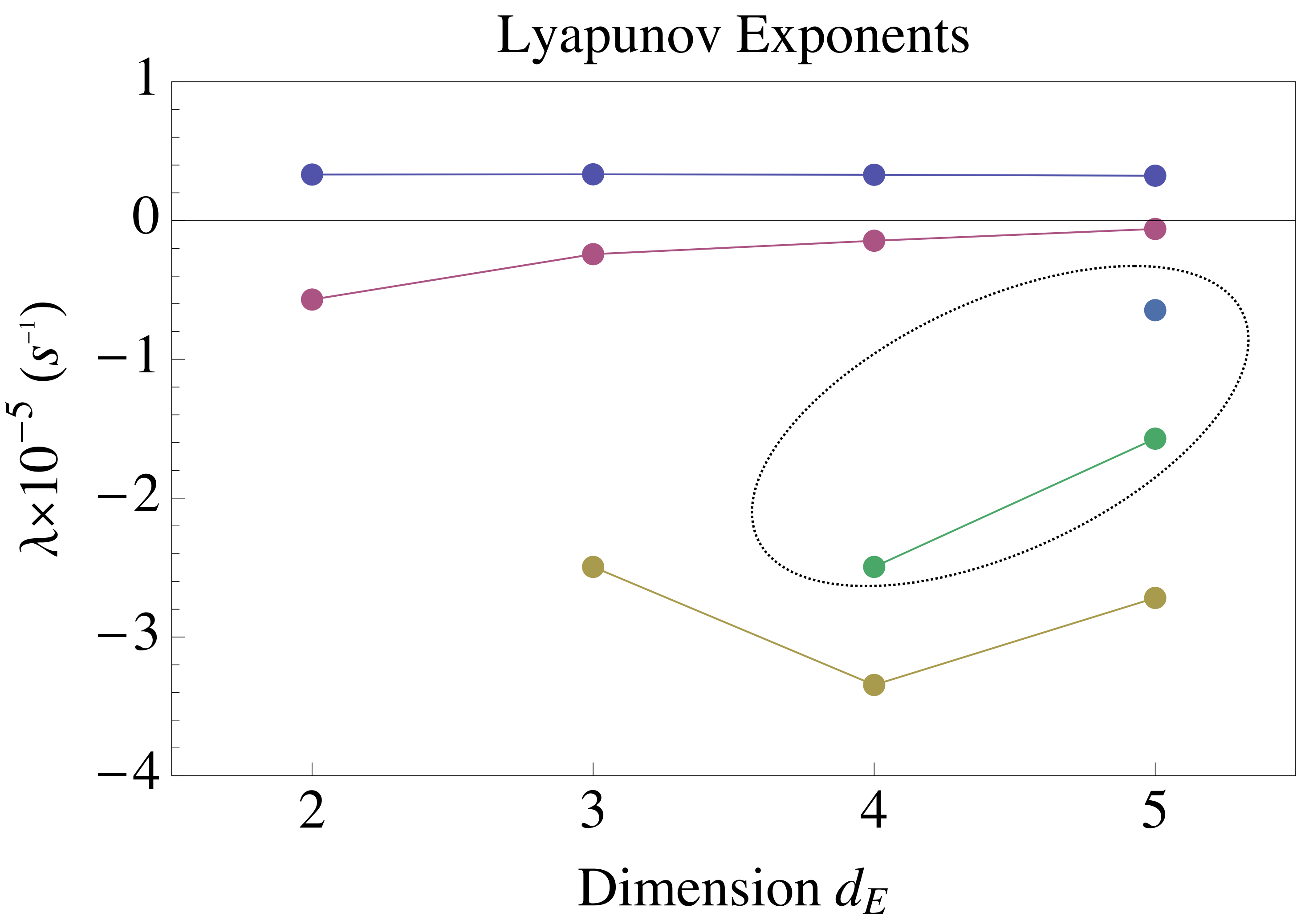

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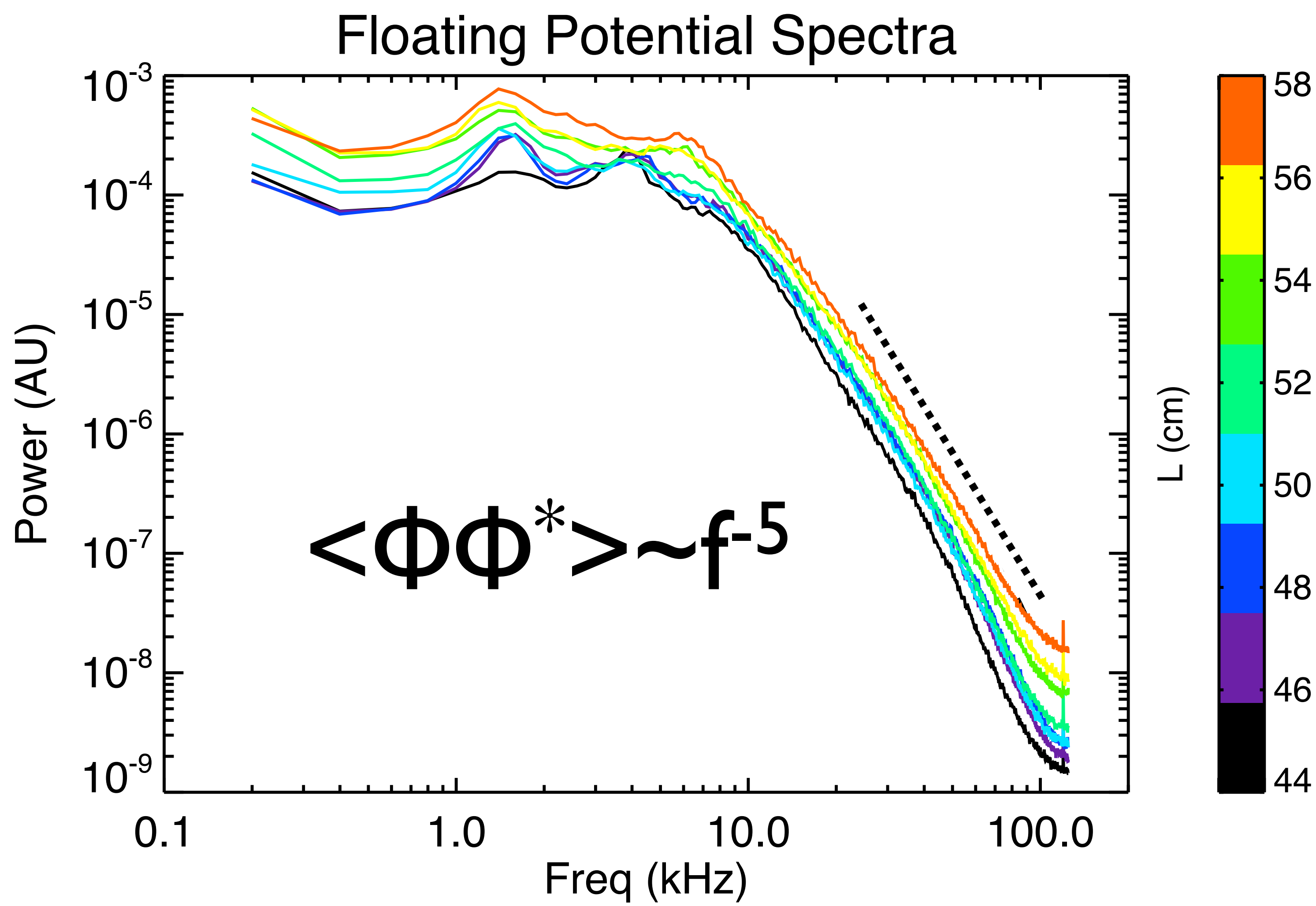
Local Statistical Analysis of Fluctuations

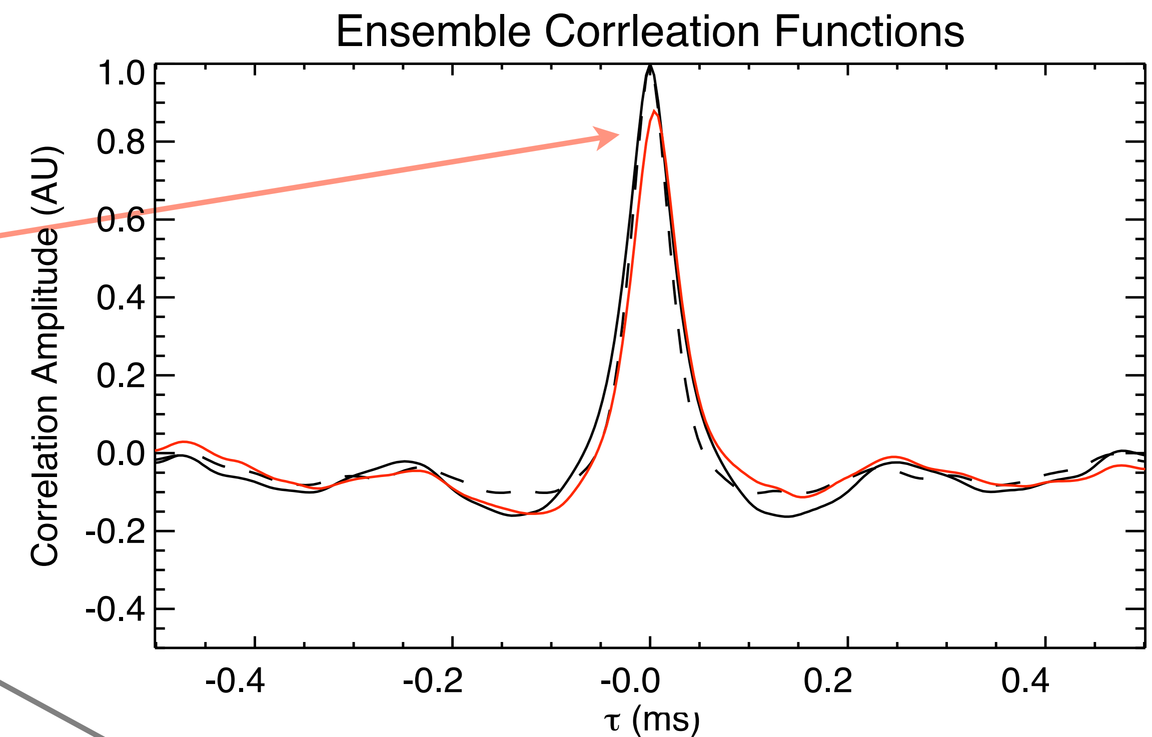
Single-Point Measurements of Potential Show Power-Law Spectrum Across the Plasma Radius

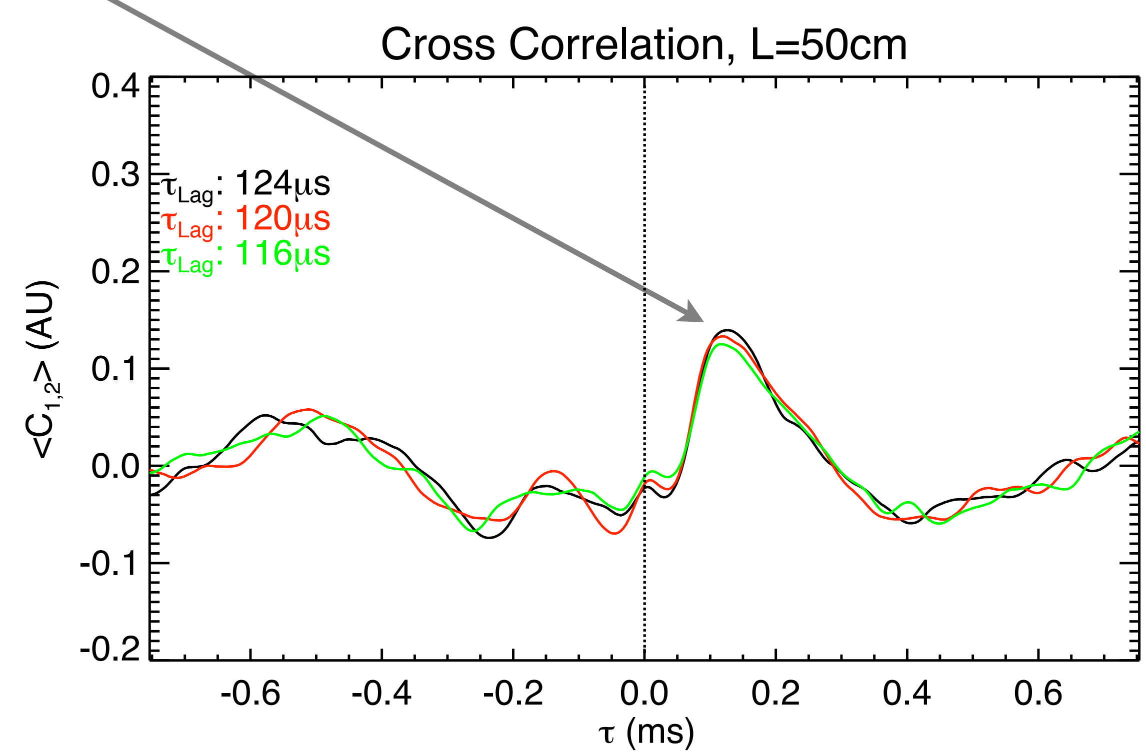
- Measurements taken at 2 cm increments across the plasma radius.
- The power-law spectrum for f>10 kHz is approximately uniform and near f^{-5}.
- The density, however, fluctuates as f^{-3}.



Fluctuations Are Correlated at Short Scales (8cm) and Decorrelate by 45 cm

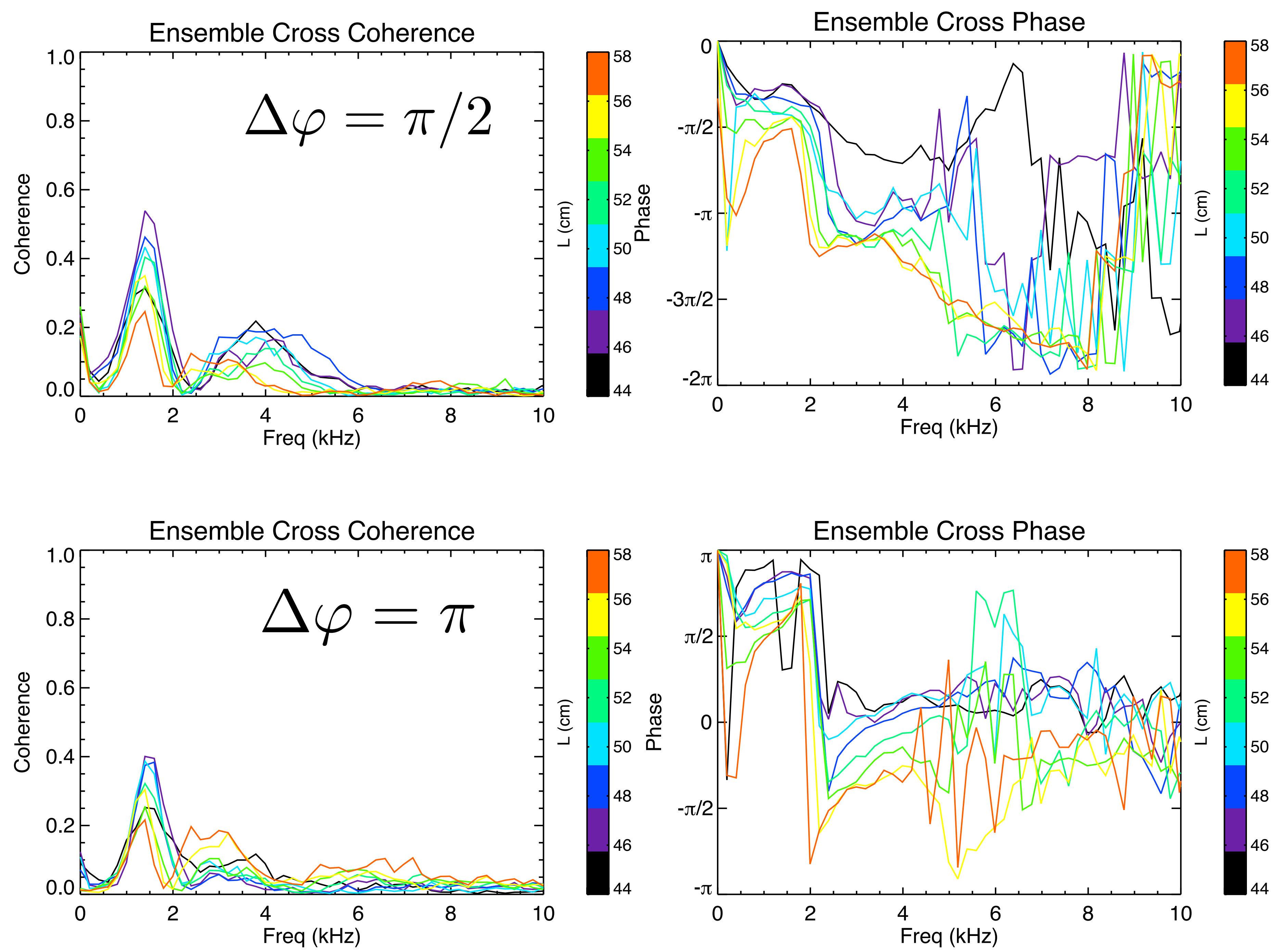
- Probes separated by Δφ=9° (~8 cm) show nearly perfect correlation with a very small positive lag time.
- Long-wavelength correlation does exist for probes separated by Δφ=90°.
- The amplitude of the correlation decreases with probe separation (Δφ=9°,90°,180°).
- Decay behaves as $e^{-x/\lambda}$ where λ~45 cm (Δφ~50°). This is 14% of the device circumference.





Ensemble Coherence and Phase Between Two Probes Reveals Azimuthal Mode Structure of Potential

- Two probes separated azimuthally by 90° and 180°.
- Hundreds of measured realizations.
- An m=1 mode exists at ~1-2 kHz.
- An m=2 mode exists at ~3-4 kHz.



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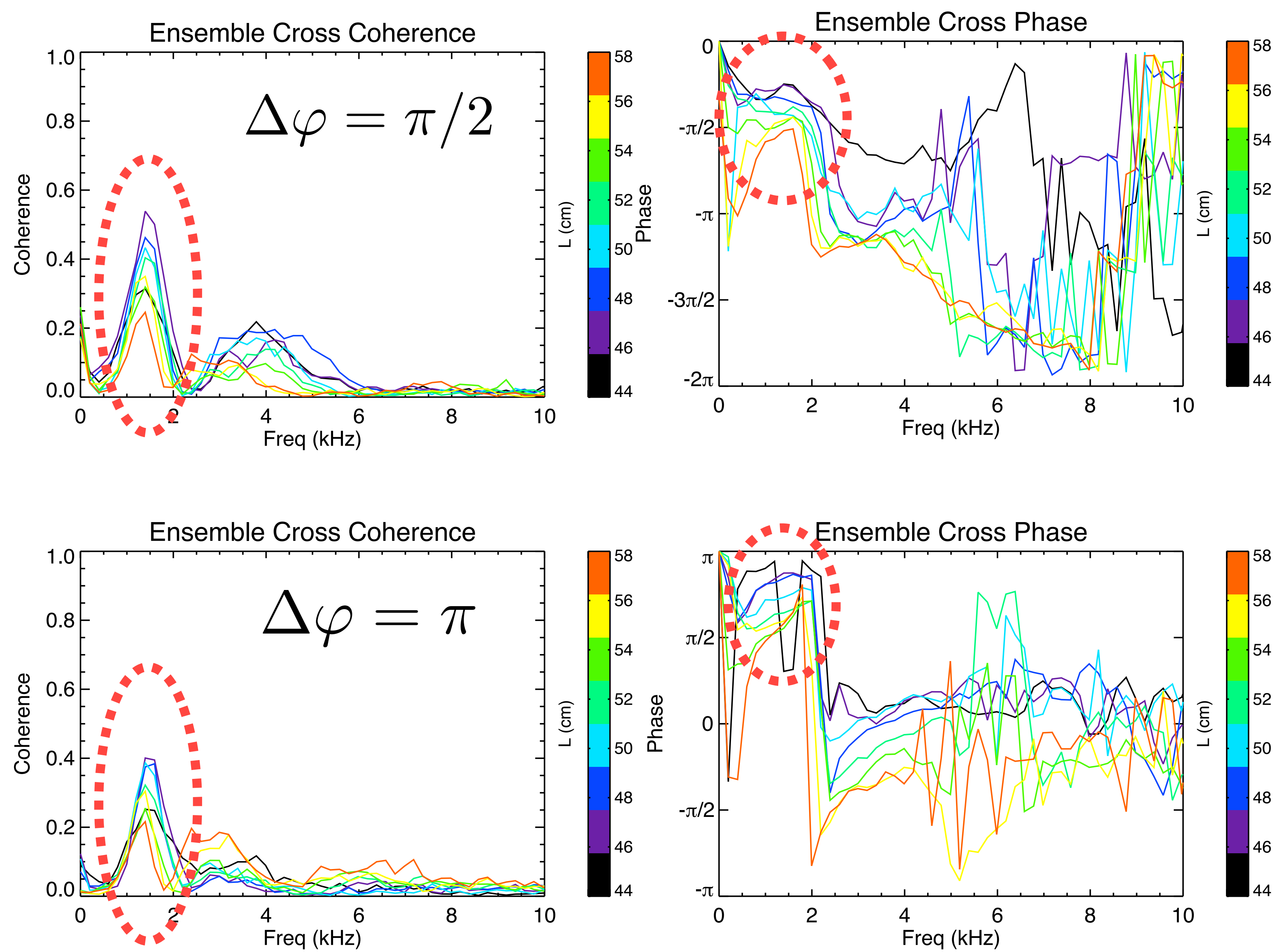

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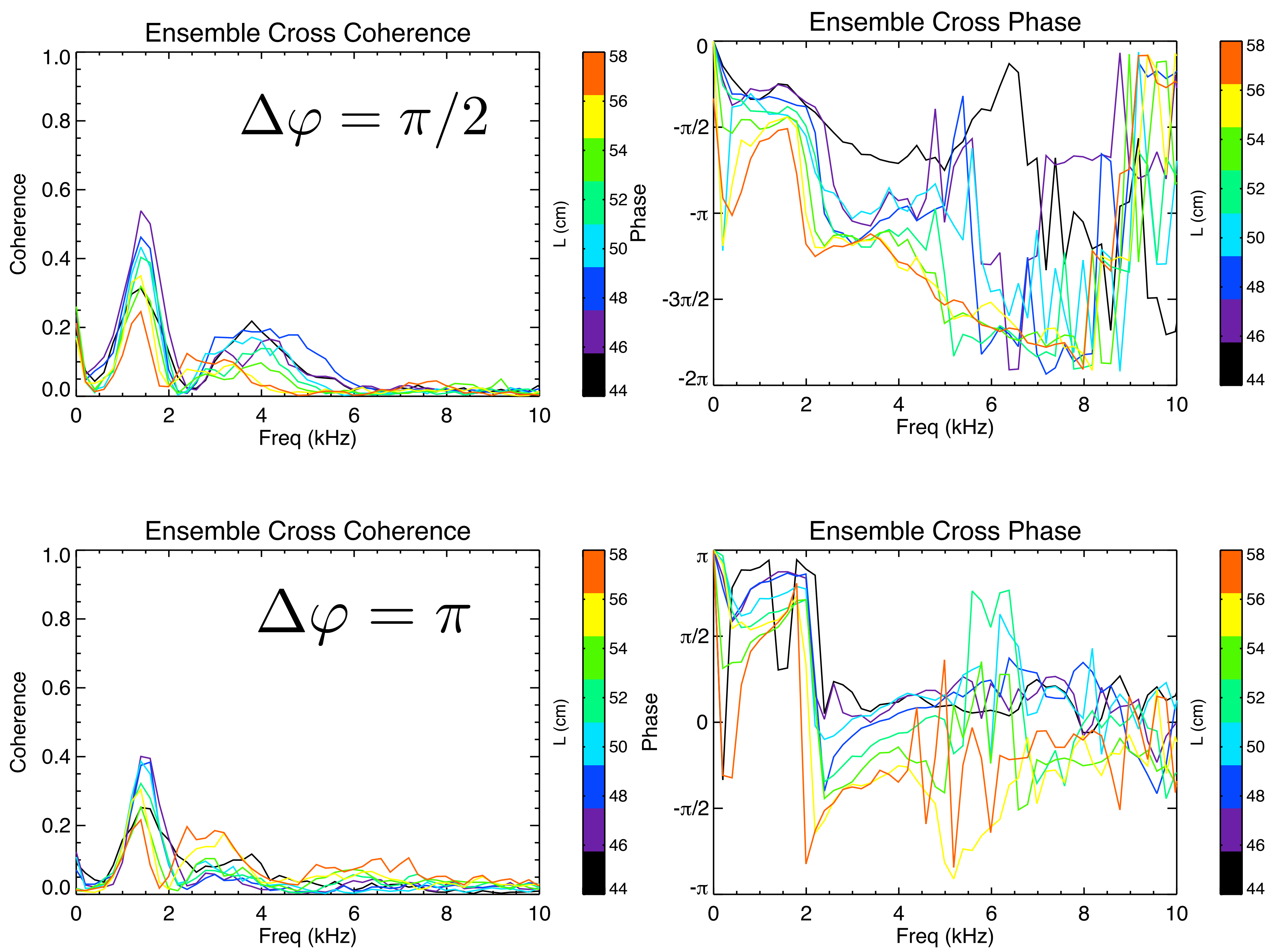

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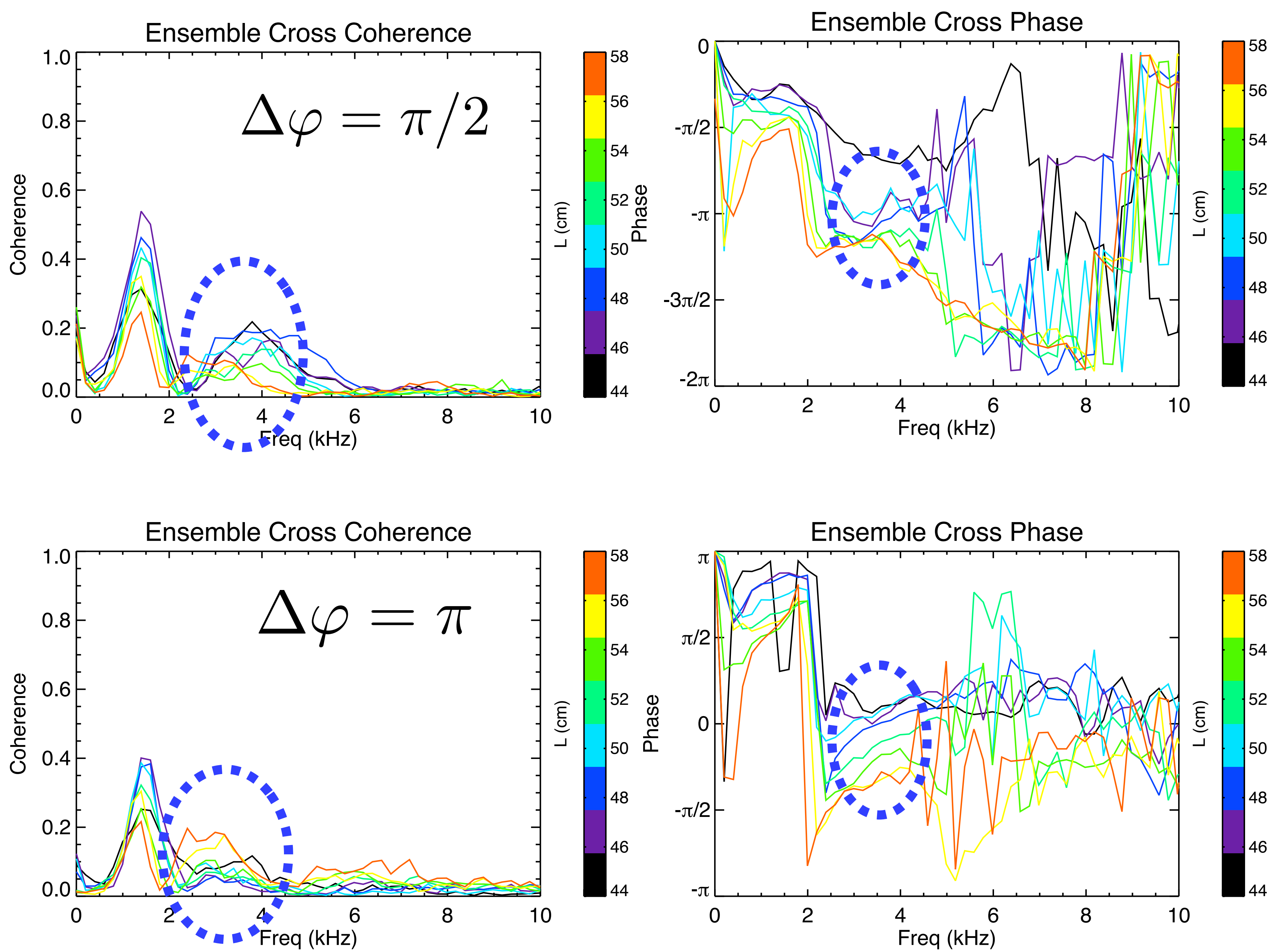

Measuring the dispersion and nonlinear structure coupling of turbulence

Auto and Cross Bispectral Analysis Measures Linear and Nonlinear Mode Dynamics

- A method developed by Ritz, Kim, et al, and most recently used in the heliac by Xia and Shats.
- Solves for the complex linear growth $\gamma_{\mathbf{k}}$ and dispersion $\omega_{\mathbf{k}}$.
- Solves for the three-wave coupling coefficient $\Lambda_Q^{\mathbf{k}}$, and determines direction of spectral energy flow.

$$\frac{\partial \phi(\mathbf{k},t)}{\partial t} = \Lambda_L(\mathbf{k})\phi(\mathbf{k},t) + \frac{1}{2}\sum_{\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2} \Lambda_Q^{\mathbf{k}}(\mathbf{k}_1,\mathbf{k}_2)\phi(\mathbf{k_1},t)\phi(\mathbf{k}_2,t)$$

Linear $\longrightarrow \Lambda_L(\mathbf{k}) = \gamma_k + i\omega_k$

*Xia and Shats PRL **91** (15), 2003.

*Kim et. al. Phys. Plasmas **3** (11), 1996.

*Ritz et. al. Phys. Fluids B **1** (1), 1989.

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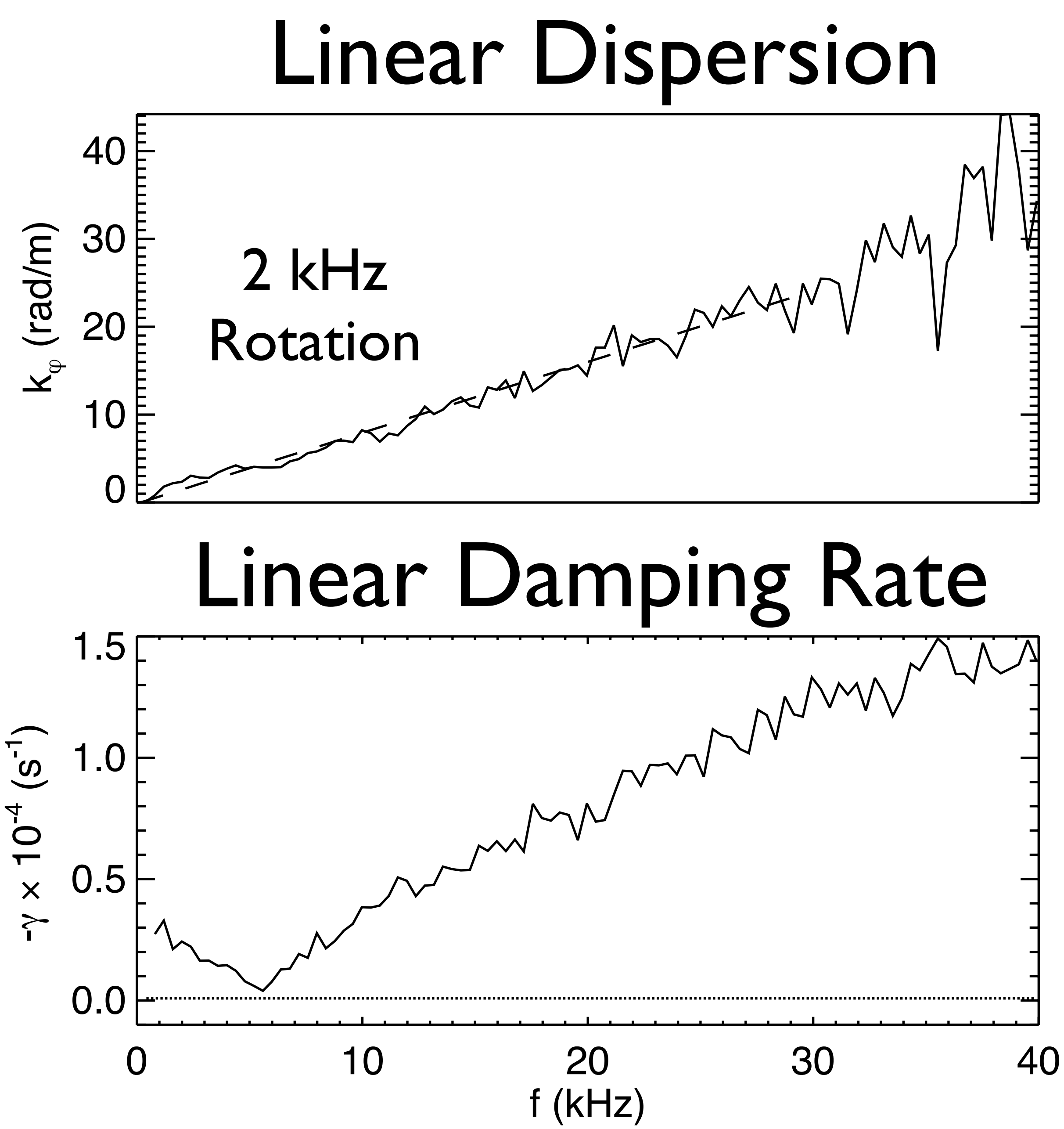
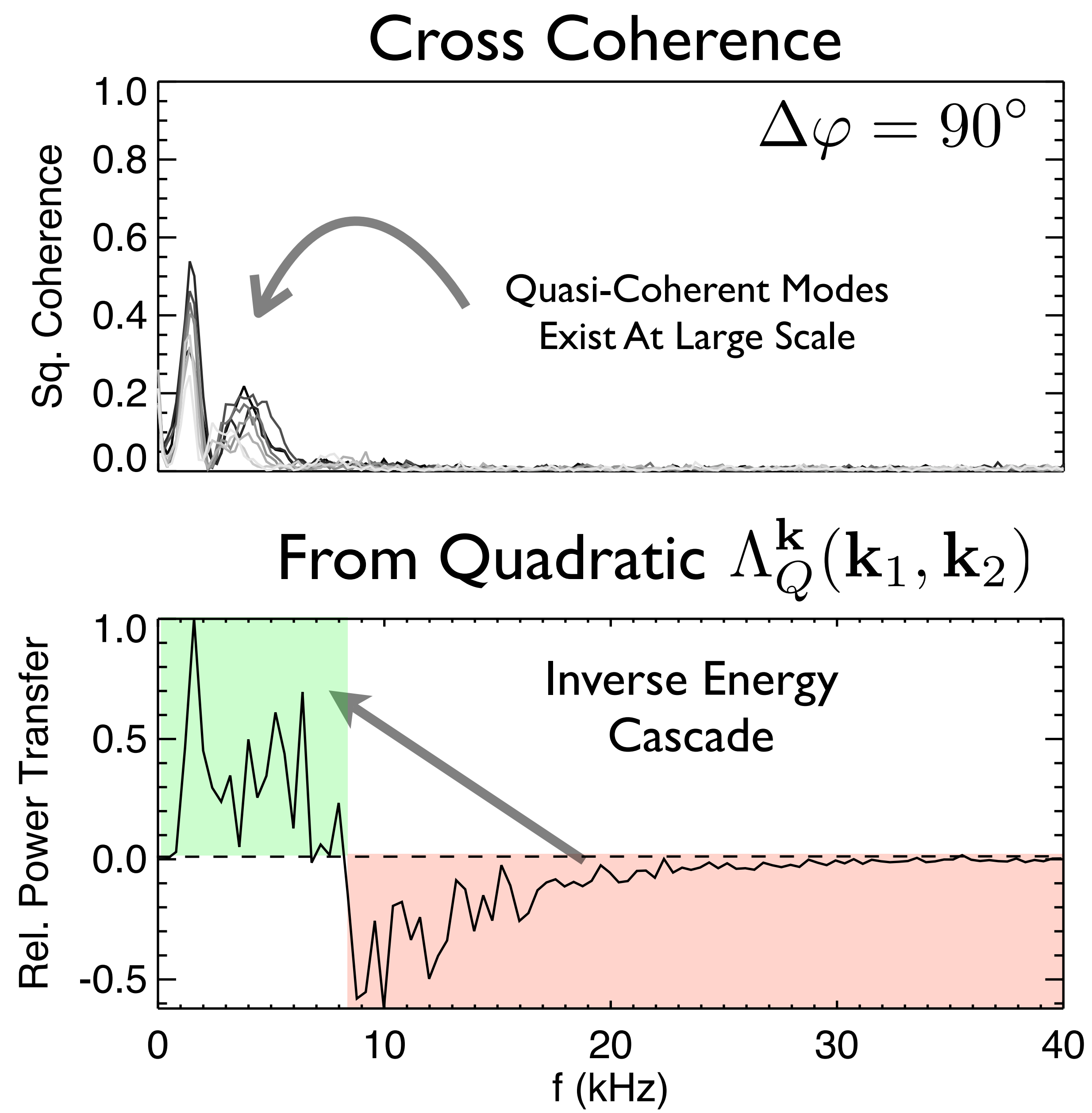
Linear Dispersion and Damping Rate Shows Small Scale Fluctuations are Damped More Strongly Than Large Structures

- The two-point calculated dispersion relation allows rigorous interchange of frequency and wavenumber.
- The damping rate is strongest for high frequency, small scale fluctuations, and marginally damped modes exist near 4-7 kHz.



Spectral Power is Transferred Nonlinearly From Small to Large Scale

- The quasi-coherent global structures exist in the frequency range f<6 kHz (or $k<1\ m^{-1}$).
- The net power transfer is found to be from small to large scale structures, into the lower wavenumbers.
- The power transfer changes sign near 8-9 kHz.



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Spectral Characteristics of 2D Turbulence

- For physical systems such as soap films, stratified fluids, geophysical flows and magnetized plasmas.
- Conserves energy and enstrophy in an *inverse energy cascade* from small to large scales.
- Kraichnan's k^{-3} Law for *forward enstrophy* cascade.*

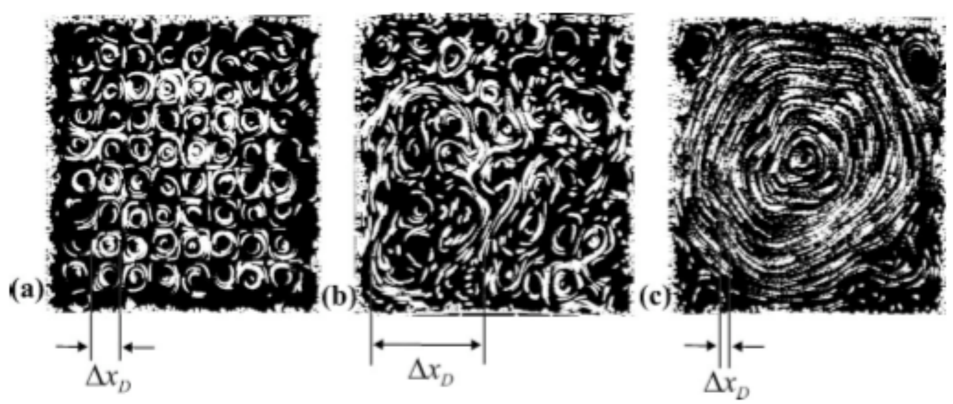
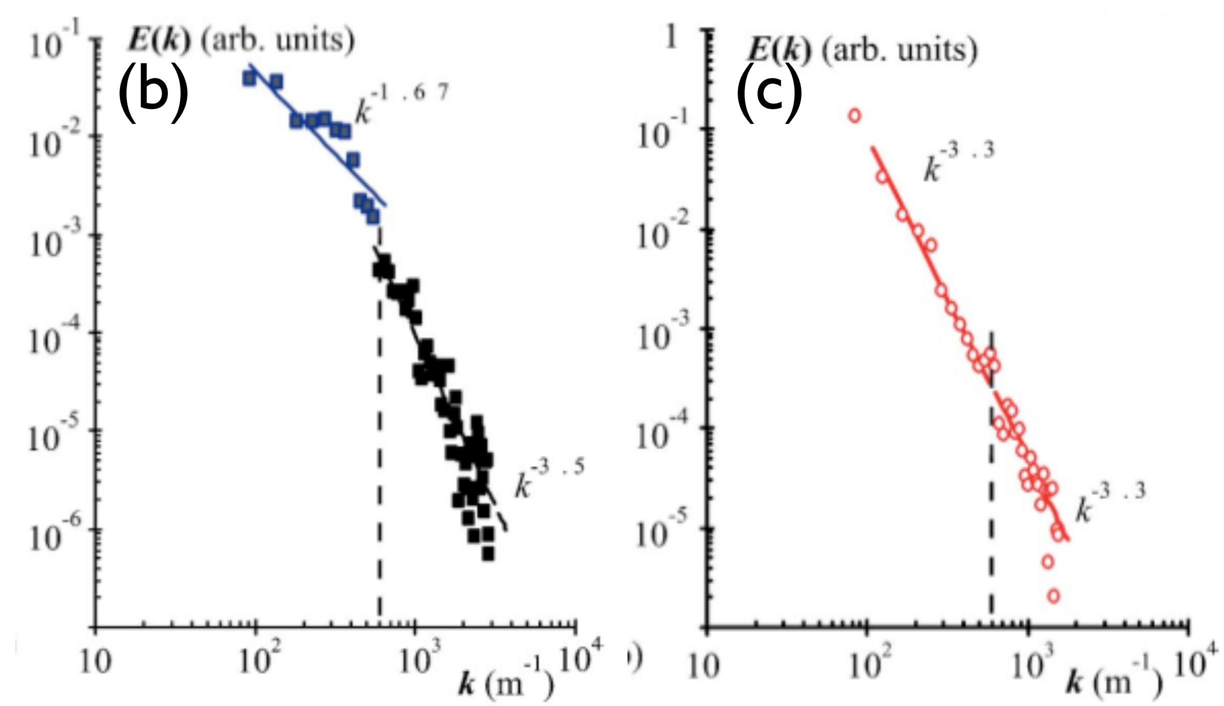


FIG. 2. Evolution of turbulence in a thin layer of electrolyte in a cell during spectral condensation. Trajectories of the tracer particles averaged over 12 frames of recorded video are shown. (a) The initial (linear) stage, $t=3$ s. (b) The inverse cascade stage, $t=25$ s. (c) The condensate stage, $t=60$ s. Δx_D represents the spatial scale of the trace particle transport during three stages of the flow evolution.

Shats, *et. al. Phys. Rev. E*, **71** (2005)



*Robert H. Kraichnan, *Phys. Fluids* **10** (7) 1967

The Power-Law Trend of the Fluctuation Spectra are Consistent With 2D Turbulence

- Turbulence in 2D fluid conserves both the energy and enstrophy.
- In the drift-interchange limit, low frequency fluctuations are the sum of an adiabatic, and an electrostatic contribution to the energy.

$$E_k \sim \left(\frac{\tilde{n}_k}{n_0}\right)^2 + k^2 \rho_S^2 \left(\frac{e\tilde{\Phi}_k}{T_e}\right)^2$$

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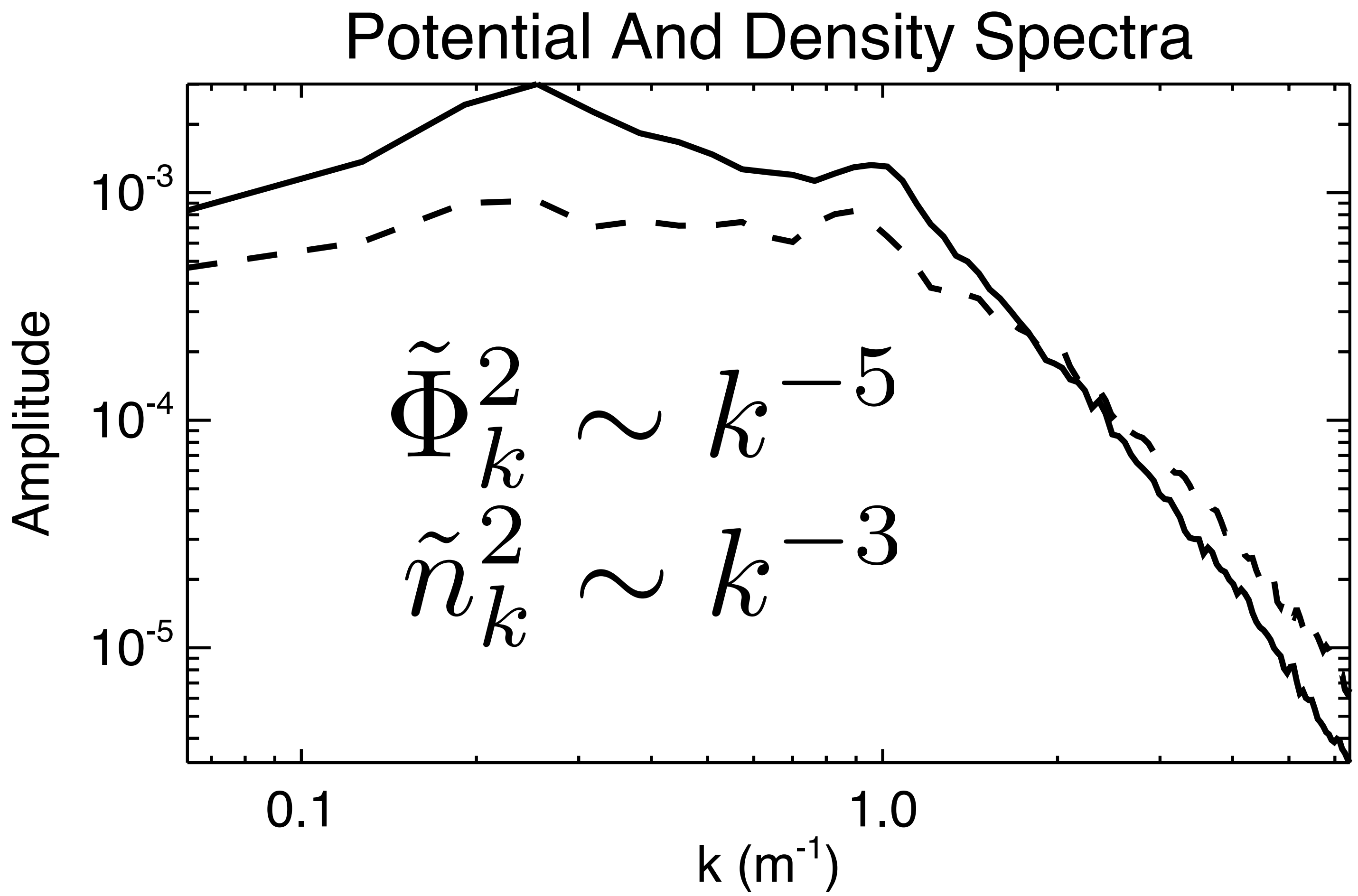
The Power-Law Trend of the Fluctuation Spectra are Consistent With 2D Turbulence

- Power-law trend in density and potential agree with expected scalings.
- The energy spectrum obeys a power law of k^{-3} for k > $2m^{-1}$ consistent with the 2D forward enstrophy cascade.
- The transition in slope occurs near k~$2m^{-1}$, where the power transfer changes sign.



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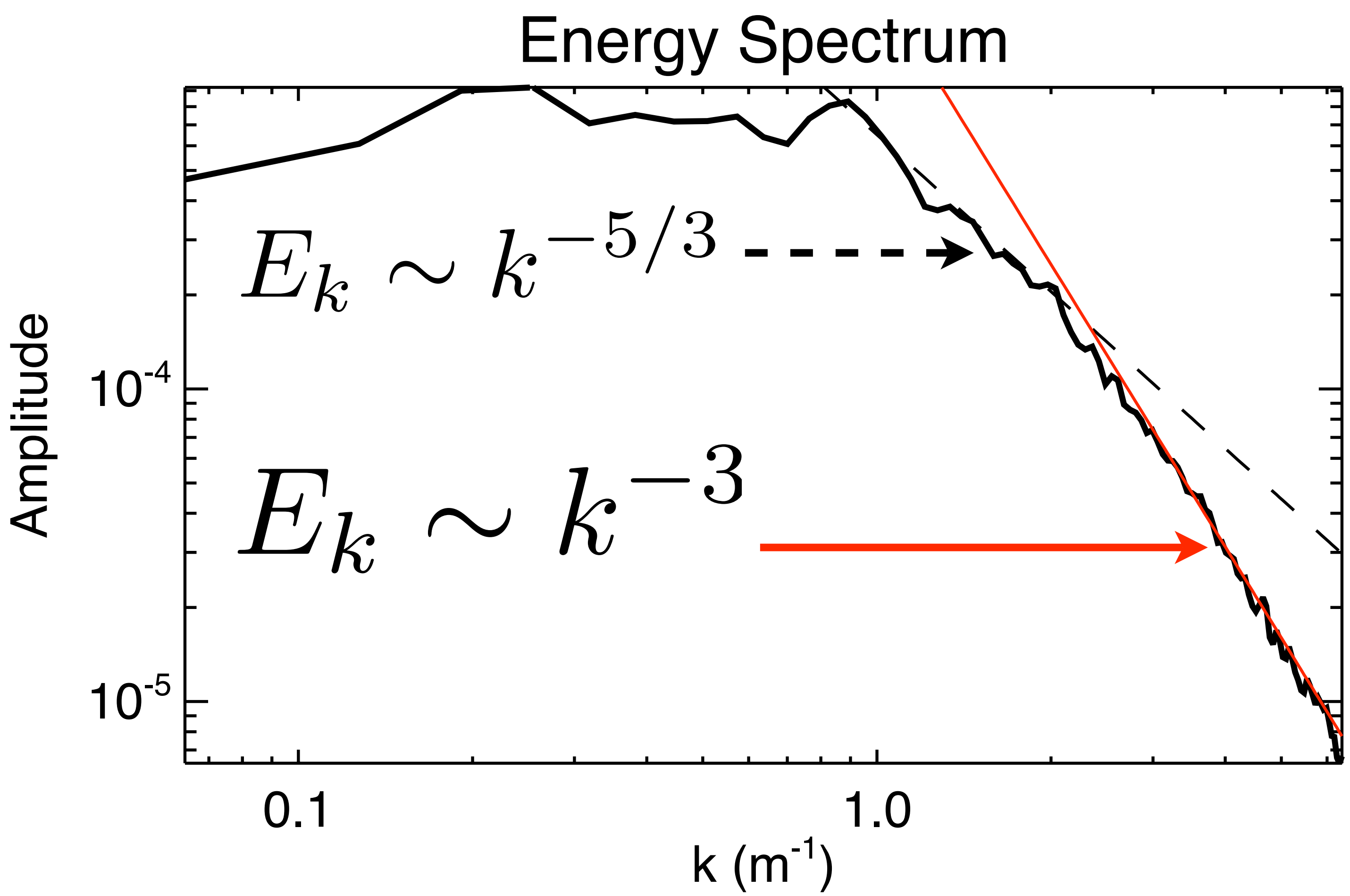

Conclusions

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Thank You