Understanding and Controlling Turbulent Mixing in a Laboratory Magnetosphere

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(Acknowledging the work from many former students and collaborators including Darren Garnier, Jay Kesner, Max Roberts, Ben Levitt, Brian Grierson)

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With Sincere Apologies…

Mike Mauel is chair of Invited Session TI3 (now):

Non-neutral Plasmas, Fusion, and Beams:
The Legacy of Ron Davidson

Professor of Physics
Director, PFC
(1978-1991)

Director, PPPL

Editor-in-Chief
Abstract

In a laboratory magnetosphere, plasma is confined by a strong dipole magnet, and complex nonlinear processes can be studied and controlled in near steady-state conditions. Because a dipole’s magnetic field resemble the inner regions of planetary magnetospheres, these laboratory observations are linked to space plasma physics. Unlike many other toroidal configurations, interchange and entropy modes dominate plasma dynamics, and turbulence causes self-organization and centrally-peaked profiles as the plasma approaches a state of minimum entropy production.

We report progress in understanding and controlling turbulent mixing through a combination of laboratory investigation, modeling, and simulation. Topics discussed:

(i) Extending the global extent of local regulation of the interchange and entropy mode turbulence through current injection,

(ii) Measurement and interpretation of the statistical properties of stationary turbulence, and

(iii) Advancements in the nonlinear simulation of turbulence control in a dipole plasma torus.
Two Laboratory Magnetospheres: Plasma Experiments without Field-Aligned Currents

LDX: High Beta Levitation & Turbulent Pinch

CTX: Polar Imaging, Current Injection, Rotation

24 Probes 1 m Radius
Toroidal Confinement with Closed-Field Lines: Interchange and Entropy Modes

- Axisymmetric magnetically dipole guarantees omnigeneous particle drifts.
- The only high-$\beta$ toroidal magnetic configuration that satisfies the Palumbo condition: the divergence of the perpendicular plasma current vanishes.
- Absence of parallel currents in a dipole-confined plasma is significant: many tokamak instabilities are not found in a dipole plasma torus, e.g. kink, tearing, ballooning, and drift modes.
- Instead, interchange and entropy modes dominate plasma dynamics, and particle and power source profiles determine the level of turbulence.
- Turbulent transport causes centrally-peaked profiles and self-organization, as the plasma approaches a state of minimum entropy production.
- Axisymmetric interchange/entropy mode turbulence exhibit 2D inverse cascade at long wavelengths.
Closed Field-Line Plasma Dynamics

- How do we know dynamics is “interchange” dominated?
  - Direct laboratory measurement of $\delta \Phi$, in all cases, but when $\omega_{be} \gg \omega_d$

- What are the consequences of “interchange” dynamics?
  - 2D inverse cascade couples fluctuations to largest scales
  - “Weak gradients” with $\omega^* \sim \omega_d$
  - Profile consistency, turbulent pinch, …
  - Self-organization toward state of minimum entropy production, $\eta \sim 2/3$
Turbulent Intensity is Observed to Peak at Long Wavelengths (Inverse Mode-Mode Cascade)

Measured Interchange Modes in Dipole Torus

Convective Structures are Dynamic

With \( T_e \gg T_i \) (CTX and LDX) modes (usually) propagate in electron drift direction
Induced Field-Aligned Currents in Magnetospheres

\[ U_{con} \approx \frac{c^2}{8\pi \Sigma_p} \cdot \frac{L_{ps}}{L_{ps}} \cdot \varepsilon \left( \frac{p_{tot}^{(12)}}{p_{tot}^{(12)}} \right) \]

Figure 3. Dynamo forces, auroral current system, and resulting convection under frictional control by the ionosphere, after Boström (1964).

Fig. 7. Electric field vectors (rotated 90° counter clockwise) calculated from SuperDARN data averaged over 03:30–04:30 UT, 1 November, 2001. The electric potential contours, DMSP and Oersted tracks and the sunlight terminator are overlayed. The extremes in potential are located at the blue (−ve) and red (+ve) dots. The electric field vectors are bold at locations where radar returns were received.

Interchange Motion is Regulated by Ionosphere, or External Circuits, or …

\[
\int \frac{ds}{B} \nabla_\perp \cdot \mathbf{J}_\perp = \begin{cases} 
0 & \text{Closed, insulated, field lines} \\
2(J_{||}/B)_{\text{poles}} & \text{Ionospheric current} \\
\sum_j I_j \delta(\psi - \psi_j) \delta(\phi - \phi_j) & \text{External circuits}
\end{cases}
\]

Steady MHD Convection in Space

\[
\mathbf{J}_\perp = \frac{\hat{\mathbf{b}} \times \nabla P}{B} \quad \text{(space)}
\]

\[
\frac{2J_{||}}{B_{\text{pole}}} = \nabla_\perp P \cdot \hat{\mathbf{b}} \times \nabla_\perp \int \frac{ds}{B}
\]

\[
\nabla_\perp \cdot \Sigma_p \nabla_\perp \Phi \approx -J_{||}(\hat{\mathbf{b}} \cdot \hat{\mathbf{n}}) \quad \text{(poles)}
\]

Dynamic Drift-like Motion in Lab

\[
\hat{\mathbf{b}} \cdot \nabla \Phi = 0
\]

\[
\mathbf{J}_\perp = \frac{\hat{\mathbf{b}} \times \nabla P}{B} - \frac{nM_i}{B^2} \nabla_\perp \frac{d\Phi}{dt}
\]

\[
\int \frac{ds}{B} \nabla_\perp \cdot \mathbf{J}_\perp = 0
\]

\[
\nabla_\perp \cdot \Sigma \nabla_\perp \frac{\partial \Phi}{\partial t} \approx -\nabla_\perp P \cdot \hat{\mathbf{b}} \times \nabla_\perp \int \frac{ds}{B}
\]

Ion Inertial Currents

Integrated Plasma Dielectric

Entropy & Drift-Interchange Modes

(For CTX and LDX with $T_e \gg T_i$)

\[
\frac{\partial \tilde{N}}{\partial t} + \frac{dh_n}{dy} \frac{\partial \tilde{\Phi}}{\partial \varphi} + \frac{4}{y^5} \frac{\partial \tilde{P}_e}{\partial \varphi} = 0
\]

Collisionless heat flux due to Electron magnetic drift

\[
\frac{\partial \tilde{P}_e}{\partial t} + y^{4\gamma} \frac{dh_g}{dy} \frac{\partial \tilde{\Phi}}{\partial \varphi} + \gamma \frac{4}{y^5} \left( \frac{y^{4\gamma} h_g}{h_n} \right) \left[ 2 \frac{\partial \tilde{P}_e}{\partial \varphi} - \left( \frac{y^{4\gamma} h_g}{h_n} \right) \frac{\partial \tilde{N}}{\partial \varphi} \right] = 0
\]

\[
\rho^2 \left( \frac{\partial}{\partial t} + \nu_i \right) \left[ \frac{\partial}{\partial y} \left( h_n \Sigma_\psi \frac{\partial \tilde{\Phi}}{\partial y} \right) + h_n \Sigma_\varphi \frac{\partial^2 \tilde{\Phi}}{\partial \varphi^2} \right] + \frac{4}{y^5} \frac{\partial \tilde{P}_e}{\partial \varphi} = 0
\]

ion-neutral damping

Linear Braginskii interchange motion
Gradient Drive for Turbulent Transport: Comparing to the Familiar Tokamak...

(a) Dipole Interchange-Entropy Modes

Minimum Entropy Production

Weak gradients: $\omega_p^* \sim \omega_d$

Stable by compressibility and field line tension

(b) Tokamak ITG-TEM Modes

Steep gradients: $\omega_p^* \gg \omega_d$

Stable by average curvature and magnetic shear

X. Garbet, Comptes Rendus Physique 7, 573 (2006)
Quasilinear Flux using 2D Bounce-Averaged Fluid Equations with Drift-Kinetic Closure

(a) Particle Flux  
(b) Temperature Flux  
(c) Entropy ($P\delta V'$) Flux

Interchange-Entropy Mode Dispersion Agrees with Observations

(a) Entropy Mode Dispersion: $\Delta W_p \sim \Delta (PV^{5/3}) \sim 0$

(b) "Warm Core" with Electron Drift

(c) "Cool Core" Reversed Rotation

Drift-Kinetic Heat moves toroidally from Warm to Cool Flux-Tubes

Entropy Mode Rotation

\[
\frac{\partial \tilde{p}_e}{\partial t} + \frac{1}{\delta V} \frac{\partial}{\partial \psi} (p \delta V \gamma) \frac{\partial \tilde{\Phi}}{\partial \varphi} - 2\gamma \langle \kappa_{\psi} \rangle (T_e / e) \left[ 2 \frac{\partial \tilde{p}_e}{\partial \varphi} - T_e \frac{\partial \tilde{n}}{\partial \varphi} \right] \approx 0, \quad \omega \approx \frac{1}{(\rho_s m_\perp)^{2/3}} \begin{cases} \exp(i2\pi/3) \left| \eta - 2/3 \right|^{1/3} & \text{if } \eta < 2/3 \\ \exp(i\pi/3) \left( \eta - 2/3 \right)^{1/3} & \text{if } \eta > 2/3 \end{cases}
\]
Entropy Modes Reverse with $\eta$ (Pellet Injection)

(a) Line Density and Photodiode Array Profile Times

(b) μWave Interferometer

Pellet Trajectory

Probe Array

$\eta > 2/3$

$\eta < 2/3$

$\eta > 2/3$
Entropy Modes Reverse with Pellet Injection

\( \eta > \frac{2}{3} \)

\( \eta < \frac{2}{3} \)
Global Entropy Eigenmodes

Mode Toroidal Rotation ($\omega/m\omega_d$)

Mode Growth Rate ($\gamma/m\omega_d$)

Density
Temperature
$\omega_d/\rho^2u_d$

Normalized Radius ($L/L_0$)

$\eta \approx 0.22$
$\eta \approx 0.67$
$\eta \approx 1.7$

$\Delta$ $m = 1$
$\Diamond$ $m = 2$
$\Box$ $m = 8$
Global Entropy Eigenmodes

(a) $\eta = 1.68$

(b) $\eta = 0.67$

(c) $\eta = 0.22$

Mode Toroidal Rotation $(\omega/m\omega_d)$

Mode Growth Rate $(\gamma/m\omega_d)$

Normalized Radius $(L/L_0)$
Summary and Applications

• Global flux-tube averaged gyro-fluid description of flute-type instabilities describes drift-interchange and entropy modes

• Long wavelength eigenmodes and real frequencies like observations in CTX and LDX
  • Quasilinear theory describes up-gradient turbulent pinches
  • Linear theory can model local current-injection feedback (Roberts, *PoP* 2015)
  • Li pellet injection reduces $\eta \rightarrow 0$ and reverses toroidal propagation of fluctuations


• Mode-mode and 2D interchange cascade may explain the discrepancy between observations dominated with low-$m$ eigenmodes and linear high-$m$ eigenmodes with large growth rates.

• Flux-tube averaging makes possible “whole-plasma” nonlinear turbulence simulations.
Single-Point Regulation of Interchange Turbulence with Current-Collection Feedback

Local Regulation of Interchange Turbulence with Current-Collection Feedback

**Application:** Toroidal Confinement without $B_t$ may Speed Fusion Development using *much smaller* Superconducting Coils

($Q_{DT} \sim 10$ Magnet Systems Compared at Same Scale)


**Plasmas Volumes**
- $837 \text{ m}^3$
- $42,000 \text{ m}^3$

**Parameters**
- $P_{\text{fus}} = 410 \text{ MW} \quad W_p = 1.1 \text{ GJ} \quad W_b = 51 \text{ GJ} \quad I_t = 164 \text{ MA}$
- $P_{\text{fus}} = 39 \text{ MW} \quad W_p = 0.06 \text{ GJ} \quad W_b = 1.6 \text{ GJ} \quad I_d = 25 \text{ MA}$

(a) Conventional Fusion Experiment (Gain = 10)

(b) Dipole Fusion Experiment (Gain = 10)

30-fold size/energy reduction (!)