Characterizing Interchange Turbulence In Dipole Confined Plasmas

B.A. Grierson, M.W. Worstell, S. Stattel, M.E. Mauel

APS DPP 2006, Philadelphia, PA

http://www.apam.columbia.edu/ctx/ctx.html

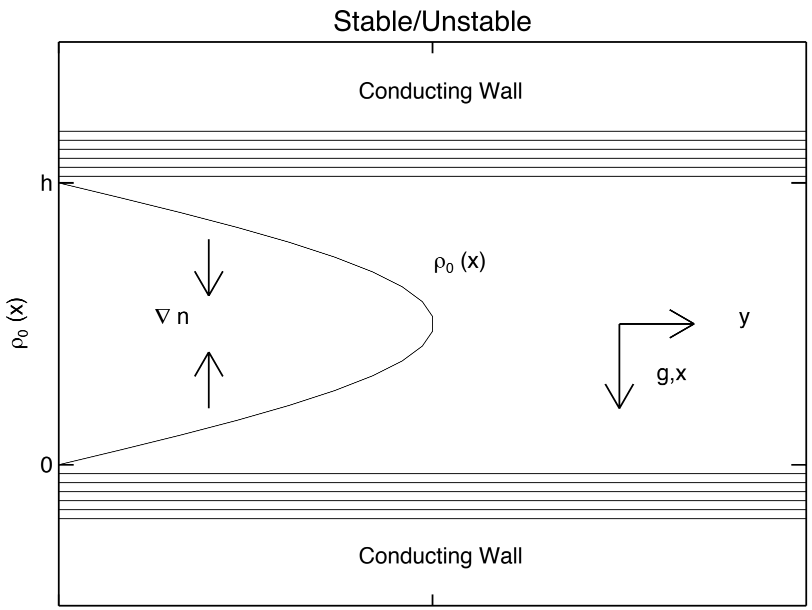
COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Abstract

The dipole magnetic field has closed field-lines without magnetic shear, and this confinement concept allows large fluting instabilities. When dipole-confined plasma is produced with ECRH, fast Hot Electron Instabilities (HEI) appear at low densities, and slower turbulent fluctuations occur at higher densities. The global mode structure of the fast HEI instability and centrifugal interchange are understood. However, the characteristics of the turbulent interchange fluctuations (that occur between HEI bursts and when the HEI is suppressed by fueling) are less well understood. These low frequency, non-stationary fluctuations exhibit a power-law like turbulence spectrum and intermodal coupling. Correlation analysis, modal decomposition Hilbert methods, time-frequency spectrograms, and bicoherence are used to characterize interchange turbulence in a dipole and to form a basis for understanding nonlinear plasma mixing.

Fixed Boundary Interchange Instability

- Fluting Instability
- Drift-resonant fluctuations $\omega \sim m\omega_d$.
- Interchange of flux tubes in a dipole is associated with significant compression. $B \sim 1/L^3$.
- Classic Mechanism:



Ideal MHD

$$nM_i \frac{d\mathbf{v}}{dt} = -\nabla P + \mathbf{J} \times \mathbf{B}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{J} = 0$$

Model

$$\nabla P = 0,\ \mathbf{J} \times \mathbf{B} = n\nabla h\ (s.s.)$$

$$\nabla h = M_i g \hat{x}$$

$$\mathbf{v} \equiv \mathbf{v}_{\mathbf{E}\times\mathbf{B}} = \hat{z} \times (\nabla\Phi)/B$$

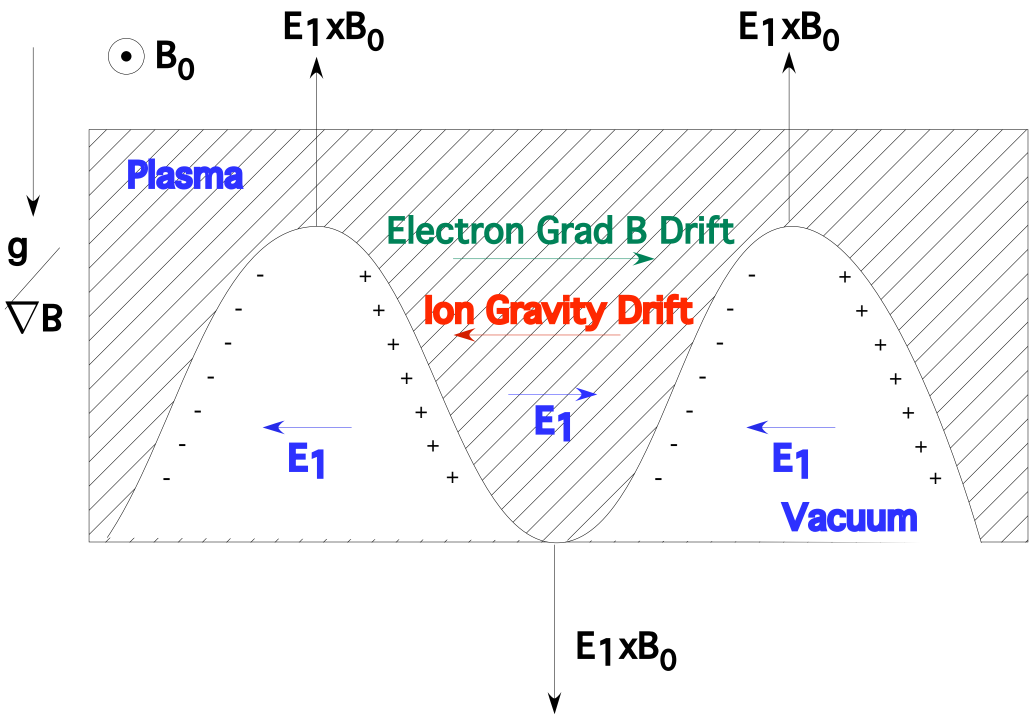
$$n(x,y,t) = \bar{n}(x) + \delta n(x,y,t)$$

$$\Phi = \Phi(x,y,t)$$

$$\partial/\partial y \rightarrow ik \quad \partial/\partial t \rightarrow -i\omega$$

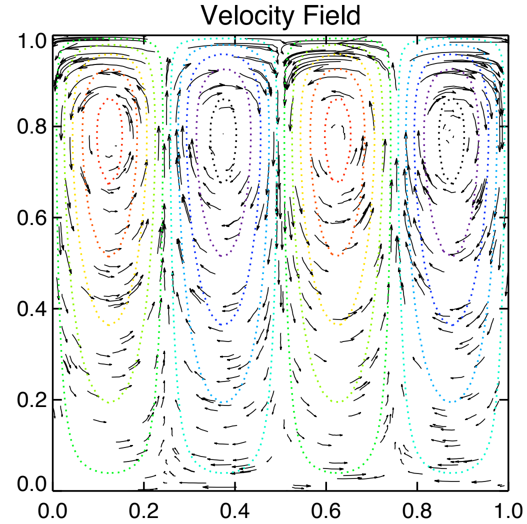
$$\epsilon = \frac{nM_i}{B^2}$$

Linear Interchange Solution.

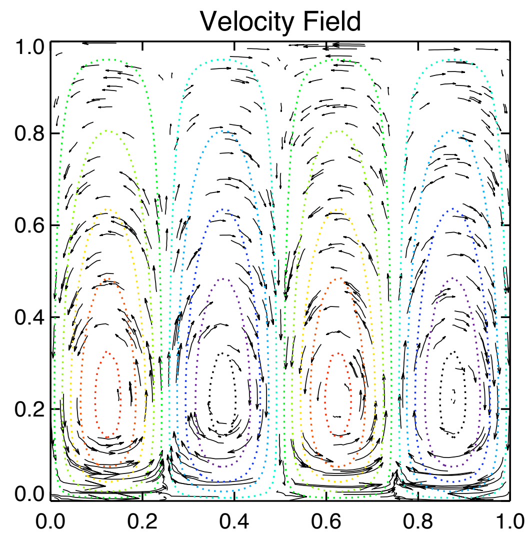


Interchange unstable when **g**, ∇n are antiparallel

Stable



Unstable



Neglecting $(v \cdot \nabla)v$ Eigenproblem

$$\nabla \cdot \epsilon \frac{\partial}{\partial t} \nabla \Phi = \nabla \cdot \left(\frac{n}{B} \hat{z} \times \nabla h \right) \rightarrow \frac{\partial}{\partial x} \bar{\epsilon} \frac{\partial \Phi}{\partial x} - \bar{\epsilon} k^2 \left(1 + \frac{g}{\omega^2} \frac{d \ln \bar{n}}{dx} \right) \Phi = 0$$

$$\omega^2 \approx - \left(\frac{k^2}{k_\perp^2} \right) g \frac{d}{dx} \log \bar{n}$$

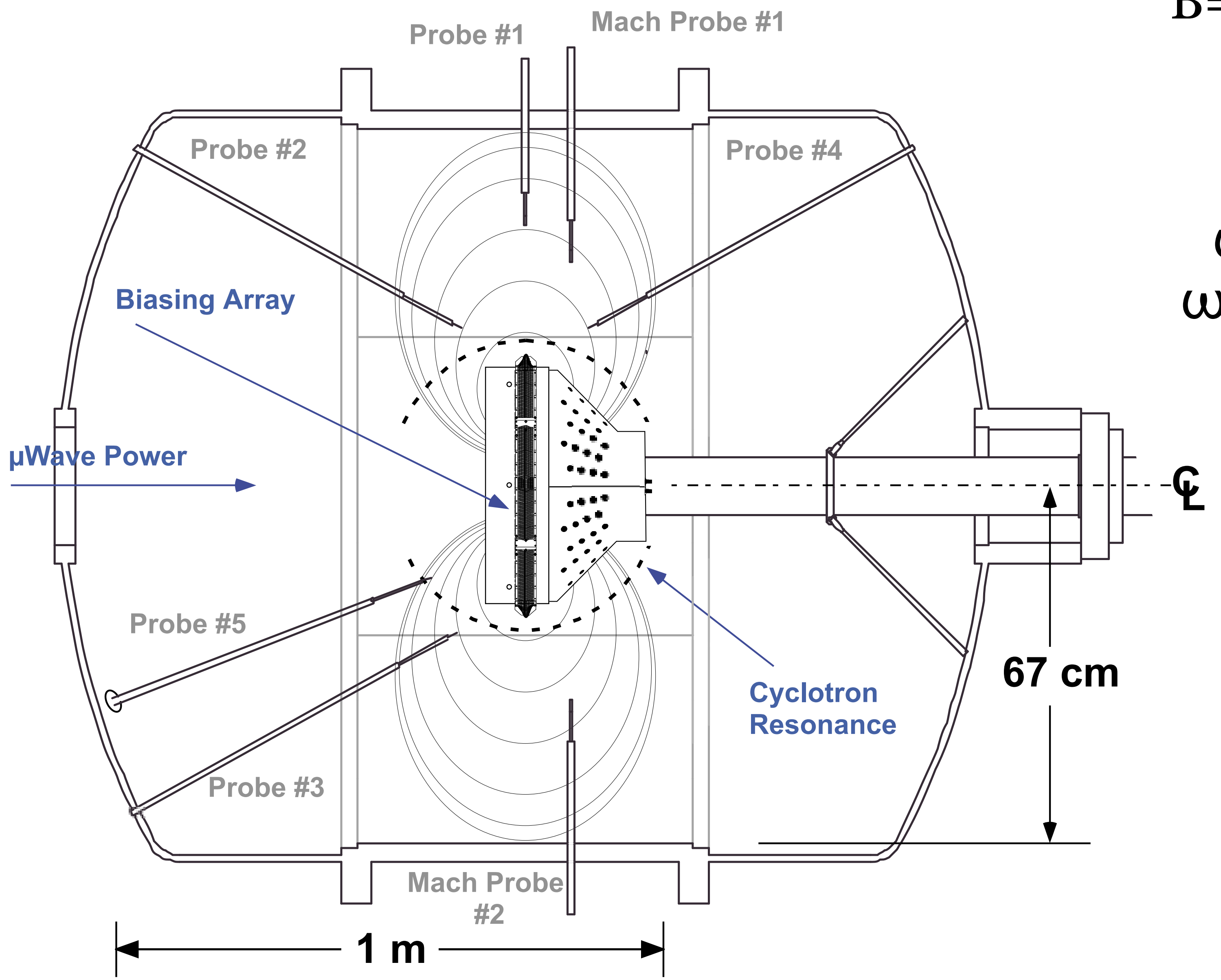
Sign of ω^2 determines stable/unstable mode.

Turbulence In Dipole

- Global mode structure of Hot Electron Instability in low density plasma has been characterized.*
- The low frequency fluctuations between HEI bursts and in 'High Density' have not been characterized.
- Correlation studies between measurements at different locations (L,λ,φ) in the plasma are required to extract correlation times, lengths, and mode structure.

* B. Levitt, D. Maslovsky, M.E. Mauel, *Phys. Plasmas* **9**, 2507 (2002)

CTX



1kW ECRH H_2 plasma
B=1kG-50G ($1/R^2$)

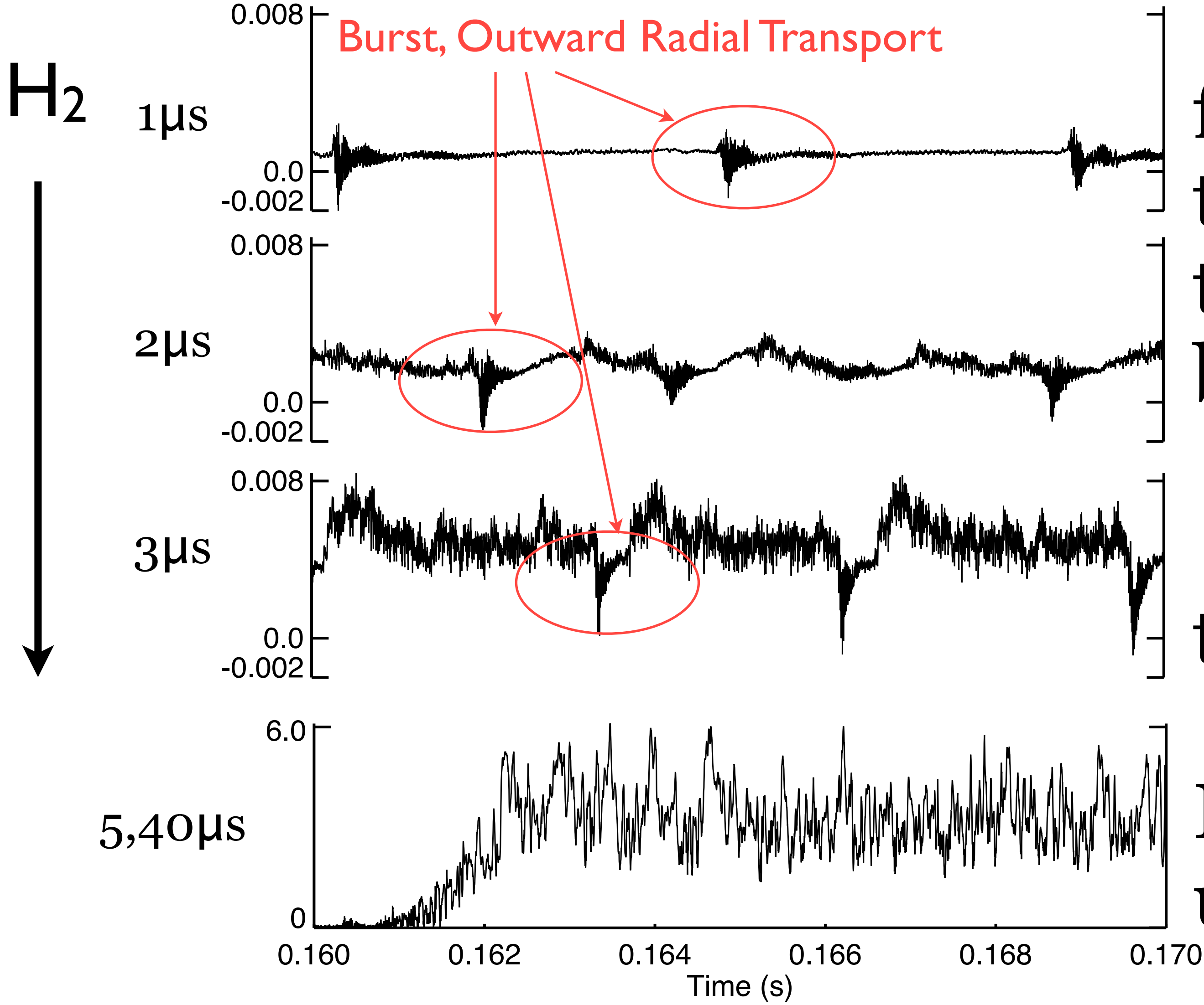
$\omega_c >> \omega_b >> \omega_d$

$\omega_c \sim 0.1\text{-}10$GHz
$\omega_b \sim 10\text{-}1000$MHz
$\omega_d < 5\text{-}10$MHz

Multiple moveable probes allows simultaneous measurement of plasma quantities at many different locations.

Dipole Plasma Density

I_{sat}~1000 times more in High Density

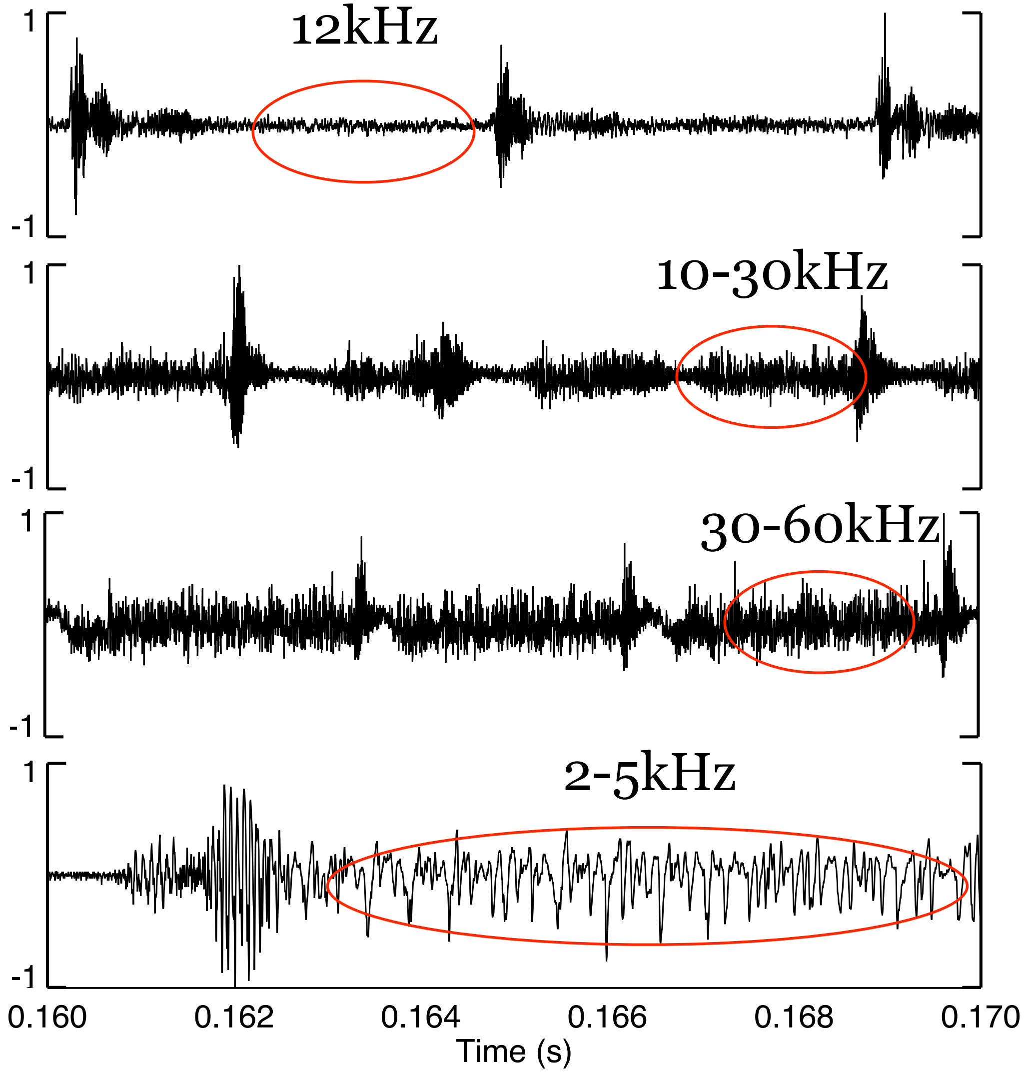


In 'Low Density', fueling decreases the characteristic time between HEI bursts.

Fueling stabilizes the HEI bursts.

Fluctuations order unity.

Floating Potential Fluctuations



Fueling in low density causes the intensity and frequency of fluctuations to increase.

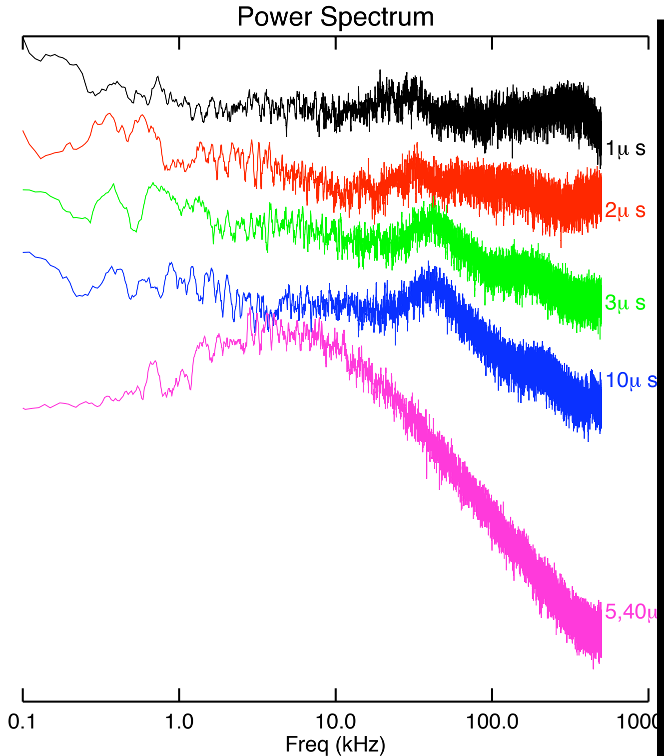
The time-scale of fluctuations decreases in high density.

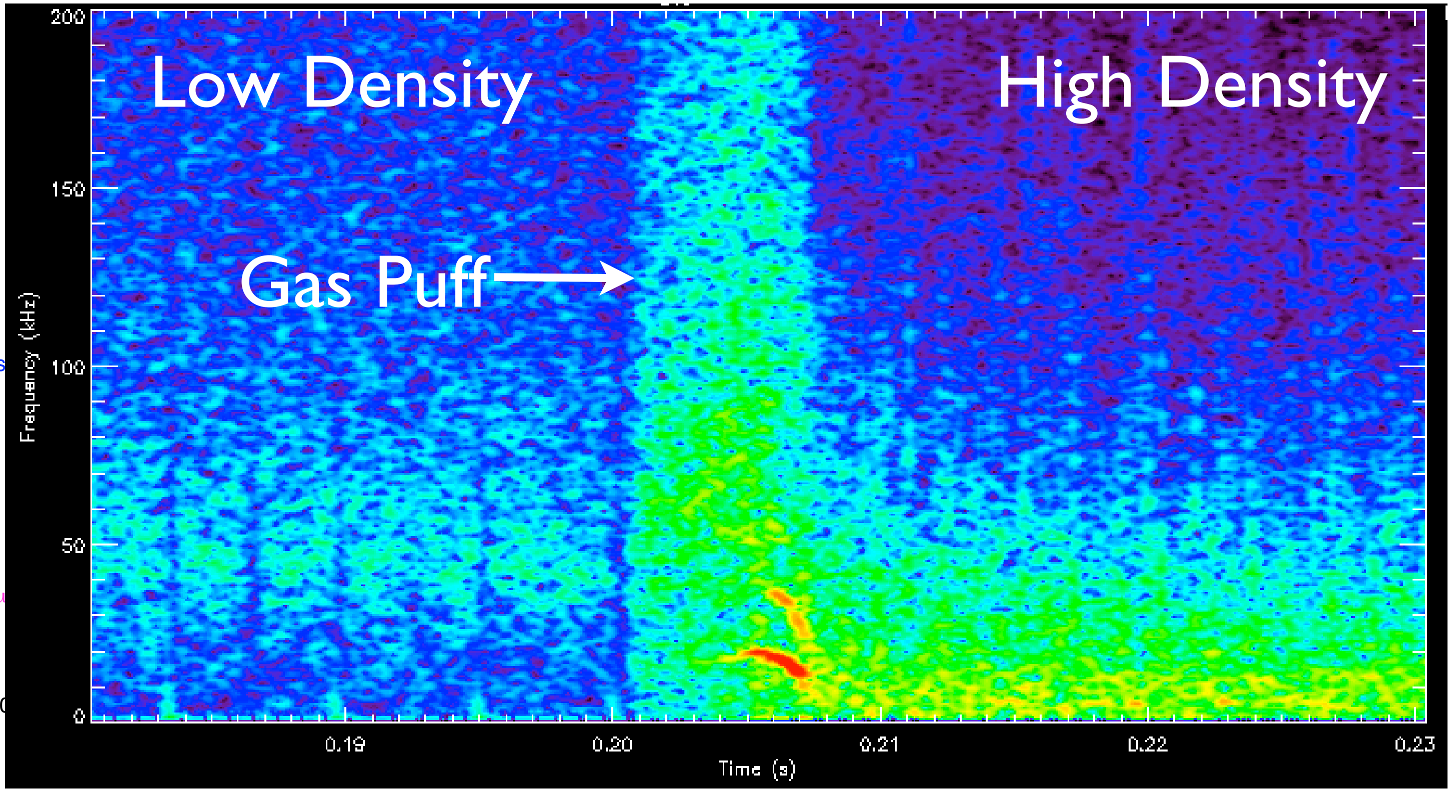
Spectra

- Increased gas pressure causes change in the spectral characteristics of the fluctuations.

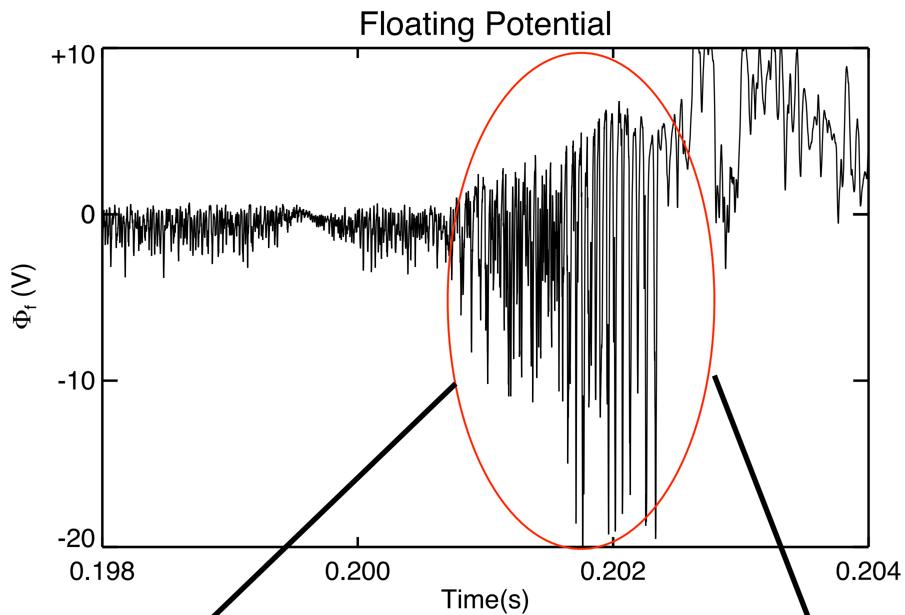
- Causes a trend towards power-law like spectra.

Spectrogram reveals the dynamic change in floating potential fluctuations.

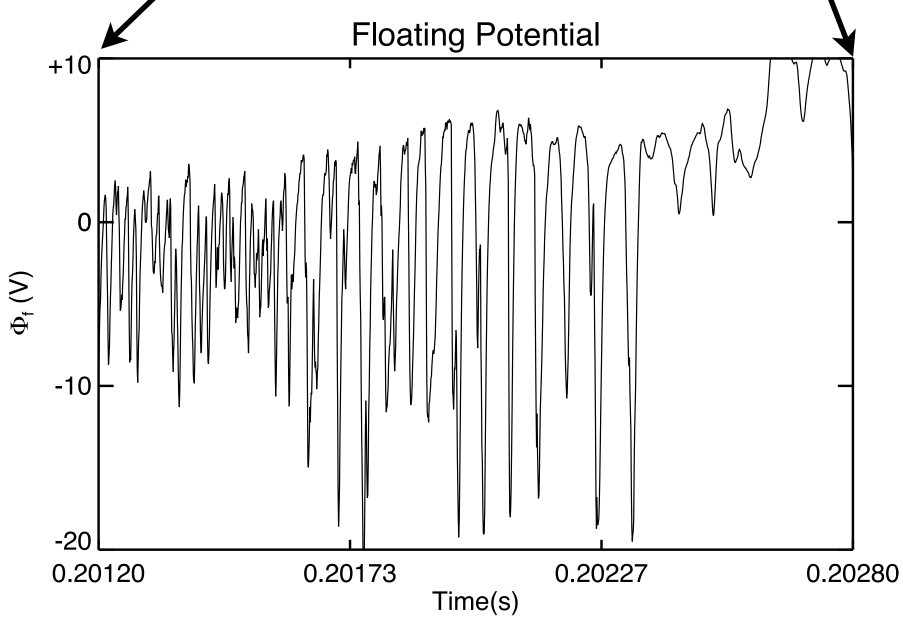


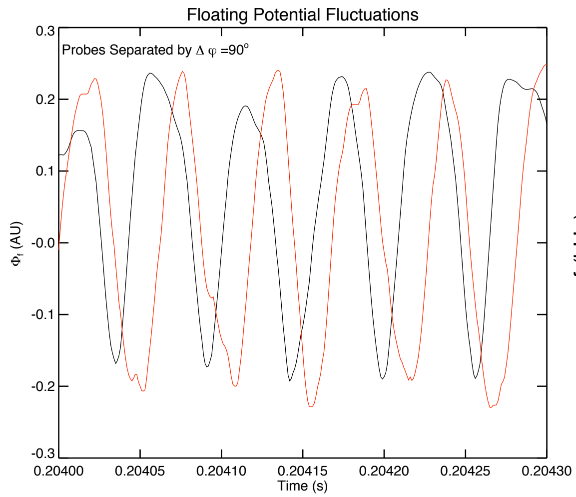


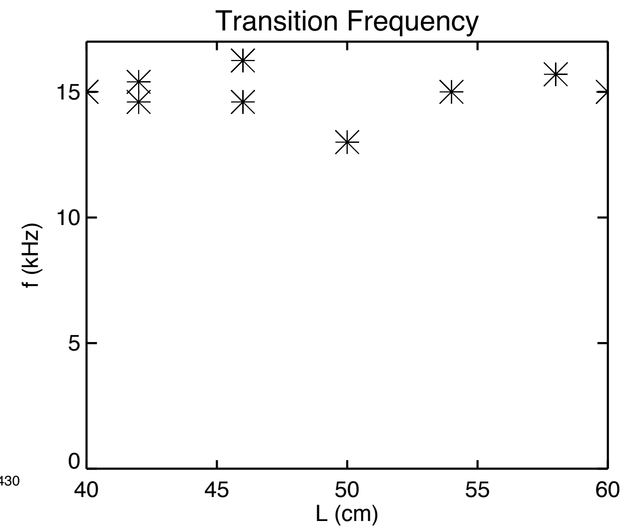
Transition to High Density



- 15kHz m~1, rigidly-rotating mode with $k_{||}$~0 marks the transition to the High Density regime.
- The waveform has a nonlinearity, hence the m~1, where probes separated azimuthally record a 55° phase shift.







Non-axisymmetric Blob!

Fourier Statistics

$$S(t) \rightarrow_{FFT} \rightarrow \hat{S}(\omega)$$

$$\hat{C}_{1,2}(\omega) \equiv \hat{S}_1(\omega)\hat{S}_2^*(\omega)$$

$$\alpha_{1,2} \equiv tan^{-1}\left(\frac{\Im[\hat{C}_{1,2}(\omega)]}{\Re[\hat{C}_{1,2}(\omega)]}\right)$$

$$< \alpha_{1,2} > \equiv \frac{\int \alpha_{1,2}(\omega)|\hat{C}_{1,2}(\omega)|d\omega}{\int |\hat{C}_{1,2}(\omega)|d\omega}$$

$$< \gamma_{1,2}^2 > \equiv \frac{< |\hat{C}_{1,2}(\omega)|^2 >}{< \hat{S}_1 >< \hat{S}_2 >}$$

Classic Fourier methods require signals to be stationary and linear, otherwise spurious harmonics will be generated to match nonlinear signals.

In high density plasmas, the ensemble cross phase $<\alpha_{1,2}>\approx 0$ (random phase), and squared cross-coherence $<\Upsilon^2_{1,2}>\approx 1\%$ due to a non-existing linear mode.

Correlation analysis and mode decomposition can provide information about the dynamics.

Bicoherence

- Transform a time series to the frequency domain.
- Create the Bispectrum over many records (ensemble average)
- Form power-weighted Bispectrum (bicoherence) after M samples have be taken.
- 95% confidence for $b^2 > 3/M$ *

$$S(t) \rightarrow_{FFT} \rightarrow \hat{S}(\omega)$$

$$< A >= \frac{1}{M} \sum_{i=1}^{M} A_i$$

$$\hat{B}(\omega_1, \omega_2) =< \hat{S}(\omega_1)\hat{S}(\omega_2)\hat{S}^*(\omega_1 + \omega_2) >$$

$$\hat{b}^2(\omega_1, \omega_2) = \frac{|\hat{B}(\omega_1, \omega_2)|^2}{| < \hat{S}(\omega_1)\hat{S}(\omega_2) > |^2 | < \hat{S}(\omega_1 + \omega_2) > |^2}$$

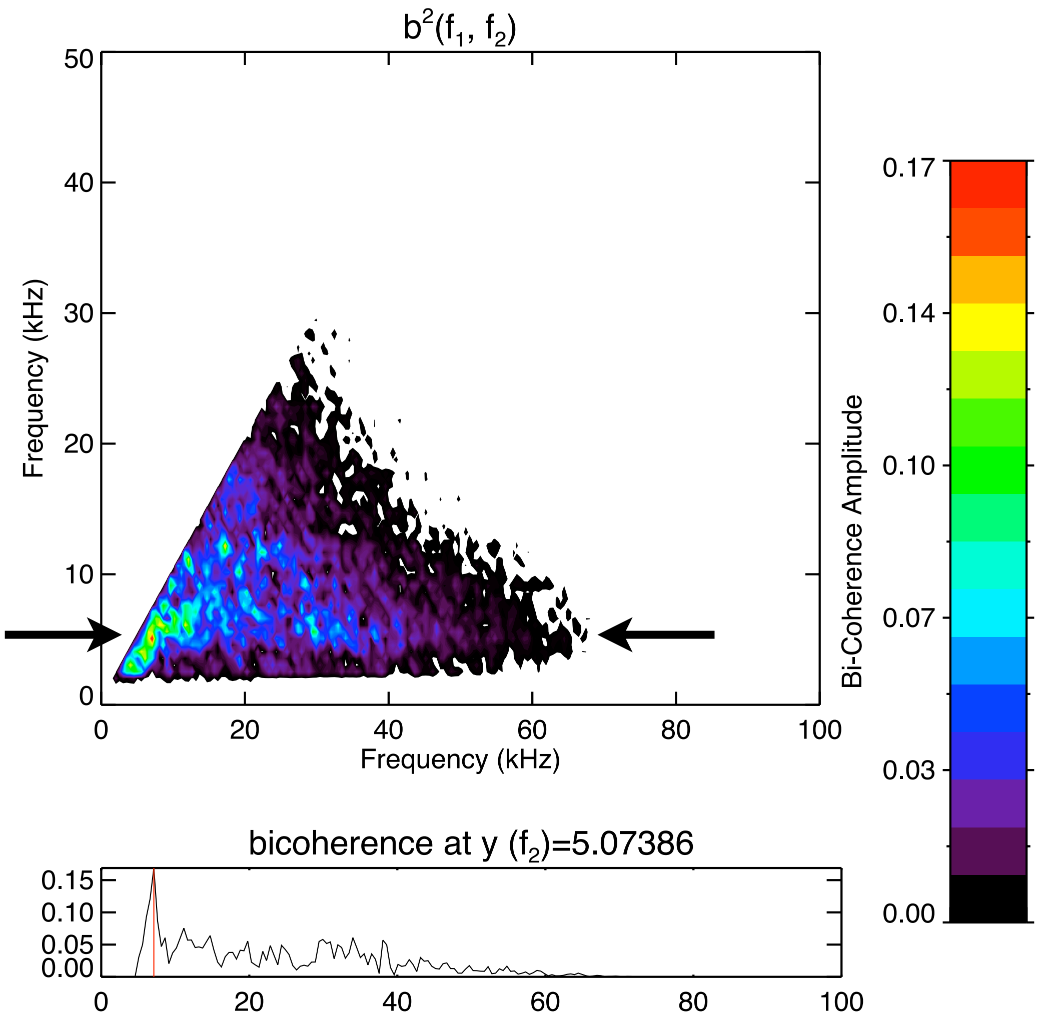
*V.Nosenko, J.Goree, and F.Skiff, Phys. Rev. E 73 016401 (2006)

The Analysis Procedure

- For the following figures, 730 records have been taken to calculate the bicoherence.
- The records overlap by 75% to accurately measure the biphase evolution.
- The frequency pair where the Max Bispectrum occurs is tracked in time, as well as the amplitude (BiAmplitude).
- The BiAmplitude is qualitative, and measures the intensity of mode-mode coupling in time.
- The frequency pairs record where, in frequency-space, the coupling occurs in time.

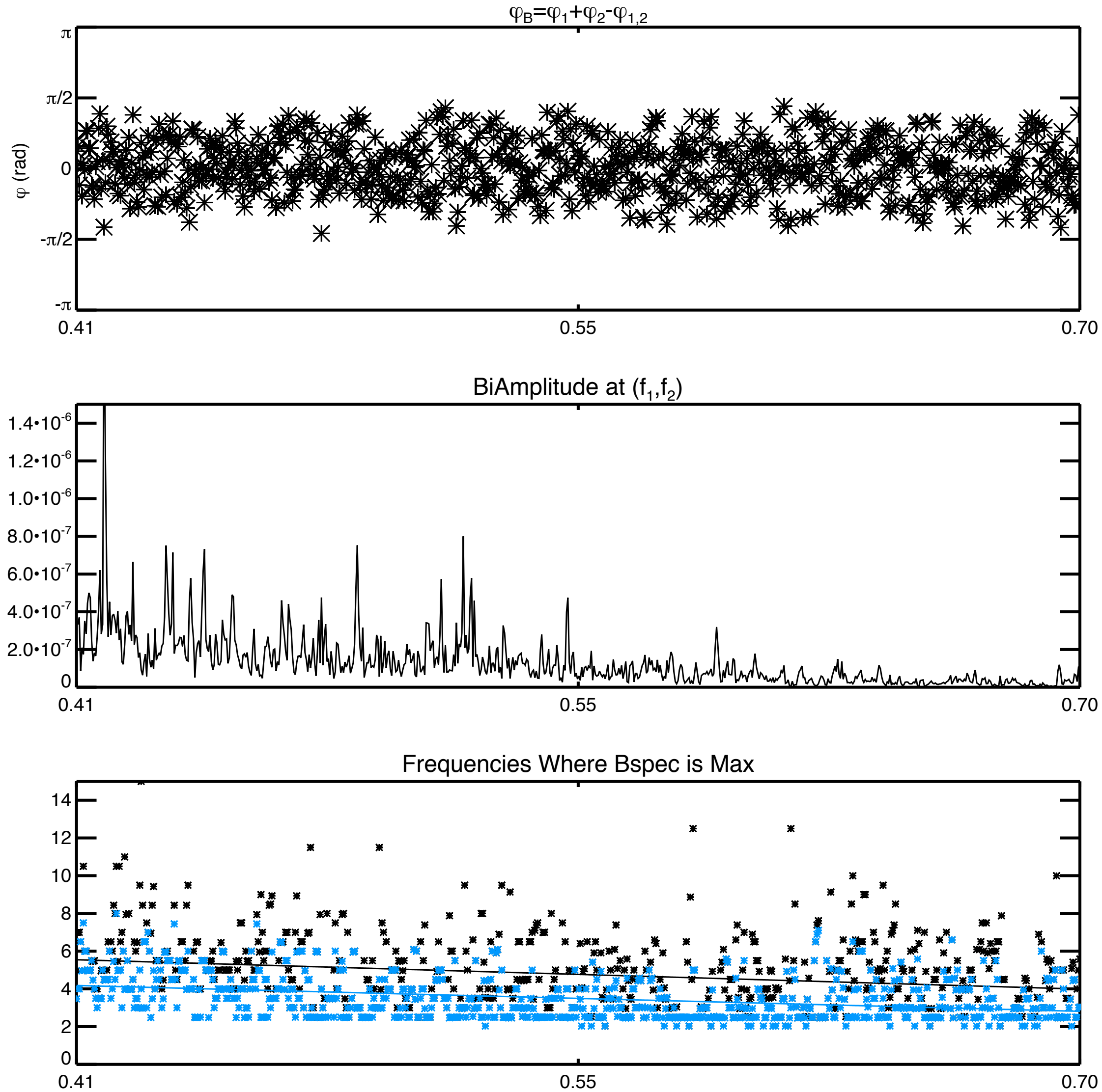
We are using a Fourier Mode technique to measure non-stationary fluctuations. Dominant frequencies evolve during the discharge, making the bicoherence a 'smeared-out' statistical measure.

Bicoherence



- Bicoherence Max at $(f_1,f_2)=(7,5)$kHz indicating mode-mode coupling (not harmonic).
- Rich coupling exists above statistical cutoff (0.004) across many frequency pairs (triangle-like region).
- 5kHz mode coupled to 7-40kHz modes (arrows).

Tracked BiAmplitude, Phase



- Tracked pairs maintain a biphase close to zero (phase coupled)
- BiAmplitude displays intermittency, but decreases over time.
- Frequencies at Max BiSpectrum decrease with slope -5.3, -4.7.

Correlation Functions

The time correlation function is defined as:

$$C_{1,2}(\tau) = \frac{\int_0^T S_1(t)S_2(t-\tau)dt}{\sqrt{\int_0^T S_1^2(t)dt \int_0^T S_2^2(t)dt}}$$

and can determine lag time and correlation time between two signals. $(\tau_{Lag}, \tau_{Corr})$

The lag time is the time by which signal 2 lags signal 1.

$$\tau_{Lag} > 0 \rightarrow \text{Signal 2 Lags Signal 1}$$

Correlation In Time

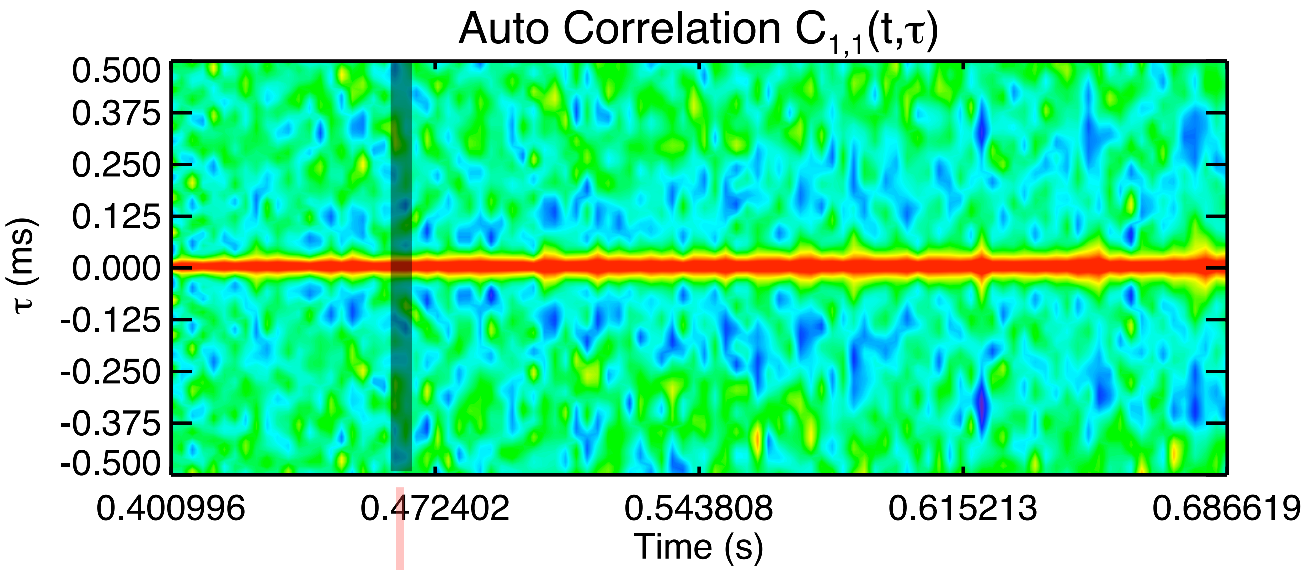
Multiple correlation functions can produce 'correlation in time' contour plots.

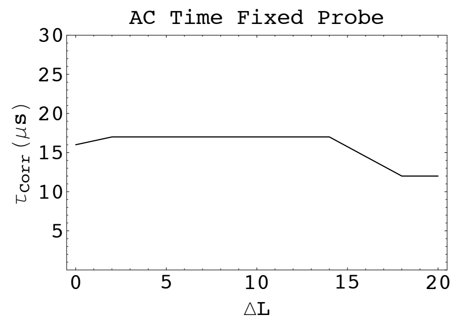
$$C_{1,2}(t,\tau) = [C_{1,2}^{(1)}(\tau), C_{1,2}^{(2)}(\tau), \ldots, C_{1,2}^{(M)}(\tau)]$$

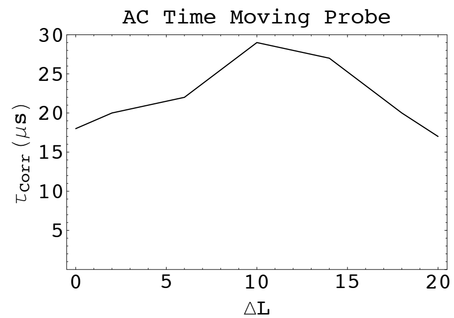
Ensemble correlation can be formed.

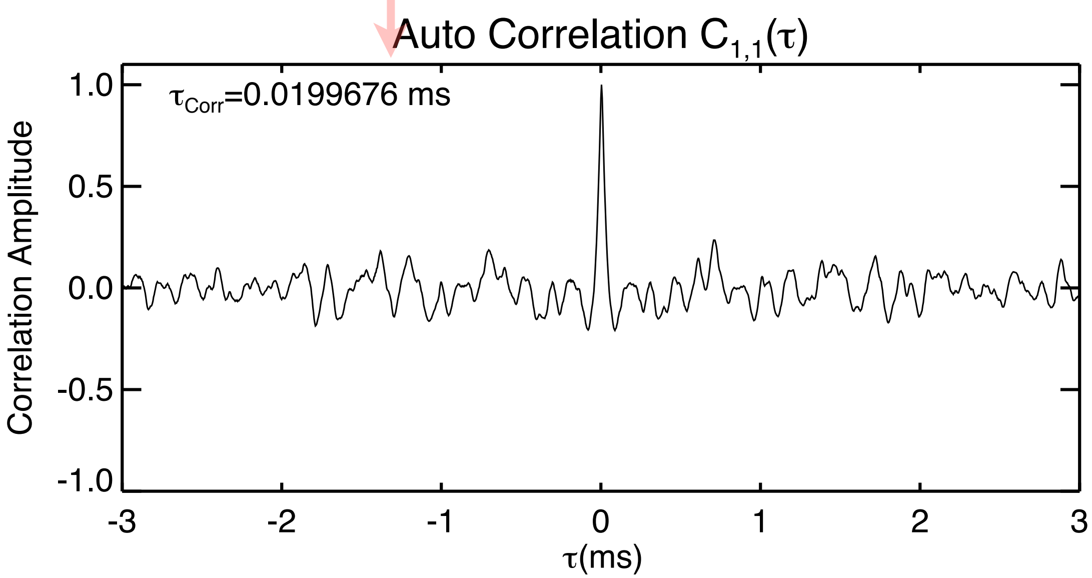
$$< C_{1,2}(\tau) >= \frac{1}{M} \sum_{i=1}^{M} C_{1,2}^{(i)}(\tau)$$

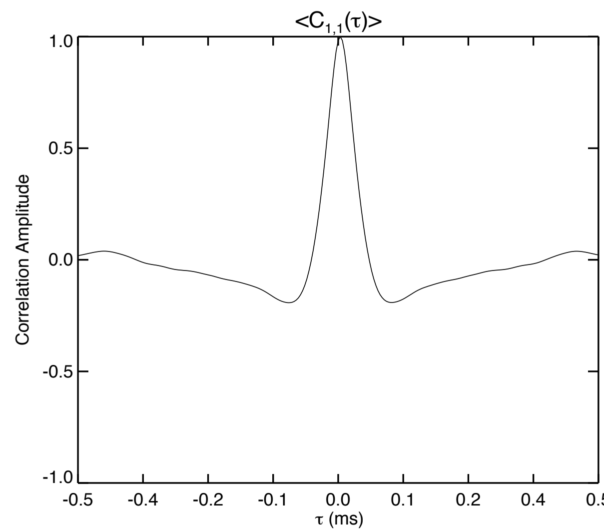
High Density Correlation





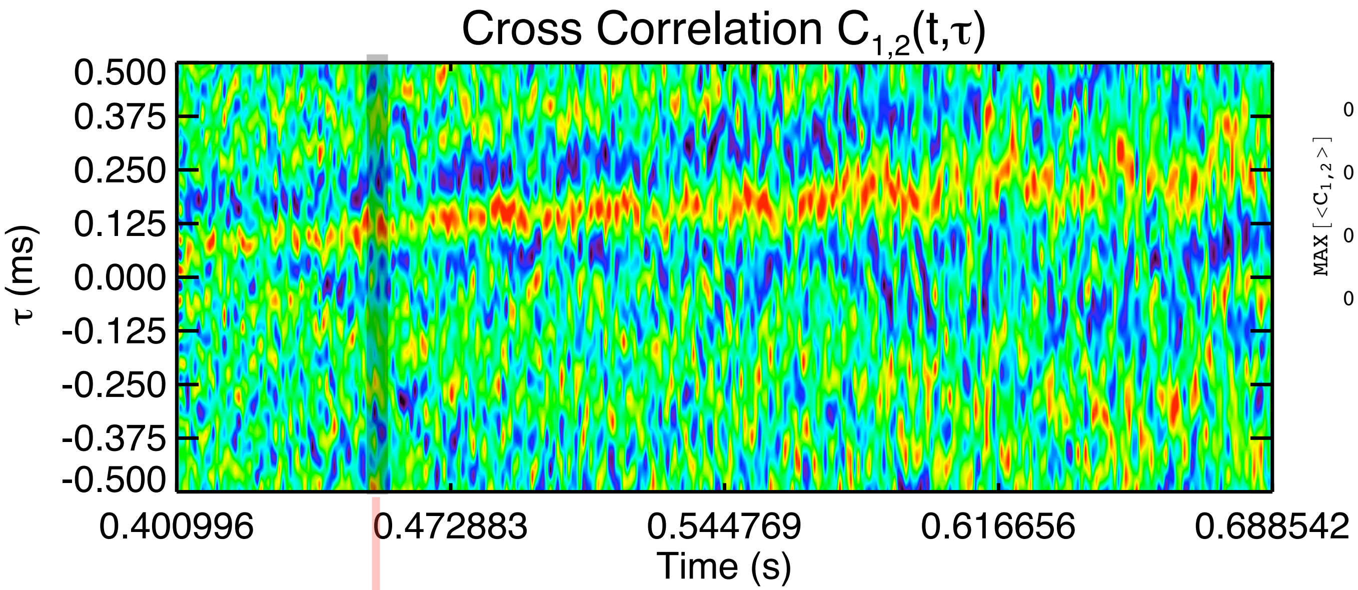


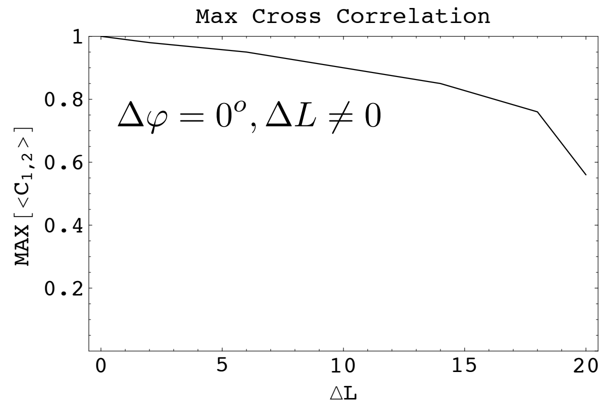




$$< \tau_{Corr} > \approx 0.02ms$$

High Density Correlation

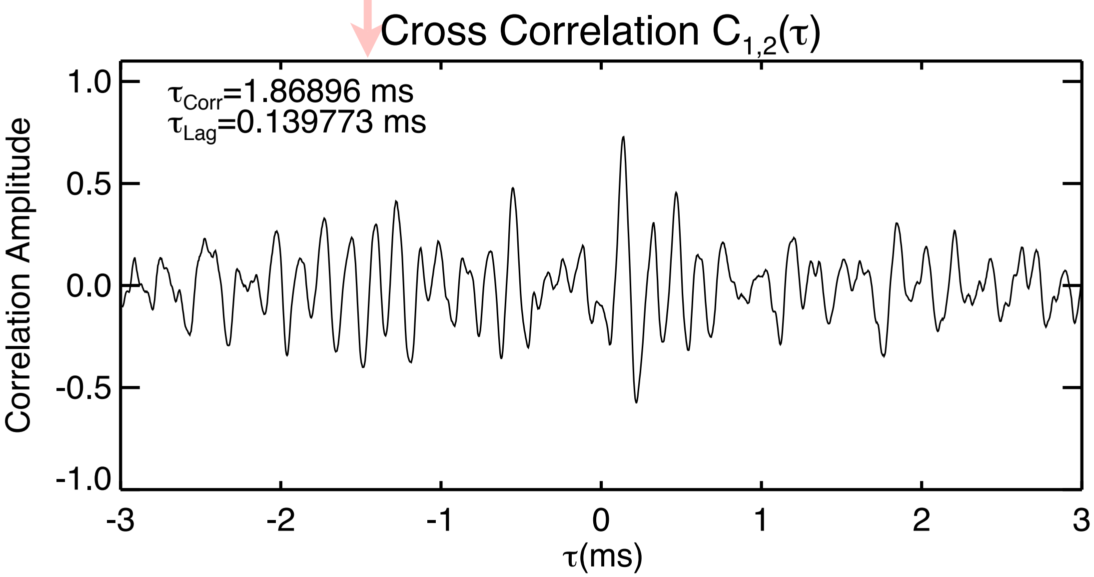


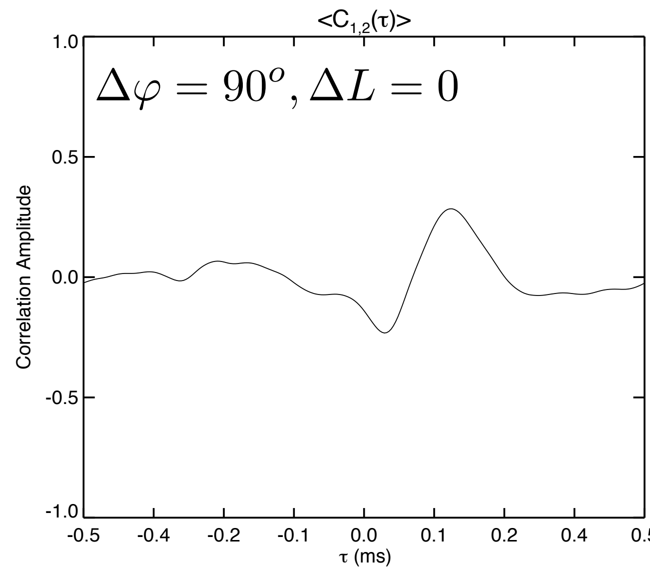


$$MAX[< C_{1,2}(\tau) >] = 0.284$$

$$< \tau_{Lag} >= 0.157ms$$

$$< \tau_{Corr} >= 0.044ms$$





Hilbert Spectrum*

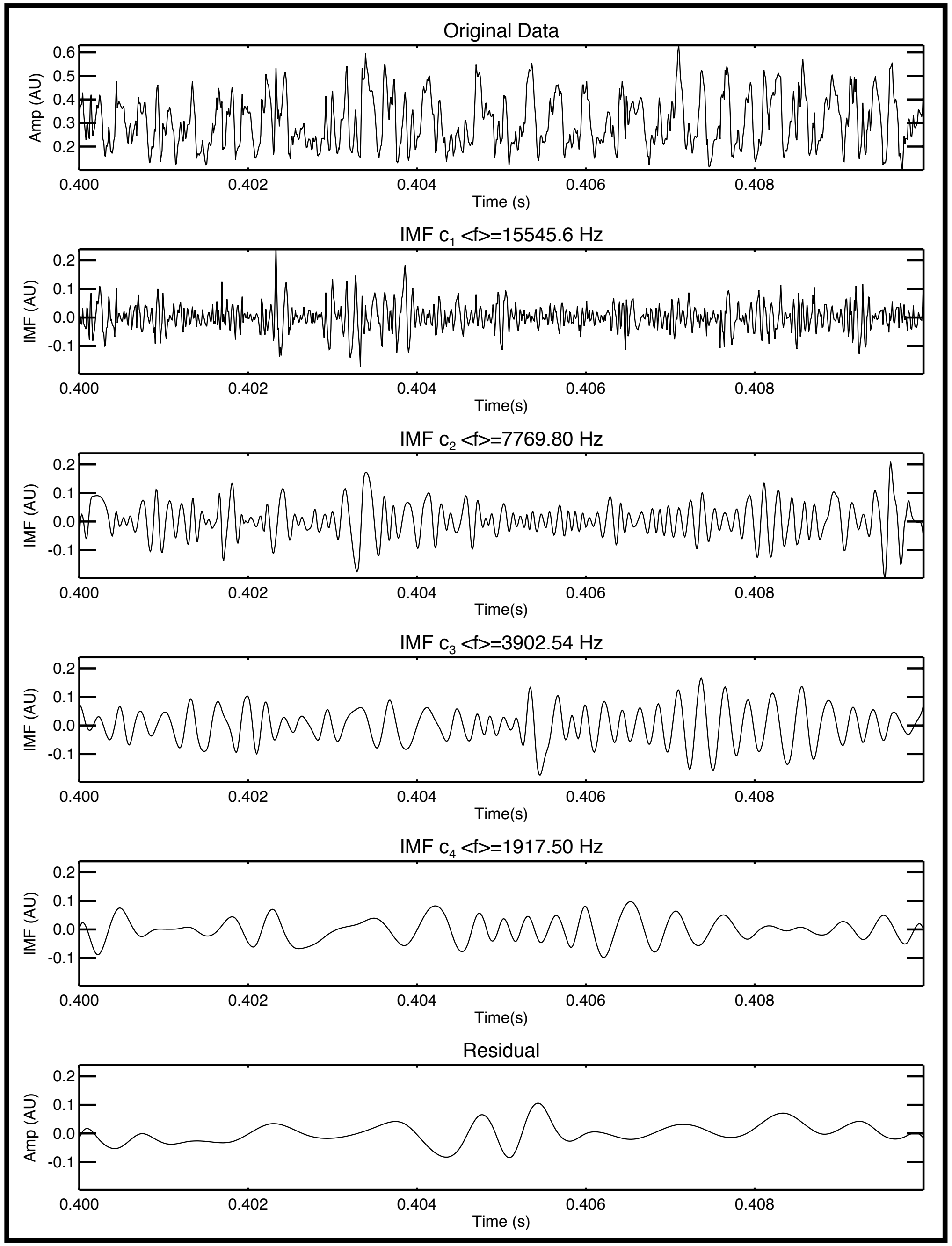
- Hilbert Transform given by: $Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt'$
- Form Analytic Function: $Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)}$
- Instantaneous Frequency: $\omega(t) = \frac{d\theta(t)}{dt}$ $\theta(t) = arctan\left[\frac{Y(t)}{X(t)}\right]$
- Instantaneous Amplitude: $a(t) = \sqrt{X(t)^2 + Y(t)^2}$
- The phase must be `unwrapped' before differentiating.

* Proc. R. Soc. Lond. A (1998) **454**, 903–995

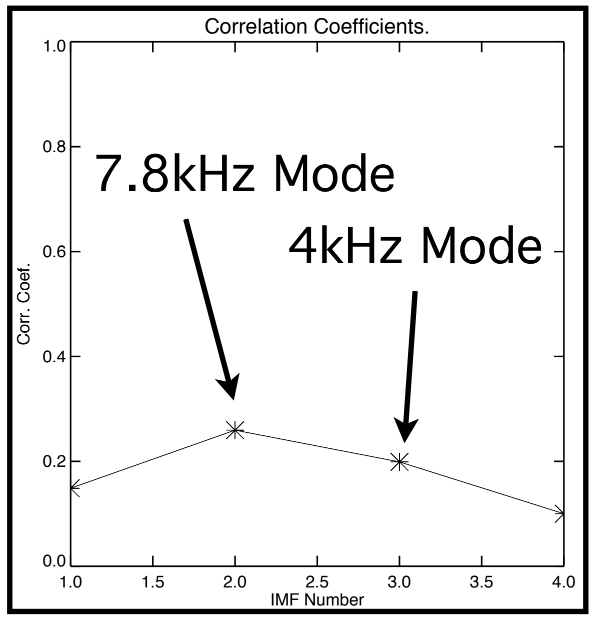
IMFs[1]

- In order to apply the Hilbert Transform, the time series must be of the class ‘Intrinsic Mode Functions’.
- Envelope functions symmetric about the local zero.
- No positive minima or negative maxima.
- Same number of zero crossings as extrema, within one.
- Formed by ‘sifting the time series’

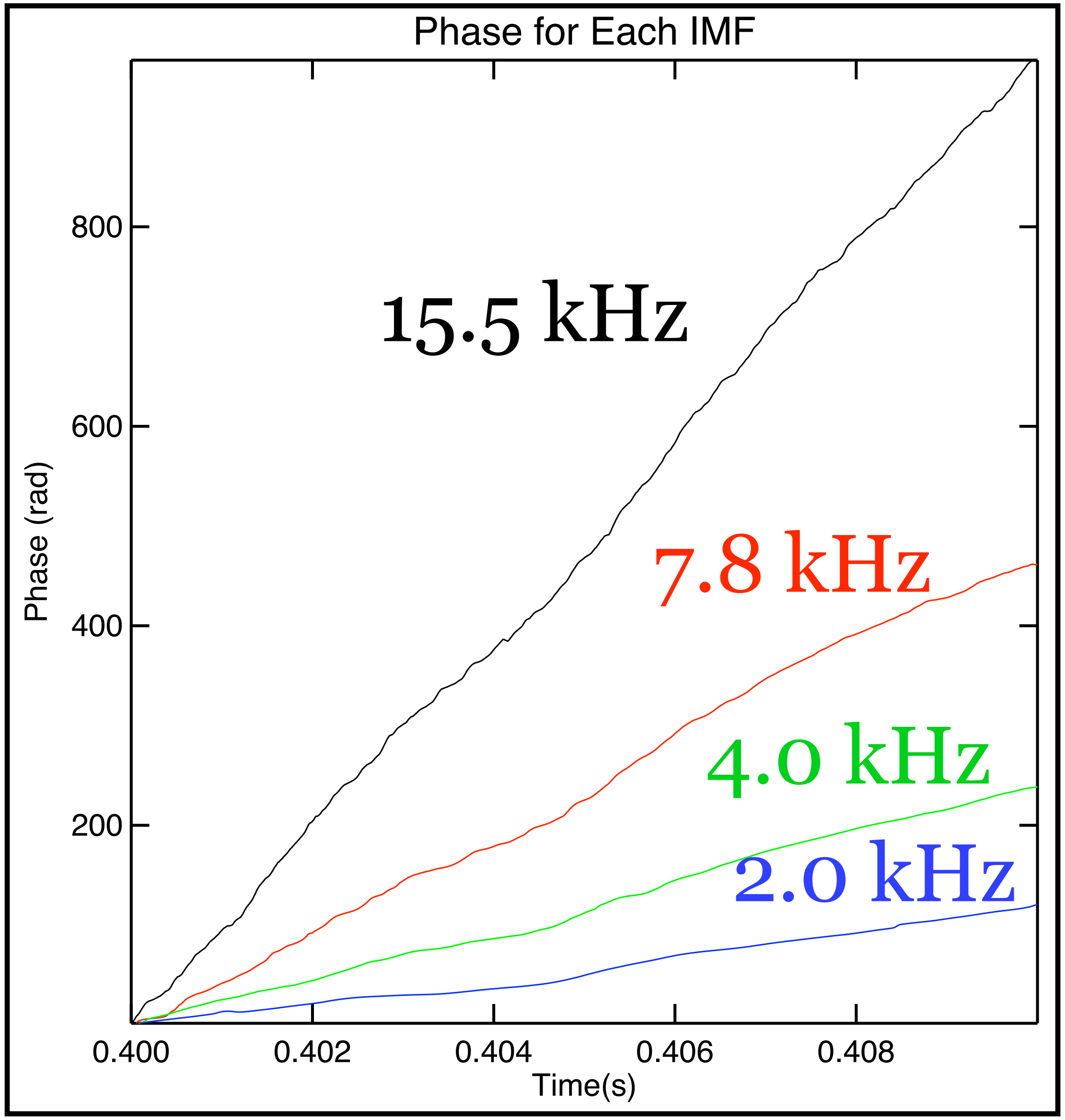
Sifting I_{sat} Data



- The data is sorted into functions with intrinsic time scales that are inherent to the data.
- Each IMF has a frequency which is approximately half the previous IMF



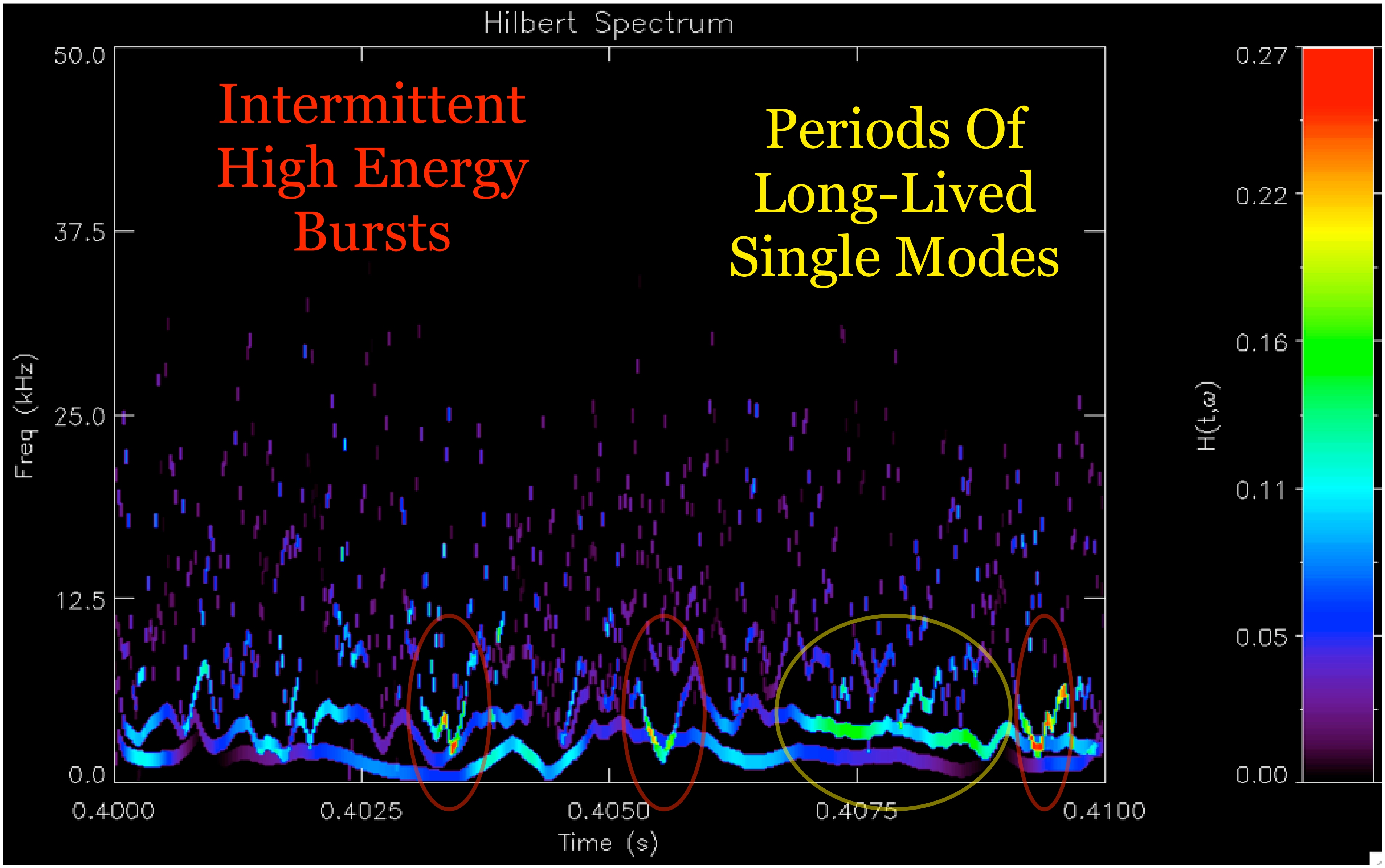
Instantaneous Phase



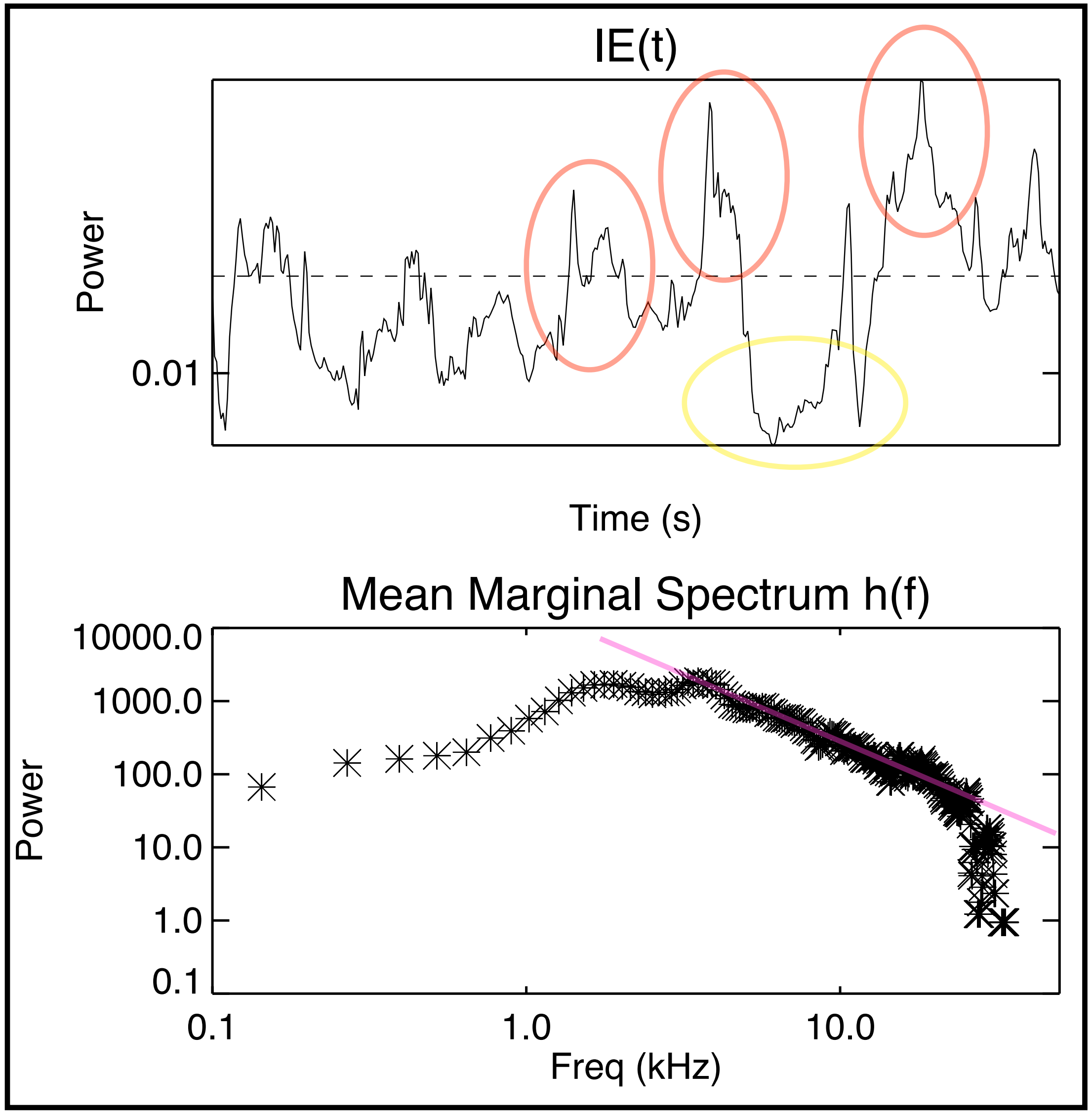
The instantaneous phase for each IMF.

The frequency regimes are well separated. A linear fit gives the average frequency.

Hilbert Spectrum



IE and Spectrum



Instantaneous Energy also records the highly energetic, intermittent bursts of activity.

$$IE(t) = \int_0^{\omega_N} H^2(t,\omega)d\omega$$

Spectrum also displays a power-law scaling, similar to FFT.

$$h(\omega) = \frac{1}{T}\int_0^T H(t,\omega)dt$$

Summary

- Using novel signal analysis techniques, the mode structure of a dipole-confined, high density, turbulent interchange mixing plasma has been investigated.
- This plasma displays a power-law frequency spectrum and intermittency, characteristic of a turbulent phenomena.
- Diagnostics have begun to probe the time-space correlation of structures in the plasma, and revealed transient correlation. This is markedly different from previous investigations.
- Hilbert Transform methods provide accurate measurements of dominant temporal modes, as well as more accurate measurement of frequency and power evolution in time than simple spectrograms.
- Bicoherence suggests that the dynamics are dominated by nonlinear phenomena.

Future Work

- Implementation of new amplifiers providing real-time visualization of polar loss current, able to diagnose mode structure with high spatio-temporal resolution.

- Improved simulation code with parallel implementation (MPI, PETSc) providing high spatial resolution and inclusion of nonlinear terms.

$$\nabla \cdot [\epsilon \mathbf{B} \times (\mathbf{V} \cdot \nabla)\mathbf{V}] = \frac{1}{B}\frac{\partial}{\partial y}\left[\epsilon\left(- \frac{\partial \Phi}{\partial x}\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial \Phi}{\partial y}\frac{\partial^2 \Phi}{\partial x \partial y}\right)\right] - \frac{1}{B}\frac{\partial}{\partial x}\left[\epsilon\left(+ \frac{\partial \Phi}{\partial x}\frac{\partial^2 \Phi}{\partial y \partial x} - \frac{\partial \Phi}{\partial y}\frac{\partial^2 \Phi}{\partial x^2}\right)\right]$$