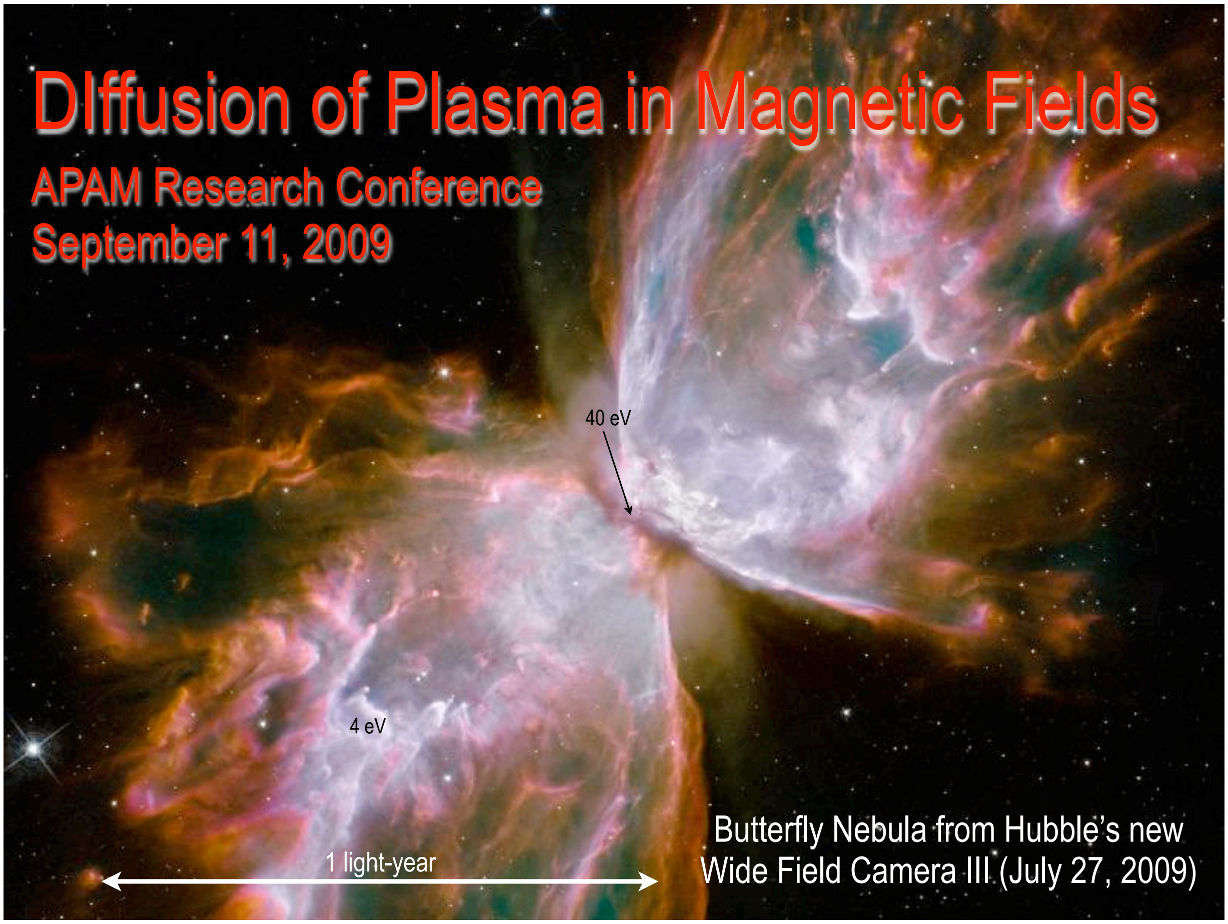
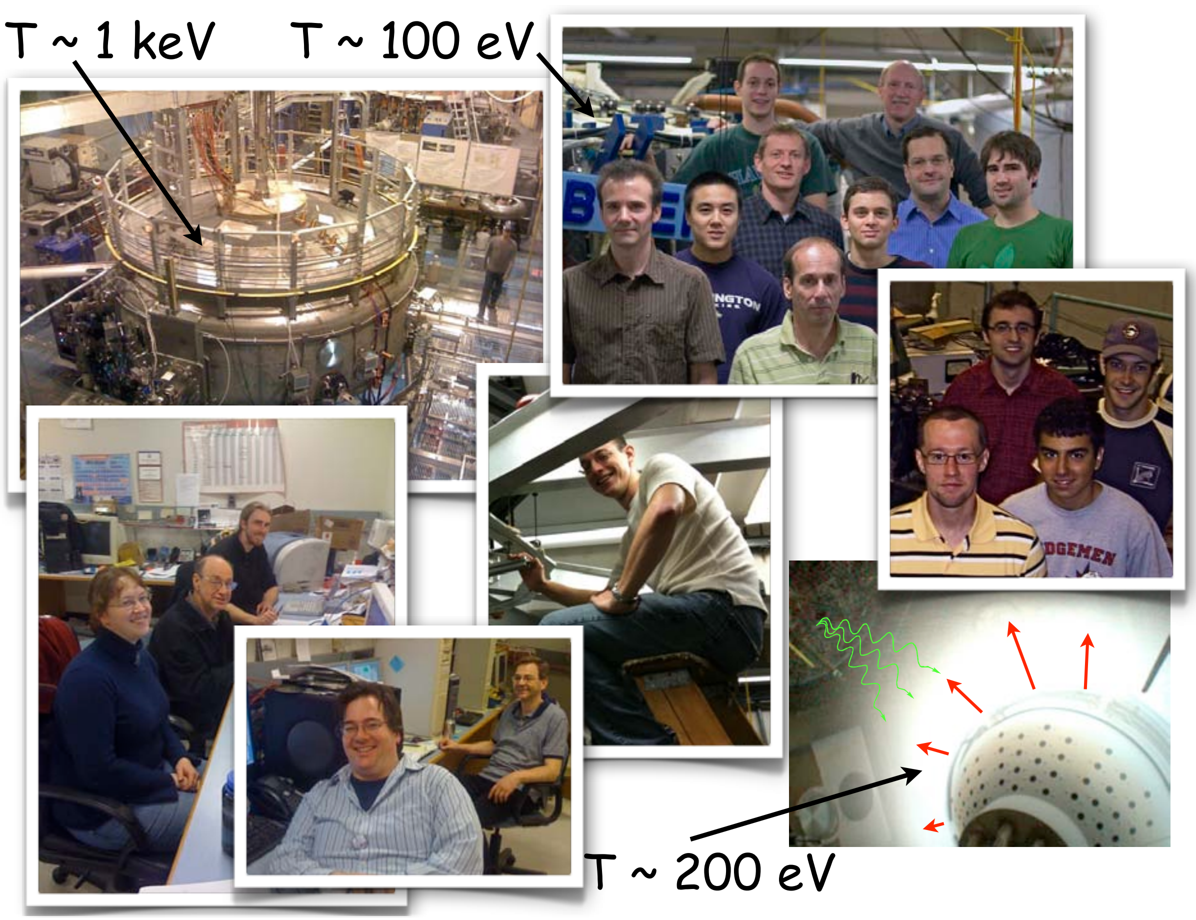

DIffusion of Plasma in Magnetic Fields
APAM Research Conference
September 11, 2009
40 eV
4 eV
1 light-year
Butterfly Nebula from Hubble's new
Wide Field Camera III (July 27, 2009)

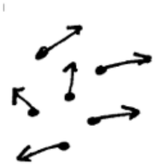

T ~ 1 keV
T ~ 100 eV
T ~ 200 eV

"Plasma Diffusion in Magnetic Fields"

Outline

- Gaseous diffusion vs. magnetized plasma diffusion
- B. B. Kadomtsev, *Plasma Turbulence*, 1965
- T. Birmingham, *JGR*, 1969
- First observation of strong turbulent pinch in laboratory
  (Please be patient: *shown on last slide!*)

Gaseous Diffusion



GAS

$$\frac{\partial n}{\partial t} = -\nabla \cdot \Gamma \qquad \Gamma = -D\frac{\partial n}{\partial x}$$

$$D = \lim_{t \to \infty} \frac{\langle (x(t) - x(0))^2 \rangle}{2t}$$

$\lambda_{mfp} \sim 70\ nm$ AIR

$\sim 1\ Km$ PLASMA EXPERIMENTS

$$x(t) - x(0) = \int_0^t dt' \, v(t')$$

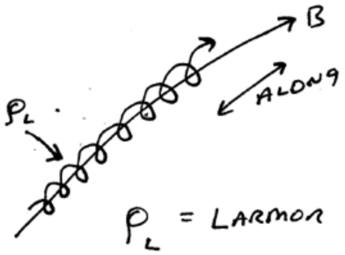
$$\therefore D = \int_0^\infty dt' \langle v(t') v(0) \rangle$$

$$= \langle v^2 \rangle \tau_{collision}$$

$\Longrightarrow$ MORE COLLISIONS, THEN <u>LESS</u> DIFFUSION

$$D \propto \frac{1}{PRESSURE}$$

Magnetized Plasma Diffusion



$D_{\parallel} \sim \langle v^2 \rangle \tau_{col} \sim 10^{12} D_{\perp}$

ρ_L = LARMOR RADIUS = $\frac{v}{\omega_c}$

$$\boxed{\underbrace{\omega_c \sim \frac{eB}{m} \gg \frac{1}{\tau_{collison}}}_{\text{MAGNITIZED}}}$$

$$D_{\perp} \sim \lim_{t \to \infty} \frac{\langle (x(t) - x(0))^2 \rangle}{2t}$$

$|x(t) - x(0)| \sim \rho_L \sim$ RANDOM COLLISION STEP SIZE

$$\therefore \; D_{\perp} \sim \frac{\rho_L^2}{\tau_{col}}$$

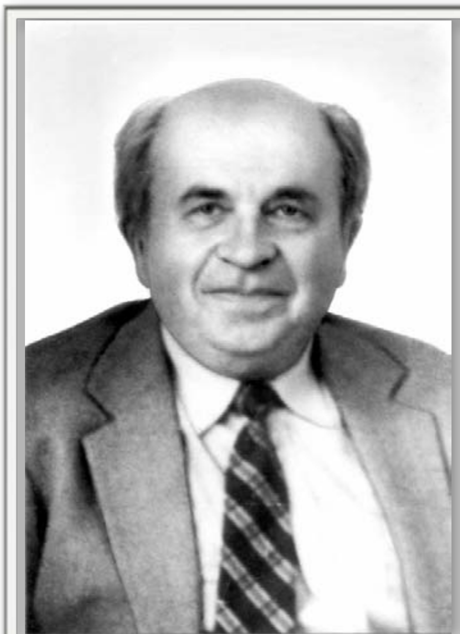
MORE COLLISIONS, THEN <u>MORE</u> DIFFUSION

BUT $D_{\perp} \propto \frac{1}{B^2}$ ← GOOD

Boris Kadomstev

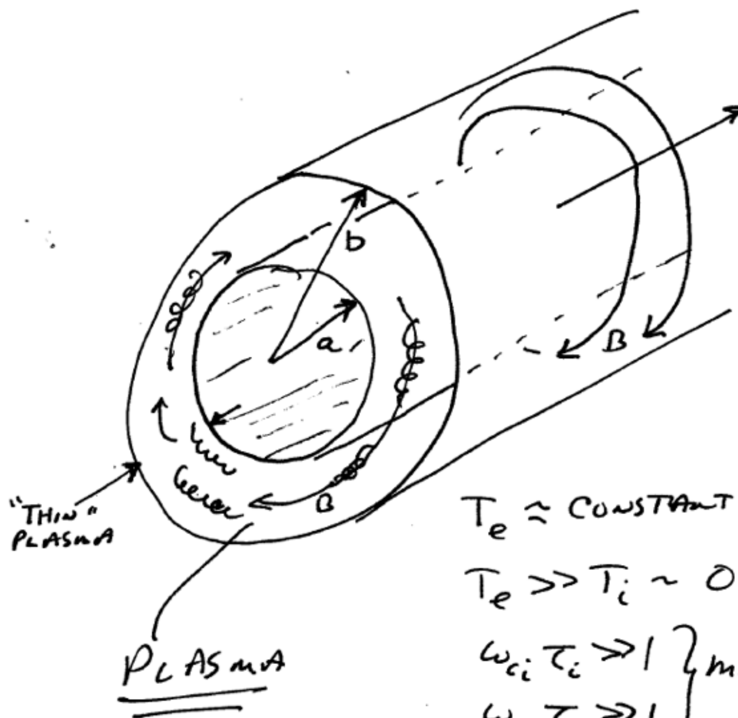
[Early on] there was no agreement whatsoever between the theory of magnetic confinement and experiment: in defiance of the idealized theoretical models of a calm inhomogeneous plasma in a magnetic field, real plasma always exhibited strong oscillations. It was difficult even to imagine how one could approach this fervid substance with explanations. It was then that the extraordinary physical intuition and imaginative thinking of B B Kadomtsev came to the fore. Very important … were the explanations given by B B Kadomtsev to the experiments on *plasma instability in a glow discharge placed in an external magnetic field* and to the experiments of M S Ioffe and colleagues concerned with detection of *trough instability* and the resulting loss of plasma. **These two works by B B Kadomtsev became milestones in the theory of controlled fusion, since they refuted the prevailing belief in the universality and inevitability of Bohm diffusion that shattered the hopes for a feasible thermonuclear reactor. B B Kadomtsev's works instilled faith in the possibility of gaining control over the processes in plasma.**

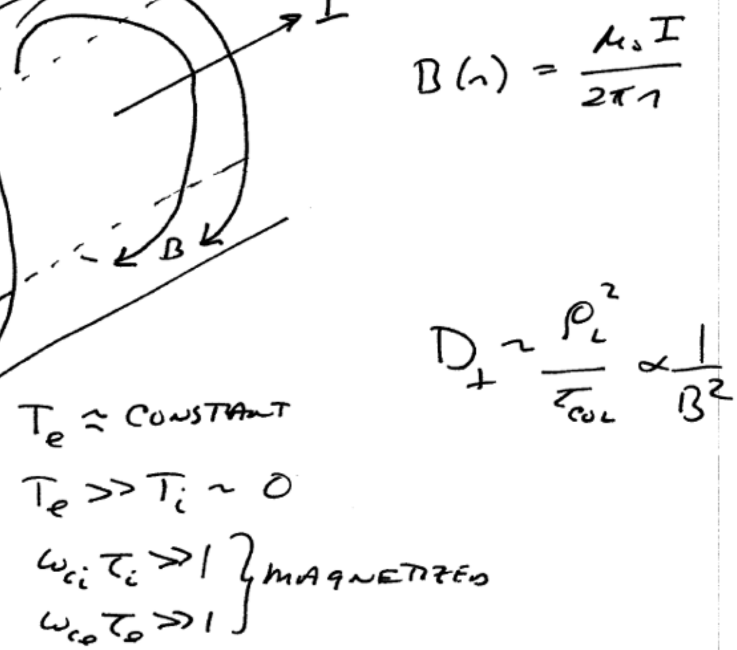
"In memory of Boris Borisovich Kadomtsev," *Physics Uspekhi* **41**, 1155 (1998), by E P Velikhov, V L Ginzburg, A V Gaponov-Grekhov, AM Dykhne, L V Keldysh, Yu L Klimontovich, V I Kogan, MB Menski|, L P Pitaevski|, V E Fortov, N A Chernoplekov, V D Shafranov.



Boris Borisovich Kadomtsev
(09.11.1928 – 19.08.1998)

Magnetized Plasma Torus





Increasing B

Bostick and Levine, 1955

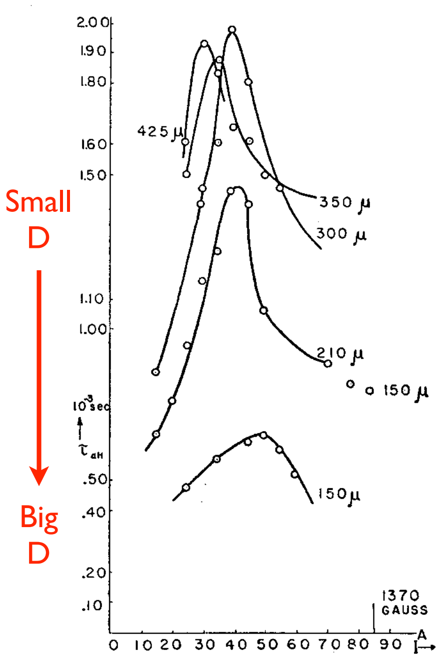


FIG. 8. Results of measurements of diffusion of He ions in helium gas in a toroid with an annular magnetic field. Ordinates are the decay times τ_{aH} which are proportional to $1/D_{aH}$, where D_{aH} is the ambipolar diffusion coefficient in a magnetic field. Abscissa is the magnetic field currents.

Hoh and Lehnert, 1960

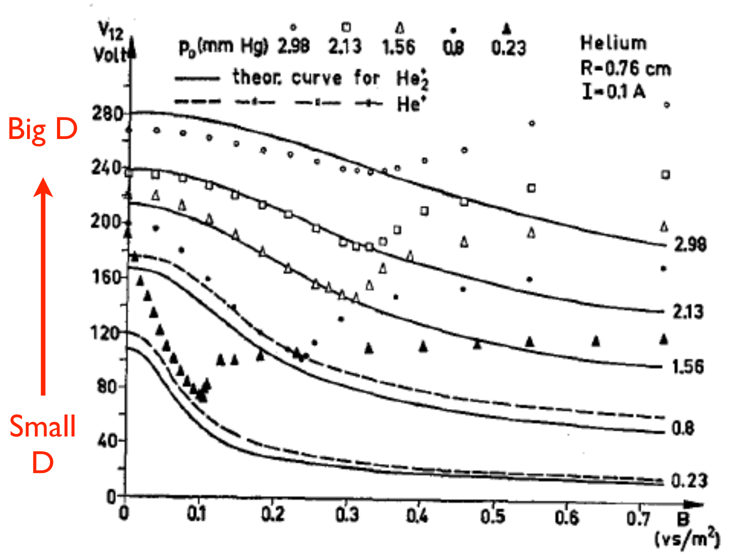


FIG. 3. Voltage drop measured across two probes 0.345 m apart as a function of the magnetic field. The full curves are calculated from the theory for molecular ions and the dashed curves for atomic ions.

Electric Fluctuations (!)

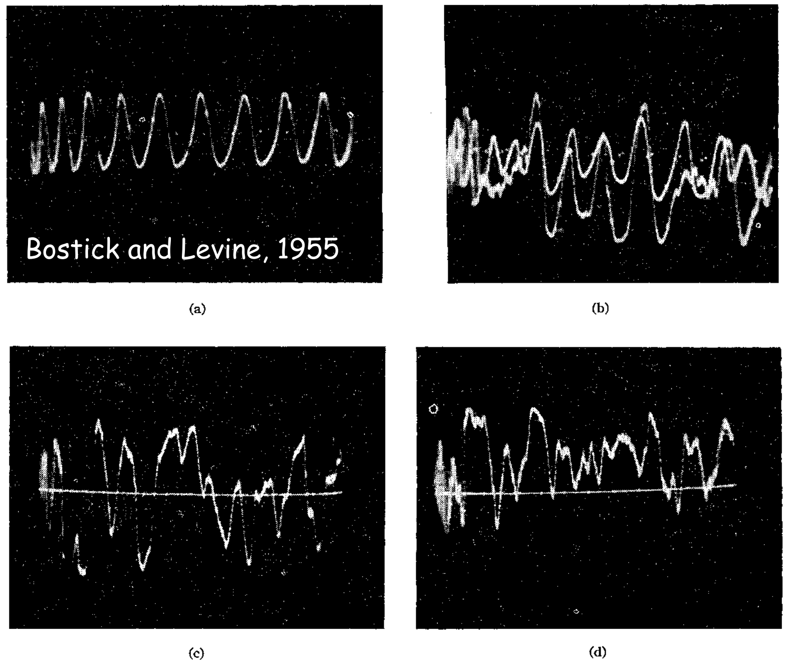
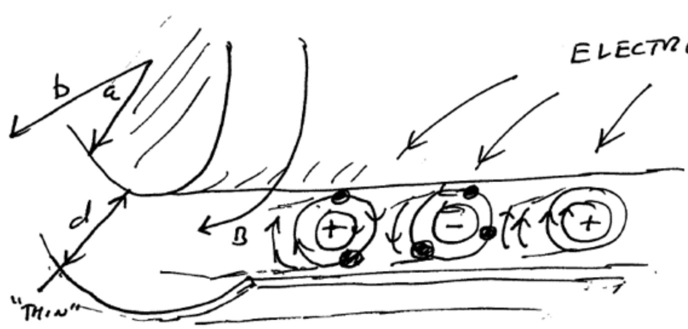


FIG. 2. (a) Argon at 0.5 mm Hg, $H=350$ gauss; frequency of oscillations is 6500 cps. (b) Argon at 0.5 mm Hg, $H=460$ gauss, $H=530$ gauss. (c) Argon at 0.2 mm Hg, $H=530$ gauss. (d) Argon at 0.2 mm Hg, $H=690$ gauss.
Current wave forms picked up by a probe in a plasma produced in the toroid shown in Fig. 1 at the indicated values of pressure and magnetic field in argon gas. The gas is excited by a dc potential of 600 volts between two probes which are diametrically opposite across the toroid. These wave forms are attributed to plasma waves of the magneto-hydrodynamic type. Note increase in higher-frequency components with increase in the magnetic field. Time is the abscissa and time marker dots are 100 μsec apart.

Kadomstev's Analysis (1965)



ELECTRIC CONVECTION OF PLASMA-FILLED FLUX TUBES (FLUX RINGS!)

"SLOW" FORCE BALANCE

$$\bar J \times \bar B - \nabla P - \frac{n M_i}{\tau_i}\bar V = n M_i \frac{d\bar V}{dt} \approx 0$$

OHM'S LAW

$$\bar E + \bar V \times \bar B = \left(\frac{m_e}{e^2 n \tau_e}\right)\bar J \approx 0$$

$$\bar V = \frac{\bar E \times \bar B}{B^2} \qquad \bar E = -\nabla \Phi$$

$$\nabla \cdot \bar J = 0$$

$$\frac{\partial n}{\partial t} + \nabla \cdot n\bar V = 0$$

MAGNETIZED PLASMA MOVES AS TUBES OF MAGNETIC FLUX

$$\frac{d}{dt}\int d\bar A \cdot \bar B = \int d\bar A \cdot \frac{\partial \bar B}{\partial t} + \oint d\bar \ell \cdot \underbrace{\bar V \times \bar B}_{\bar E}$$

$$= \int d\bar A \cdot \left(\frac{\partial \bar B}{\partial t} + \nabla \times E\right) = 0$$

Linear Instability

Kadomstev's Analysis (1965)

$$\tilde{E} = -\nabla\tilde{\Phi} \qquad \tilde{\Phi}(r,z,t) = \tilde{\Phi}\sin\left(\frac{\pi r}{d}\right)e^{-j(\omega t - kz)}$$

$$\int \frac{dl}{B}\left(\frac{\partial n}{\partial t} + \nabla\cdot n\bar{v} = 0\right)$$

$$\int \frac{dl}{B}\left(\nabla\cdot\bar{J} = 0\right)$$

$\int \frac{dl}{B} n = N$

$N =$ FLUX TUBE NUMBER

$= \langle n \rangle \delta V$

$$-j\omega = -(D_+ - D_c)\left(k^2 + \left(\frac{\pi}{d}\right)^2\right)$$

UNSTABLE IF

$D_+ < D_c$

$$D_c \equiv -\frac{D_a}{a}\frac{d^2}{2\pi^2}\frac{\partial}{\partial r}\log(N)$$

SINCE $D_+ \sim \frac{1}{B^2}$

LARGE B FIELD DESTABILIZES INTERCHANGE INSTABILITY

Nonlinear Transport

Kadomstev's Analysis (1965)

$$\Gamma = -D_\perp\frac{\partial n}{\partial r} + \langle\tilde{n}\tilde{v}\rangle$$

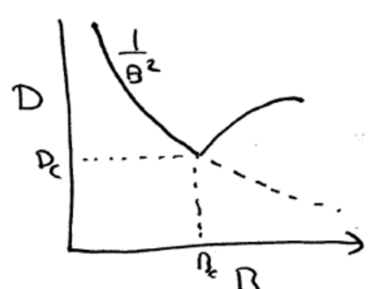
$$= -(D_+ + D_E)\frac{\partial n}{\partial r}$$

$$D_E = \begin{cases} 0 & \text{IF } D_+ > D_c \\ 2\left(\frac{E}{B}\right)^2\tau_{DIFF} & \text{IF } D_+ < D_c \end{cases}$$

$|V_E| \sim \left|\frac{E}{B}\right|$

$D_E \sim V_E^2\,\tau_{DIFF}$

$$\left(\frac{E}{B}\right)^2 = 4k^2D_\perp^2\left(1 - \frac{D_+}{D_c}\right)$$



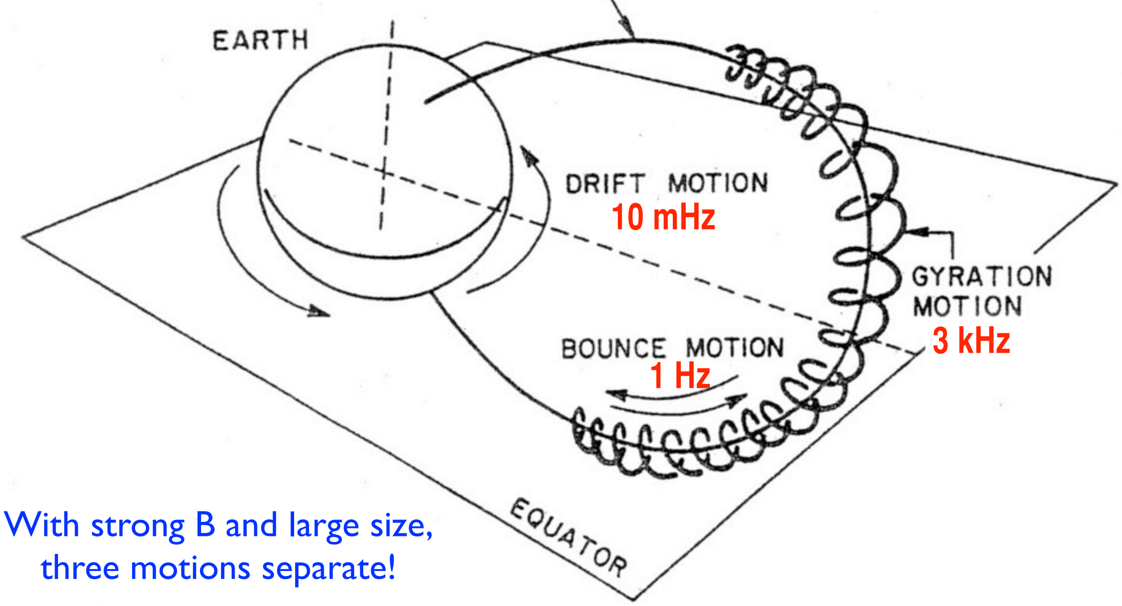
NOTE D_E (STILL) $\propto \frac{1}{B^2}$

GOOD

Plasma physics works!!!

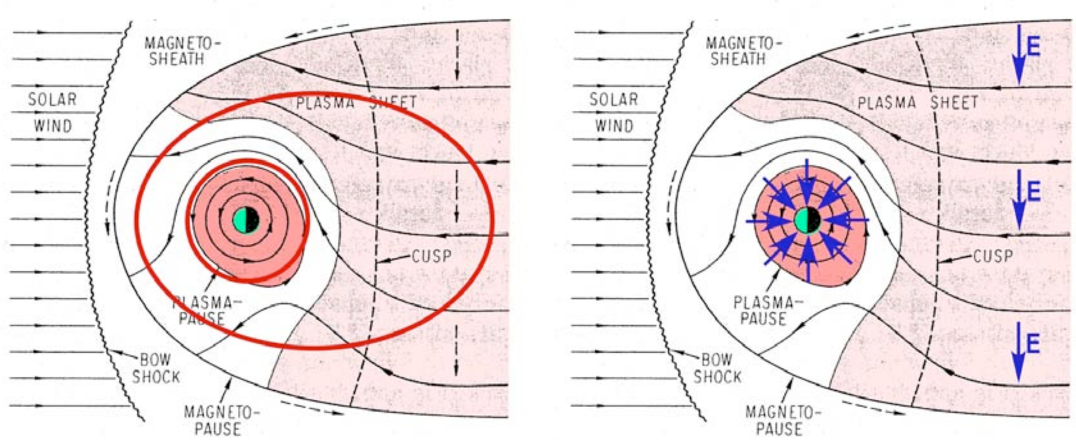
Rad Belt Dynamics Characterized by Adiabatic Invariants: Gyration (μ), Bounce (J), and Drift (ψ)

For outer zone e⁻ (~ 0.5 MeV)...



With strong B and large size, three motions separate!

Perturbed ψ Caused by Global Fluctuations of Geomagnetic Cavity (Easily Measured!)



$$\delta A_\phi \sim \overbrace{\frac{L}{4}\left(\frac{R_e}{R_m}\right)^3}^{\text{Axisymmetric}} - \underbrace{\frac{4}{30}\frac{L^2}{R_e}\left(\frac{R_e}{R_m}\right)^4 \cos\phi}_{m=\pm 1} + \ldots$$

$$\delta\Phi \sim -\overbrace{E_c\left(\frac{R_e^2}{L}\right)}^{\text{Axisymmetric}} + \underbrace{E_c L \sin\phi}_{m=\pm 1} + \ldots$$

Nakada and Mead, *JGR* (1965)

T. Birmingham, *JGR* (1969)

JOURNAL OF GEOPHYSICAL RESEARCH, SPACE PHYSICS VOL. 74, NO. 9, MAY 1, 1969

Convection Electric Fields and the Diffusion of Trapped Magnetospheric Radiation

THOMAS J. BIRMINGHAM

Collisionless Random Electric Convection

$$\frac{\partial\langle\bar{Q}\rangle(\alpha, M, J, t)}{\partial t} = \frac{\partial}{\partial\alpha}\left[\overline{D_{\alpha\alpha}}\frac{\partial\langle\bar{Q}\rangle}{\partial\alpha}\right] \quad (5)$$

$$\boxed{\overline{D_{\alpha\alpha}} \approx \frac{c^2\mu^2}{4\alpha^2}(\pi)^{1/2}\tau_c\mathfrak{A}} \quad (18)$$

α = magnetic flux, ψ

dipole field. We describe **E** by the potential V

$$V = \frac{A(t)r}{\sin^2\vartheta}\sin\phi \quad (2)$$

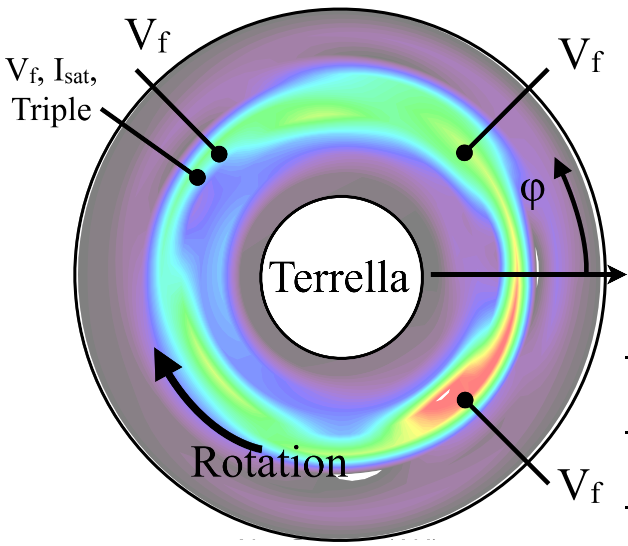
A being a positive, time-dependent amplitude. The form equation 2 is the fundamental ($m = 1$) asymmetric mode in *Fälthammar*'s [1965] Fourier expansion of a general longitudinally dependent potential. Since $r\sin^{-2}\vartheta$ and ϕ are both constant on dipole field lines, **B** lines are equipotentials, and **E·B** is zero. In the $\vartheta = \pi/2$, equatorial plane

A reasonable direction to proceed, in view of the paucity of direct experimental evidence of electric fields and their time variations, is to assume that the autocorrelation $\langle\delta A(t - \tau)\,\delta A(t)\rangle$ has the form

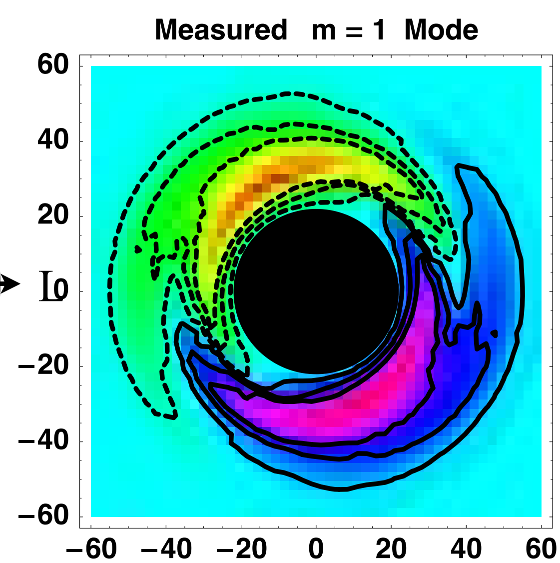
$$\langle\delta A(t - \tau)\,\delta A(t)\rangle = \mathfrak{A}\exp -\frac{\tau^2}{\tau_c^2} \quad (16)$$

from dawn to dusk, and is random on the time scale on which the solar wind executes time variations of large spatial extent. (The correlation time τ_c is thus typically one hour.)

CTX: Measurements of Fluctuating N(ψ,ϕ,t) and Φ (ψ,ϕ,t)



Density



(ϕ ~ Stream function)

Interchange Particle Diffusion

$$\dot{\psi} = \nabla\psi \cdot \mathbf{V} = \frac{\partial \Phi}{\partial \varphi} = -RE_\varphi$$

$$D = \lim_{t\to\infty} \int_0^t dt \langle \dot{\psi}(t)\dot{\psi}(0)\rangle \equiv \langle \dot{\psi}^2 \rangle \tau_c$$

Correlation Time

T. Birmingham, *JGR* (1969)

$$\boxed{D = R^2 \langle E_\varphi^2 \rangle \tau_c}$$

Flux tube particle number

$$\frac{\partial N}{\partial t} = \langle S \rangle + \frac{\partial}{\partial \psi} D \frac{\partial N}{\partial \psi}$$

Dipole Magnetic Flux Tubes (Rings!)

$$\boxed{N \equiv \langle n \rangle \delta V}$$

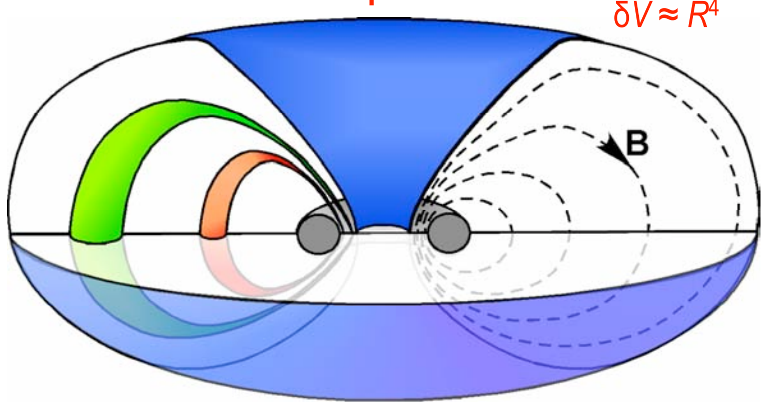
- Flux tube volume:
  - $\delta V = \int ds/B \approx R^4$
- Natural profiles:
  - $n\,\delta V$ = constant
  - $P\,\delta V^\gamma$ = constant
  - **Density and pressure profiles are strongly peaked!**

➡ Density, pressure, and temperature at edge and at core are **not equal**.

Dipole

$B \approx 1/R^3$

$\delta V \approx R^4$



"Naturally Peaked" Profiles in LDX:

$$\begin{aligned}
\delta V_{edge}/\delta V_{core} &\approx 50 \\
n_{core}/n_{edge} &\approx 50 \\
P_{core}/P_{edge} &\approx 680 \\
T_{core}/T_{edge} &\approx 14
\end{aligned}$$

Turbulent Particle Pinch

(Magnetic geometry linked with particle transport)

$$\frac{\partial N}{\partial t} = \langle S \rangle + \frac{\partial}{\partial \psi} D \frac{\partial N}{\partial \psi}$$

Look!

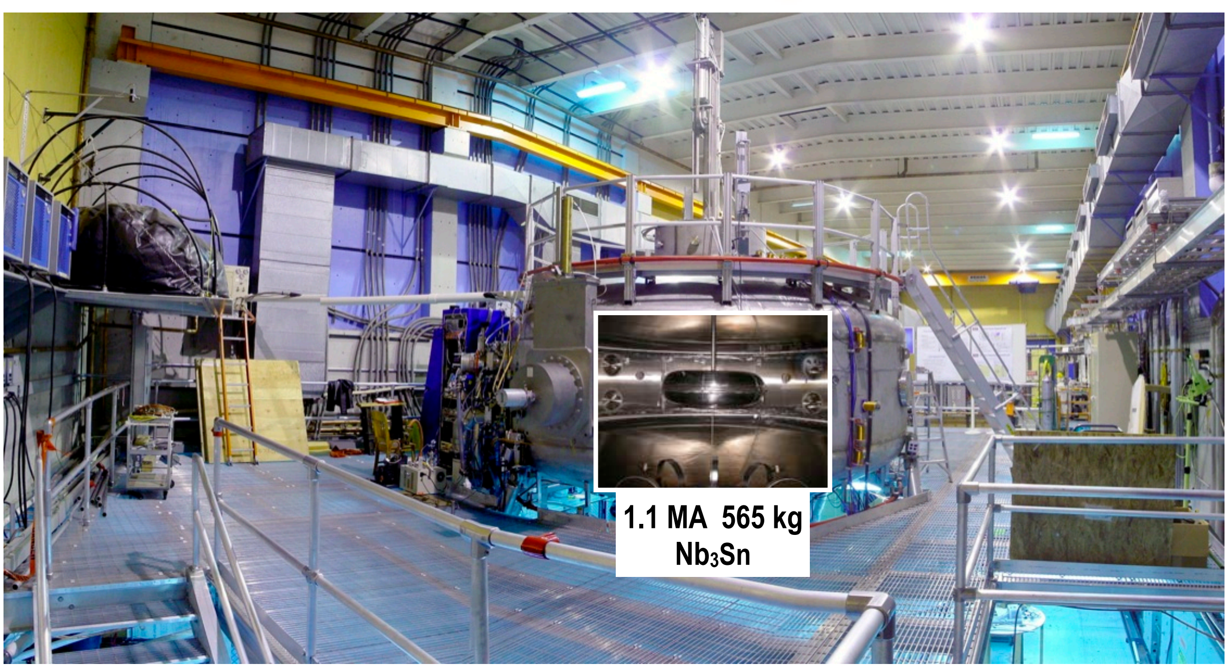
$$\Gamma_\psi = -D\frac{\partial N}{\partial \psi} = -D\delta V \frac{\partial \langle n \rangle}{\partial \psi} + \boxed{V_\psi \langle n \rangle}$$

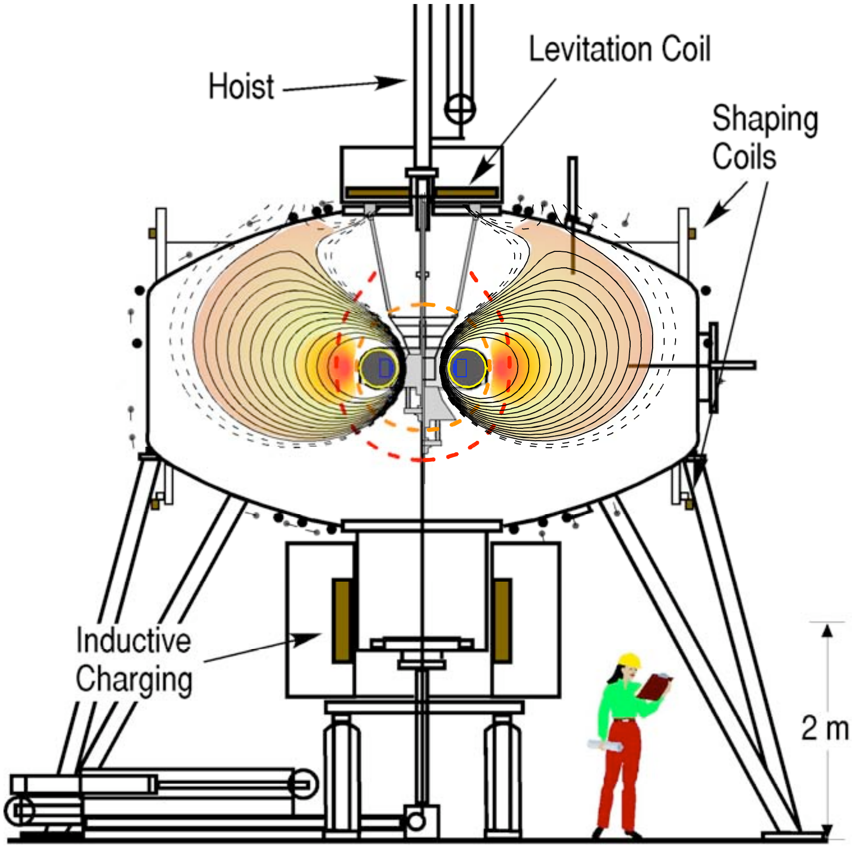
$$\text{where } V_\psi \equiv -D\frac{\partial \delta V}{\partial \psi}$$

LDX:
$D \approx 0.047$ Weber^2/s
V (pinch) ~ 45 m/s (core) and 400 m/s (edge)

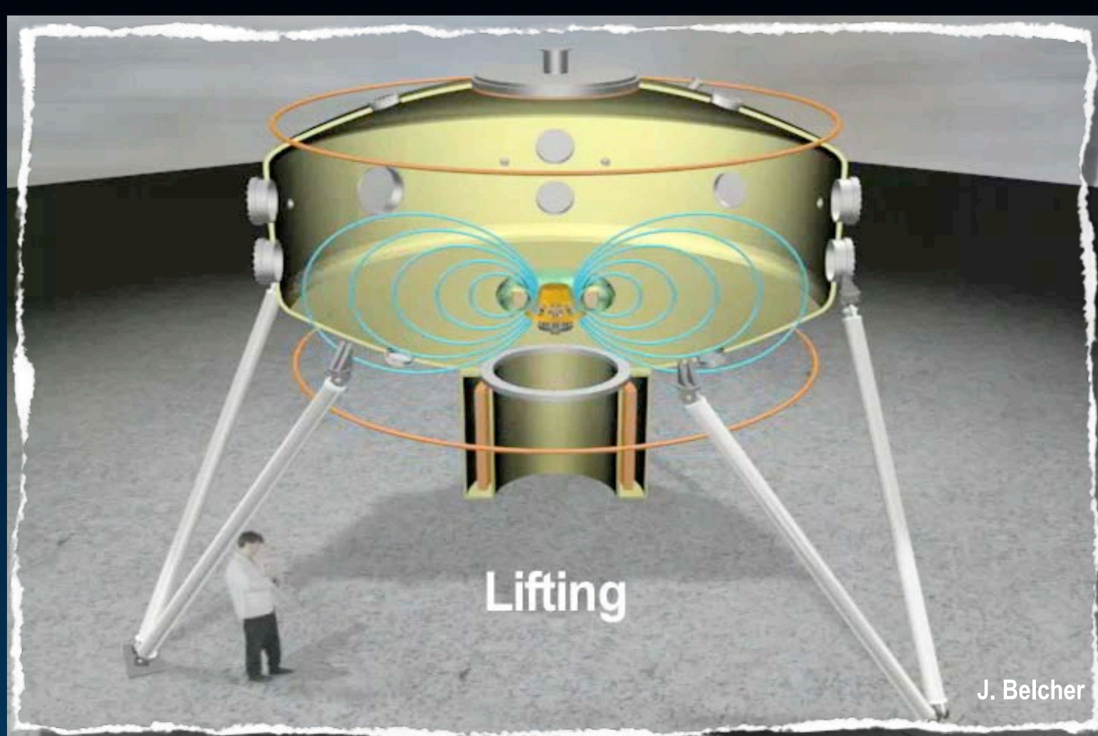
Levitated Dipole Experiment

MIT-Columbia University





Lifting, Launching, Levitation, Experiments, Catching

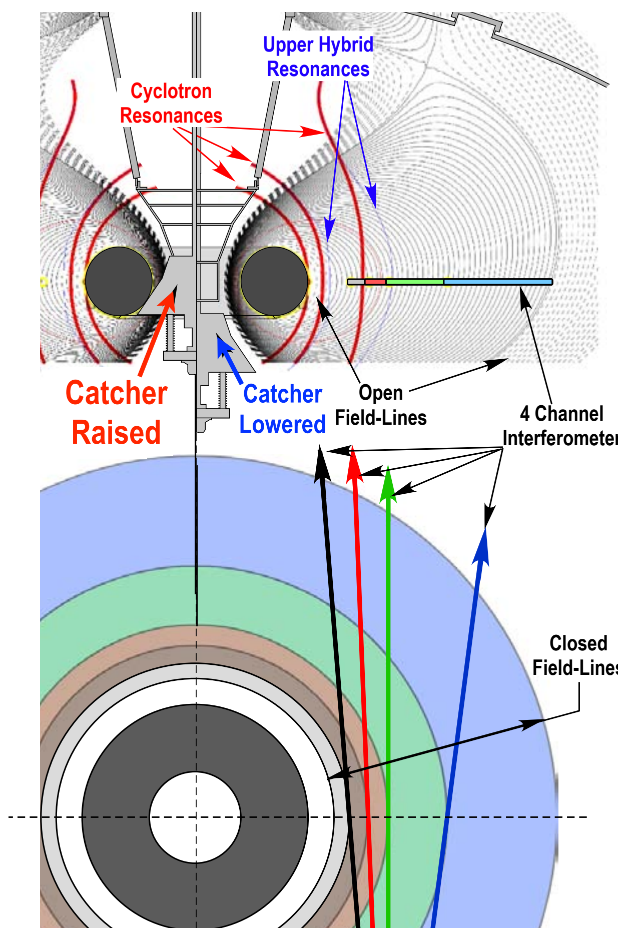


Levitated Dipole Plasma Experiments

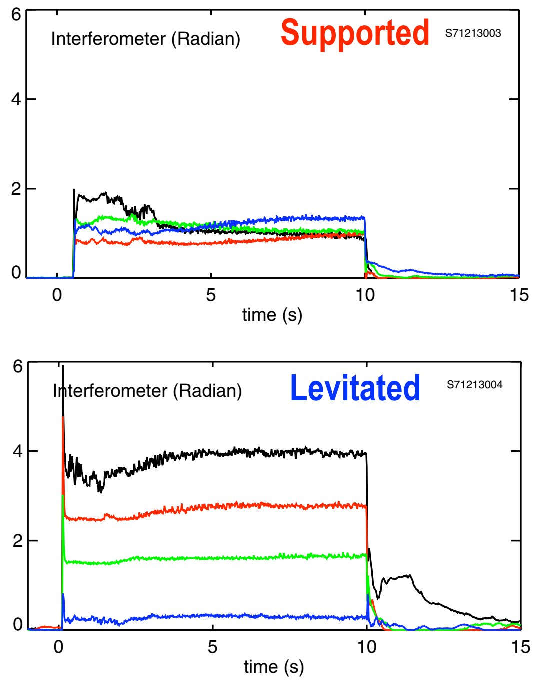
Density Profile with/ without Levitation

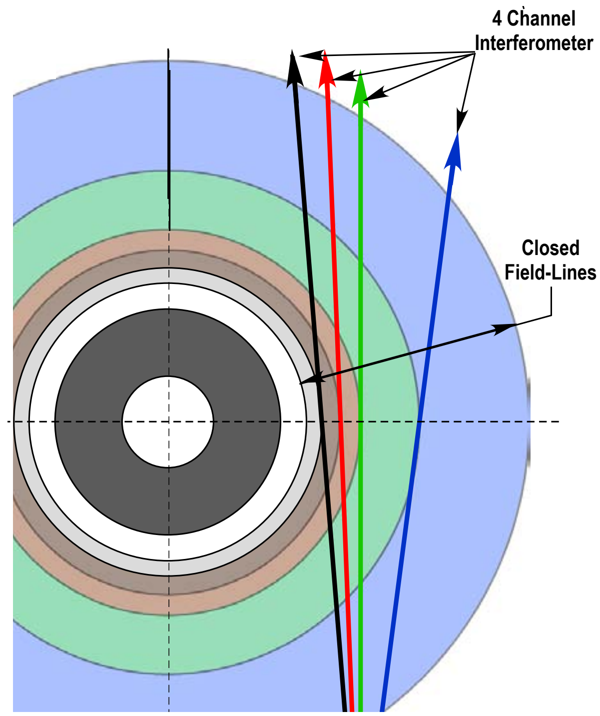
- ***Procedure***:
  - Adjust levitation coil to produce equivalent magnetic geometry
  - Investigate multiple-frequency ECRH heating
- ***Observe***: Evolution of density profile with 4 channel interferometer
- ***Compare***: Density profile evolution with supported and levitated dipole

Alex Boxer, MIT PhD, (2008)

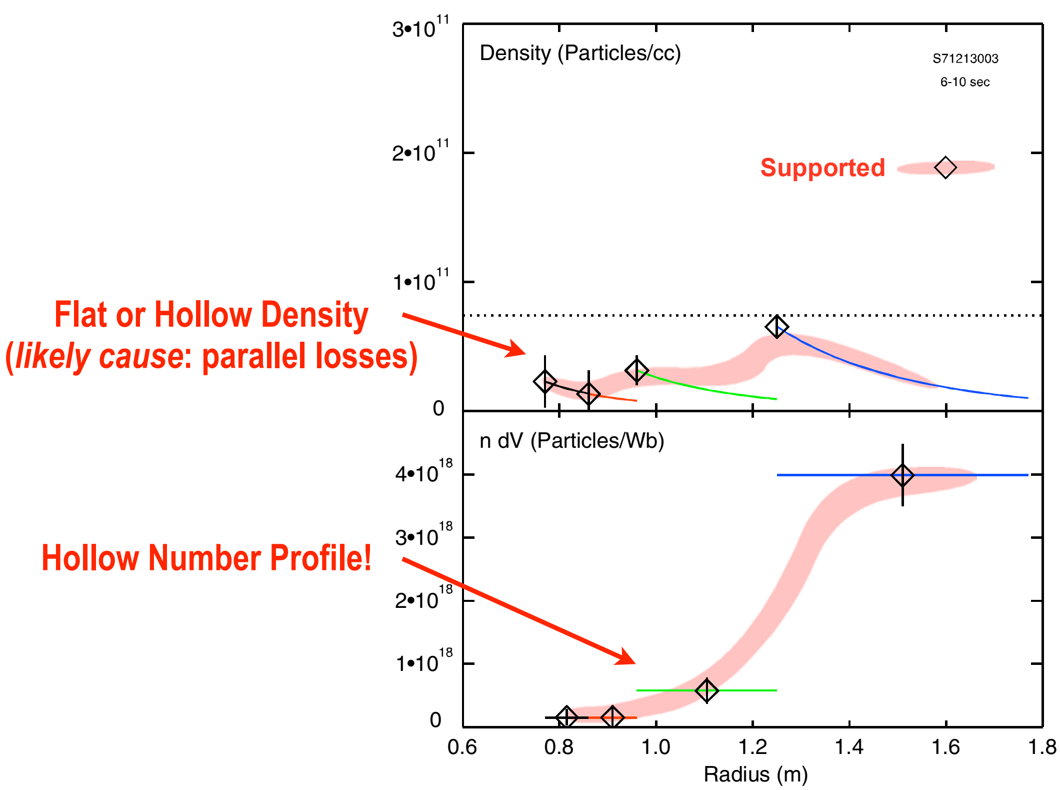


Multi-Cord Interferometer Shows Strong Density Peaking During Levitation

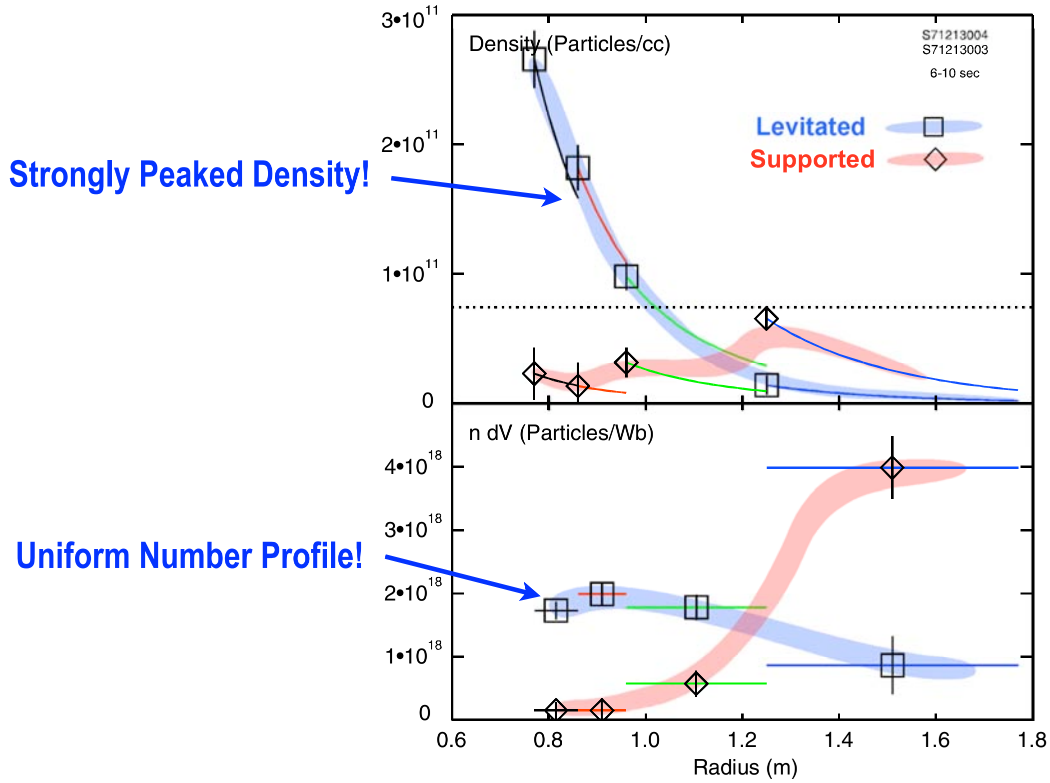




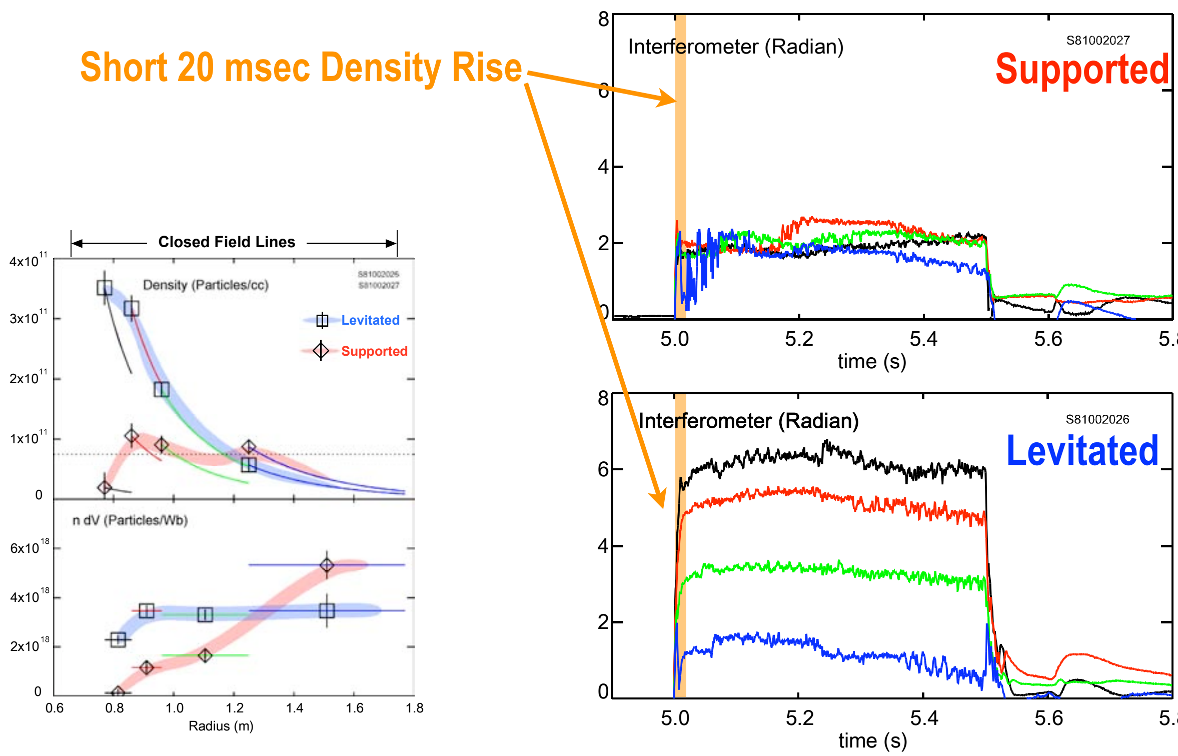
Inversion of Chord Measurements



Inversion of Chord Measurements

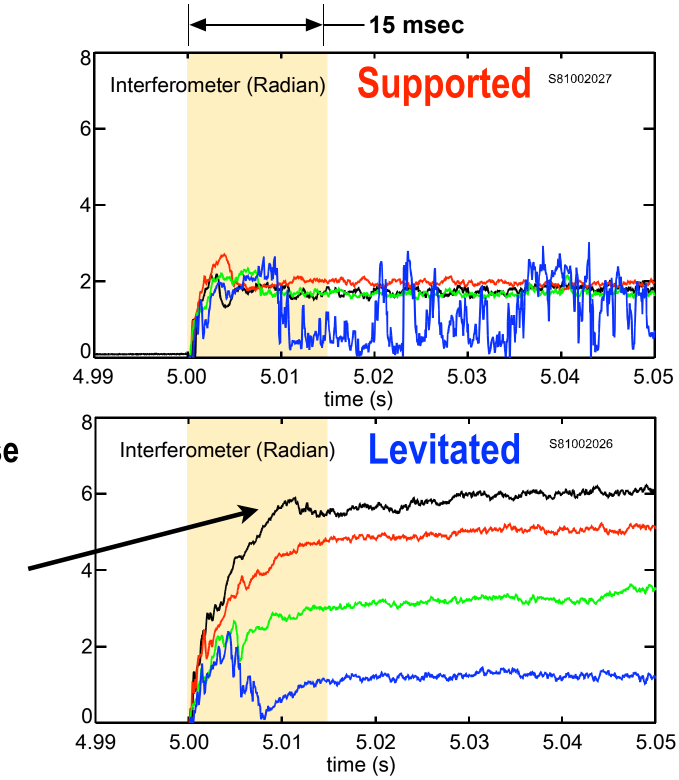


Naturally Peaked Profiles Established Rapidly



Naturally Peaked Profiles Established Rapidly

- Initially (~ 4 msec), density rises equally for supported and levitated discharges
- Only when levitated, central density continues to increase
- Natural profiles are created in 15-25 msec



Turbulent Particle Pinch

(linking magnetic geometry and particle transport)

$$\frac{\partial N}{\partial t} = \langle S \rangle + \frac{\partial}{\partial \psi} D \frac{\partial N}{\partial \psi}$$

Look!

$$\Gamma_\psi = -D \frac{\partial N}{\partial \psi} = -D \delta V \frac{\partial \langle n \rangle}{\partial \psi} + \boxed{V_\psi \langle n \rangle}$$

$$\text{where } V_\psi \equiv -D \frac{\partial \delta V}{\partial \psi}$$

This is Big

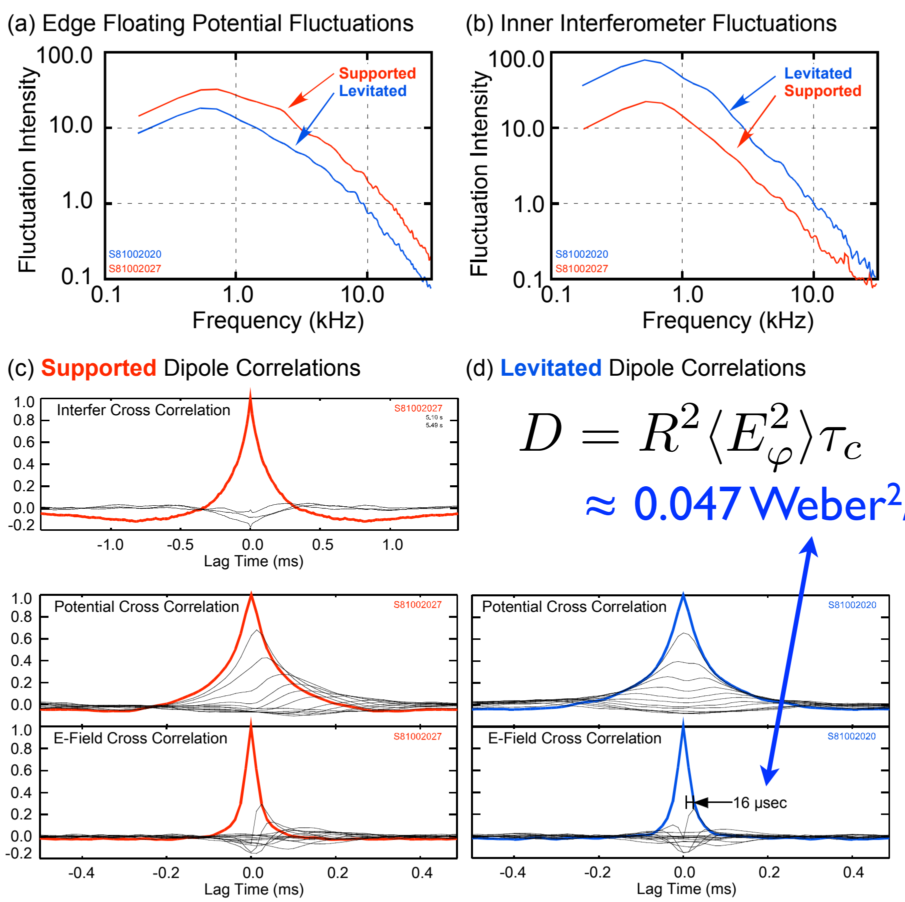
LDX:

D ≈ 0.047 Weber^2/s

V (pinch) ~ 45 m/s (core) and 400 m/s (edge)

Floating Potential Probe Array

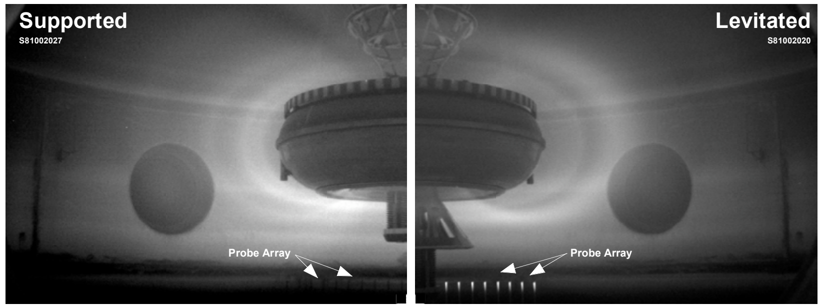
- Edge floating potential oscillations
- 4 deg spacing @ 1 m radius
- 24 probes
- Very long data records for excellent statistics!!



$$D = R^2 \langle E_\varphi^2 \rangle \tau_c$$

$\approx$ 0.047 Weber^2/s

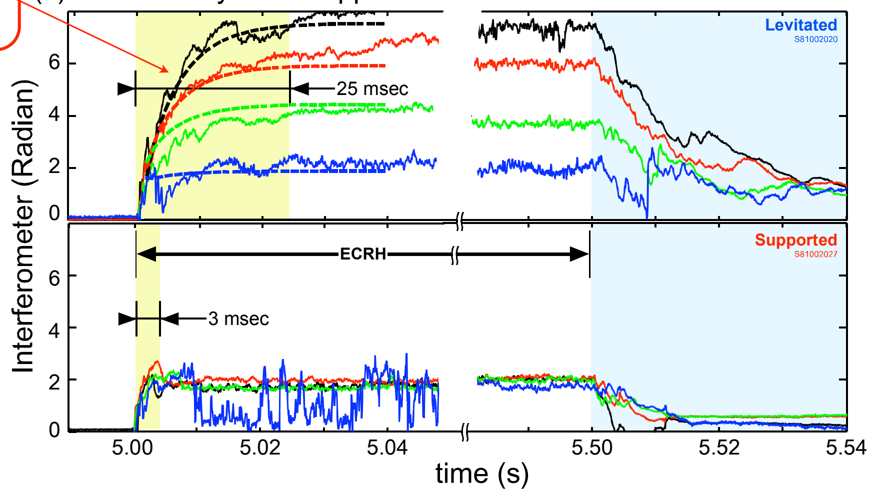
(b) Visible Light from Supported and Levitated Plasma



$$\boxed{\frac{\partial N}{\partial t} = \frac{\partial}{\partial \psi} D \frac{\partial N}{\partial \psi}}$$

with D uniform
and measured
at edge

(c) Line Density from Supported and Levitated Plasma



Summary

- The mechanics of magnetic levitation is proven reliable.
- Levitation eliminates parallel particle losses and creates strong peaking of central density and an inward turbulent pinch.
- The strength of the inward pinch is equal to that predicted by the measured electric field fluctuations at edge electric field.
- LDX has demonstrated the formation of natural density profiles in a laboratory dipole plasma and the applicability of space physics to fusion science.
- Increased stored energy consistent with adiabatic profiles: *a necessary physics requirement for dipole fusion.*